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PREMIUM REVISION HANDBOOK

Strategic Cost Management

Master Revision Handbook · Paper 16

THE INSTITUTE OF COST ACCOUNTANTS OF INDIA

EDITION
December 2026

PAPER
Paper 16 · Strategic Cost Management

STRUCTURE
Section A (60%) & Section B (40%)

COVERAGE
All 15 Modules · Fully Mapped

15	60%	40%	100%
MODULES	SECTION A	SECTION B	EXAM COVERAGE
Complete Syllabus	Strategic Cost Mgmt.	Quantitative Techniques	Complete Handbook

✓ Concept Notes ✓ Formula Boxes ✓ Case Studies ✓ Numerical Problems

✓ MCQs with Answers ✓ Quick Revision Boxes ✓ Previous Year Patterns

✓ Last-Day Revision

PUBLISHED BY
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FOR
CMA Final & Inter Students

Full Syllabus Index

PAPER 16 · STRATEGIC COST MANAGEMENT · SECTION A & B · MODULE-WISE WEIGHTAGE

SYLLABUS STRUCTURE

Section A: Strategic Cost Management for Decision Making — carries **60% weightage** and comprises **Modules 1 to 5**. **Section B: Quantitative Techniques in Decision Making** — carries **40% weightage** and comprises **Modules 6 to 15**. Total of **15 Modules** covering **100% of the Paper 16 syllabus**.

15	60%	40%
TOTAL MODULES	SECTION A	SECTION B
Complete Syllabus	Strategic Cost Management	Quantitative Techniques

100%
EXAM COVERAGE
Complete Handbook

Section A — Strategic Cost Management for Decision Making (60%)

MODULE	MODULE TITLE & KEY TOPICS	WEIGHTAGE
01	**Introduction to Strategic Cost Management** Concepts of SCM · Value Chain Analysis · Cost Control & Cost Reduction · Business Process Reengineering · Supply Chain Management	5%
02	**Quality Cost Management** Quality in Competitive Environment · Cost of Quality · Total Quality Management · Lean Accounting · Six Sigma · PRAISE · Pareto Analysis	5%
03	**Decision Making Techniques** Marginal Costing · Pricing Decisions · Transfer Pricing · Relevant Cost Analysis · Target Costing · Product & Asset Life Cycle Costing · Decision Making using Probability	25%
04	**Activity Based Management and Just in Time (JIT)** Activity Based Costing & Management · ABB · Responsibility Accounting · JIT · Throughput Accounting · Backflush Accounting · Benchmarking	10%
05	**Evaluating Performance** Variance Analyses · Planning & Operating Variances · Controllable & Non-Controllable Variances · Uniform Costing · Inter-firm Comparison · Profit Reconciliation	15%
SECTION A — TOTAL		**60%**

Section B — Quantitative Techniques in Decision Making

(40%)

MODULE	MODULE TITLE & KEY TOPICS	WEIGHTAGE
06	**Linear Programming** Formulation of LPP · Graphical Method · Simplex Method · Duality · Special Cases	—
07	**Transportation** NWCM · LCM · VAM · MODI Method · Degeneracy · Unbalanced Problem · Maximisation · Prohibited Routes	15%
08	**Assignment** Hungarian Method · Unbalanced Problem · Maximisation · Restricted Assignment · Travelling Salesman Problem	—
09	**Game Theory** Pure & Mixed Strategy · Saddle Point · Principle of Dominance · Odds Method · Graphical Method · Limitations	—
10	**Simulation** Monte Carlo Simulation · Random Numbers · Pseudo Random Numbers · Congruence Method · Queuing Problems	15%
11	**Network Analysis — PERT, CPM** Network Diagrams · Critical Path · Floats · Three Time Estimates · Probability of Completion · Crashing · Resource Levelling	—
12	**Learning Curve** Learning Ratio · Wright's Model · Experience Curve · Uses & Limitations · Factors Affecting Learning Curve	—
13	**Business Application of Maxima and Minima** Unconstrained Optimization · Constrained Optimization · Lagrangian Multiplier · Hessian Determinant · Firm's Equilibrium · Multi-plant Monopolist · Price Discriminating Monopolist	—
14	**Business Forecasting Models — Time Series and Regression Analysis** Regression Analysis (SLR, MLR) · Time Series Analysis · Exponential Smoothing · Input–Output Model · Qualitative Methods	10%

MODULE	MODULE TITLE & KEY TOPICS	WEIGHTAGE
15	**Introduction to Tools for Data Analytics** Types of Analytics · ETL · BI Tools · R · Python · SAS · Spreadsheets · Financial Analytics · Visualisation Tools	—
SECTION B — TOTAL		**40%**

WEIGHTAGE SUMMARY

Section A (60%): Decision Making Techniques carries the highest weightage of **25%**, followed by Evaluating Performance at **15%**, ABM & JIT at **10%**, and Modules 1 & 2 at **5%** each.

Section B (40%): Transportation and Simulation carry **15%** each, Business Forecasting carries **10%**, while the remaining modules are distributed within the section.

📖 Table of Contents — Complete Handbook

1. Introduction to Strategic Cost Management
2. Quality Cost Management
3. Decision Making Techniques
4. Activity Based Management and Just in Time (JIT)
5. Evaluating Performance
6. Linear Programming
7. Transportation
8. Assignment
9. Game Theory
10. Simulation
11. Network Analysis — PERT, CPM
12. Learning Curve
13. Business Application of Maxima and Minima
14. Business Forecasting Models — Time Series and Regression Analysis

15. Introduction to Tools for Data Analytics

⚡ Quick Revision — Syllabus at a Glance

- **Section A (60%):** Modules 1 to 5 — Strategic Cost Management core.
- **Section B (40%):** Modules 6 to 15 — Quantitative Techniques.
- **High-Weightage Modules:** Decision Making (25%), Evaluating Performance (15%), Transportation (15%), Simulation (15%), Business Forecasting (10%).
- **Total Coverage:** 15 Modules · 100% Weightage · Complete Paper 16 Syllabus.

CMA KNOWLEDGE

Strategic Cost Management — Master Revision Handbook

Paper 16 · December 2026 Edition

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SECTION A COVERAGE

Module 1 — Introduction to SCM

Module 2 — Quality Cost Management

Module 3 — Decision Making Techniques

Module 4 — ABM & JIT

Module 5 — Evaluating Performance

SECTION B COVERAGE

Module 6 — Linear Programming

Module 7 — Transportation

Module 8 — Assignment

Module 9 — Game Theory

Module 10 — Simulation

Module 11 — Network Analysis

Module 12 — Learning Curve

Module 13 — Maxima & Minima

Module 14 — Business Forecasting

Module 15 — Data Analytics Tools

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Section A · Weightage 60%

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Total Modules: 15

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Strategic Cost Management

Module 1 · Introduction to Strategic Cost Management

EDITION
December 2026

PAPER
Paper 16 · Strategic Cost Management

SECTION
Section A · Weightage 60%

MODULE WEIGHTAGE
5% of Section A

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading
✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

01

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 1 INCLUDES

1.1 Concepts of Strategic Cost Management in different stages of Value Chain · **1.2** Cost Control and Cost Reduction — Contemporary Techniques · **1.3** Value Analysis and Value Engineering — Business Process Re-engineering · **1.4** Supply Chain Management.

Module Learning Objectives

After studying this module, the students will be able to:

1

Understand SCM Concepts

Understand the concepts of Strategic Cost Management in different stages of the Value Chain.

2

Cost Control & Reduction

Gain in-depth knowledge on contemporary techniques of Cost Reduction and Cost Control.

3

Value Chain & Supply Chain

Develop insights into Value Chain Analysis and Supply Chain Management.

Subject Learning Objectives (SLOB) Mapped

SLOB	OBJECTIVE	CMLO LINK
1	Identify major contemporary issues and doctrines/conventions of strategic cost management applicable across multiple sectors of the economy.	CMLO 1a, 1b
2	Understand the nature of strategic competitiveness; analyse the competitive operational environment; assess industry attractiveness and isolate sources of competitive advantage/disadvantage.	CMLO 2a, 2b
3	Develop business-level cost management strategies by defining the type of advantages sought and the scope of operations required to deliver the chosen strategy.	CMLO 2c
4	Develop application-oriented knowledge to assess the likely sustainability of entrepreneurial strategies and competitive positions.	CMLO 4b
5	Equip with application-oriented knowledge of various decision-making techniques for strategic choices across enterprises.	CMLO 5b, 5c
6	Equip with application-oriented knowledge of quantitative techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling, managing cost overrun, demand estimation, production and cost analysis, etc.	CMLO 2b, 5a, 5b, 5c

Subject Learning Outcomes (SLOC) & Application Skills (APS)

SLOC(s)

- Students will acquire application-oriented knowledge to analyse and evaluate strategic cost data in a comprehensive manner to facilitate achievement of sustainable competitive advantages.
- Students will acquire appropriate capabilities to highlight the business outcome indicators to decision makers.

APS

- Students will develop skills to evaluate the performance of an entity in a comprehensive manner and formulate a strategic link to the decision-making process.
- Students will develop necessary skills to advise entrepreneurs on 'what if' and 'how to' tasks.

Module Weightage & Structure

5%	4	4
WEIGHTAGE IN SECTION A	CORE SUB-TOPICS	CASE STUDIES
Of 60% Section A total	1.1 to 1.4	Detailed analysis

3
NUMERICAL ILLUSTRATIONS
Worked solutions

SYLLABUS STRUCTURE NOTE

The syllabus comprises **Section A: Strategic Cost Management for Decision Making (60%)** and **Section B: Quantitative Techniques in Decision Making (40%)**. Module 1 carries **5% weightage** within Section A — it is a foundational module that introduces the strategic mindset required for the entire paper.

Module-wise Mapping of SLOB(s) — Research Resources

MODULE	TOPIC	ADDITIONAL RESOURCES	SLOB MAPPED
1	Introduction to Strategic Cost Management	ICMAI Study Material; Porter, M.E. — Competitive Advantage	CMLO 1a, 1b; 2a, 2b
2	Quality Cost Management	Juran, J.M. — Quality Control Handbook; Crosby, P. — Quality is Free	CMLO 2b, 2c
3	Decision Making Techniques	Drury, C. — Management and Cost Accounting	CMLO 5b, 5c
4	Activity Based Management & JIT	Kaplan & Cooper — Cost & Effect	CMLO 5a, 5b, 5c
5	Evaluating Performance	Horngren — Cost Accounting: A Managerial Emphasis	CMLO 4b, 5b

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1.1

Concepts of Strategic Cost Management in Different Stages of Value Chain

STRATEGY · SCM · THREE PILLARS · VALUE CHAIN · PORTER'S MODEL · VCA · VALUE INNOVATION

What is Strategy?

A **strategy** is an integrated set of choices for actions which positions a firm in an industry so as to generate superior financial returns over the long run. The *integrated set of choices* creates the internal environment of the firm, while the *industry* provides the external environment, and the *long run* replicates the competitive dynamics.

KEY INSIGHT

A firm's strategy aims to match its own capabilities with available opportunities. In other words, strategy defines how an organisation creates value for its customers while distinguishing itself from its competitors.

Two Broad Strategies

COST LEADERSHIP

- Low-cost carriers (Airlines) provide quality products/services at low prices.
- Winning market share by appealing to cost-conscious or price-sensitive customers.
- Requires operating at a lower cost than rivals.
- Examples: Wal-Mart, McDonald's, Big Bazaar, Air Deccan.

VS

PRODUCT DIFFERENTIATION

- Electronic giants like Apple garner premium prices.
- Building specific competitive advantages over competitors.
- Unique features create a perception of esteem value.
- Examples: Apple, Mercedes-Benz, Rolex.

Strategic Cost Management (SCM)

Strategic Cost Management (SCM) refers to cost management that specifically focuses on strategic issues such as:

- The company's cost, productivity, or efficiency advantage relative to competitors; or
- The premium prices a company can charge over its costs for distinctive product or service features.

FORMAL DEFINITION

SCM is the process of identifying, accumulating, measuring, analysing, interpreting, and reporting cost information useful to both internal and external groups concerned with the way in which an organisation utilises its resources to achieve its strategic objectives.

SCM needs to be perceived as the **application of cost management techniques** with a view to enhance the strategic posture of a firm and reduce costs. It is a process of combining the decision-making structure with cost information to reinforce the business strategy as a whole. It measures and manages costs to align them with the company's business strategy.

Four Stages of SCM

Formulation of Strategies → **Communication** across organisation → **Implementation** of tactics → **Controlling** activities

Need for SCM — Five Key Reasons

i

Updated Cost Analysis

Strategic elements are clearer and more formal than traditional cost analysis.

ii

Value Chain Link

Helps identify the cost relationship between value chain activities and the process of management to gain competitive advantage.

iii

Relevant Tactics

Used to analyse cost information to develop relevant tactics for sustainable competitive advantage.

iv

Overall Cost Structure

Provides better understanding of the overall cost structure in the quest for sustainable competitive advantage.

v

Strategic Management Process

Uses cost information specifically to govern formulation, communication, implementation and control.

FUNDAMENTAL OBJECTIVE

SCM is not just about controlling costs but also using information for strategic decision making. The fundamental objective is to gain a **sustainable competitive advantage** through cost leadership and product differentiation.

Three Pillars of SCM

1

Strategic Positioning Analysis

Determines the company's comparative position in the industry in terms of performance.

2

Cost Driver Analysis

Cost is driven by interrelated factors. Divided into **structural cost drivers** and **executional cost drivers**. Examines, measures and explains the financial impact of the cost driver.

3

Value Chain Analysis (VCA)

Firm recognises and analyses all activities and functions that contribute to the final product. Depicts how customer-value accrues along the activity chain.

Structural vs Executional Cost Drivers

STRUCTURAL COST DRIVERS	EXECUTIONAL COST DRIVERS
Scale of operation	Workforce commitment
Scope of operation	Total Quality Management
Experience / Learning curve	Plant layout efficiency
Technology applied	Product configuration
Complexity of product lines	Capacity utilization

Value Chain — Michael Porter (1985)

The **value chain** is a powerful tool for disaggregating a company into its strategically relevant activities in order to focus on the sources of competitive advantage — that is, the specific activities that result in lower costs or higher prices.

A company's value chain is typically part of a larger **value system** that includes companies either upstream (suppliers) or downstream (distribution channels), or both. This perspective forces managers to consider and see each activity not just as a cost, but as a step that has to add some increment of value to the finished product or service.

VALUE EQUATION

```
Value Created and Captured − Cost of Creating that Value = Profit Margin
```

The more value an organisation creates, the more profitable it is likely to be. As more value is provided to customers, **competitive advantage** creeps in.

Elements in Porter's Value Chain

> **PRIMARY ACTIVITIES — Directly relate to physical creation, sale, maintenance and support of a product or service**

PRIMARY ACTIVITY	DESCRIPTION	VALUE CREATION FOCUS
Inbound Logistics	Processes related to receiving, storing, and distributing inputs internally. Supplier relationships are a key factor.	Supplier relationship management; quality of inputs
Operations	Transformation activities that change inputs into outputs sold to customers. Operational systems create value.	Efficiency; quality control; automation
Outbound Logistics	Deliver the product or service to the customer — collection, storage, distribution of outputs. May be internal or external.	Speed; reliability; cost of delivery
Marketing & Sales	Processes used to persuade clients to purchase from the firm instead of competitors. Benefits offered and how well they are communicated are sources of value.	Brand; communication; pricing strategy
Service	Activities related to maintaining the value of the product or service to customers after purchase.	Customer support; warranty; maintenance

SUPPORT ACTIVITIES — Support the primary functions; each can play a role in each primary activity

SUPPORT ACTIVITY	DESCRIPTION	VALUE CREATION FOCUS
Procurement (Purchasing)	What the organisation does to get the resources it needs — finding vendors and negotiating best prices.	Cost negotiation; vendor relationships
Human Resource Management	How well a company recruits, hires, trains, motivates, rewards and retains its workers. People are a significant source of value.	Skill development; employee retention
Technological Development	Managing and processing information, protecting the company's knowledge base. Minimizing IT costs, staying current with technology.	Innovation; R&D; IT infrastructure
Infrastructure	Company's support systems and functions that allow it to maintain daily operations — accounting, legal, administrative, general management.	Governance; planning; finance

KEY TAKEAWAY

Companies use these primary and support activities as **"building blocks"** to create a valuable product or service. The way in which value chain activities are performed determines costs and affects profits.

Value Chain Analysis (VCA)

VCA is a process where a firm identifies its primary and support activities that add value to its final product and then analyses these activities to reduce costs or increase differentiation. It relies on the basic economic principle of advantage — companies are best served by operating in sectors where they have a relative productive advantage.

Competitive Advantages from VCA

₹

Cost Reduction

Making each activity in the value chain more efficient and, therefore, less expensive.

★

Product Differentiation

Investing more time and resources into activities like R&D, design, or marketing that help the product stand out.

Five Steps to Developing a Value Chain Analysis

Step 1 — Identify All Value Chain Activities

Identify each activity that plays a part in creating the finished product. Dig deeper — How many designers? How much time? What raw materials? Do the same for each support activity. Encourage cross-collaboration internally so each department can outline its logistics, operational costs and services.

Step 2 — Calculate the Cost of Each Activity

Calculate cost drivers such as rent, utilities and staff. Map each activity to delineate which parts of the value chain cost the most. *Illustration:* A value chain analysis on a ₹2.50 cup of coffee revealed only 1 penny goes to the coffee grower; the rest covers milk, stirrers, transport, rent, staff and taxes. Rent and staff are the most expensive parts.

Step 3 — Look at What Customers Perceive as Value

Customers tie value directly to a product's price tag — perception greatly impacts margins. Research shows branded and non-branded painkillers have the exact same health outcome, but the former is better perceived. Collect quantitative and qualitative data to identify statistical patterns in buying behaviour. Understand why and how customers make purchasing decisions.

Step 4 — Look at Competitors' Value Chains

Use **competitive benchmarking** in three ways: *Process benchmarking* (comparing process structure and operations), *Strategic benchmarking* (comparing high-level business strategy), and *Performance benchmarking* (comparing outcomes such as revenue, organic traffic, reviews).

Step 5 — Decide on a Competitive Advantage

For **cost advantage**: optimise and cut costs of primary and support activities — outsource, automate, find cheaper delivery. For **competitive differentiation**: capitalise on increasing value perception of products customers are most willing to pay for.

Illustrative Example: Apple Inc.

VALUE CHAIN

Apple's Primary Activities

- **Inbound Logistics:** Top 200 suppliers provide 98% of procurement expenditures for materials, manufacturing and product assembly. Strict quality standards; Apple Procurement Program ensures optimization. Supplier list revisited every year.
- **Operations:** Takes advantage of lower labour and raw material costs in Japan and China; outsourcing keeps manufacturing costs low.
- **Outbound Logistics:** Products purchased online and from company stores. Retailers get ~25% wholesale discount; gross profit of $80 for 30GB 5th Gen iPods sold through non-Apple outlets. For sales through Apple's own stores, $45 retail margin.
- **Marketing & Sales:** 2015 marketing budget boosted to $1.8 Billion. Approach reflects chosen competitive advantage — "highlighting value".
- **Service:** 1-year warranty and 90 days of support; trained Apple technicians offer guided, interactive demos.

Apple's Support Activities

- **R&D:** In 2019 alone, more than $16 billion invested; 28 new or refreshed products released in 2020.
- **Human Resource Management:** Crowned most admired company for HR in 2019; known for recruiting top candidates and poaching talent.

VALUE CHAIN

Pizza Hut's Value Chain

Primary Activities: Inbound logistics (sourcing/standardizing produce, economies of scale), Operations (licensing model with local franchise owners), Outbound logistics (in-store dining + home delivery), Marketing & Sales (large investment to compete with fast-food chains), Service (affordable and convenient pizza).

Support Activities: Infrastructure (finance, legal), Human Resources (junior, unskilled staff to keep costs down), Technological development (process to have unskilled chefs cook pizza — biggest asset), Procurement (raw food, buildings, equipment).

Result: Pizza Hut leads the market in producing affordable pizza delivered in under 30 minutes — convenience sets them apart from other competing options.

Value Innovation

Value innovation is a process in which a company introduces new technologies or upgrades designed to achieve *both* product differentiation *and* low costs. It places equal emphasis on 'Value' and 'Innovation'. First outlined in a 1997 *Harvard Business Review* article by W. Chan Kim and Renée Mauborgne, who later wrote *Blue Ocean Strategy* (2005).

RED OCEAN STRATEGY

- Compete in existing market space
- Beat the competition
- Exploit existing demand
- Make the value-cost trade-off
- Align whole system with differentiation *or* low cost

VS

BLUE OCEAN STRATEGY

- Create uncontested market space
- Make the competition irrelevant
- Create and capture new demand
- Break the value-cost trade-off
- Align whole system in pursuit of differentiation *and* low cost

Examples of Blue Ocean Strategies

Ford Motor Co. (1908)

Introduced the Model T as the car for the masses — one color, one model, reliable, durable, affordable. Market share jumped from 9% (1908) to 61% (1921), officially replacing the horse-drawn carriage.

Apple Inc. (2003)

Created the first legal music download format — iTunes. Easy to use, individual songs at reasonable prices. A win-win-win for music producers, listeners, and Apple.

Netflix (2000s)

Reinvented entertainment — first mail-order video rentals, later the first streaming video platform paid for by user subscriptions. Created new models of entertainment.

BLUE OCEAN SHIFT — THREE KEY COMPONENTS

1. Having the right perspective.
2. A clear roadmap with market-creating tools.
3. Building people's confidence at every level to drive and own the process.

⚡ Quick Revision — Section 1.1

- **Strategy:** Integrated set of choices positioning a firm to generate superior long-run returns.
- **Two broad strategies:** Cost Leadership & Product Differentiation.
- **SCM:** Cost management focused on strategic issues — relative cost advantage or premium pricing.
- **Four stages:** Formulation → Communication → Implementation → Control.
- **Three pillars:** Strategic Positioning, Cost Driver Analysis, Value Chain Analysis.
- **Cost drivers:** Structural (scale, scope, experience, technology, complexity) vs Executional (workforce, TQM, layout, capacity utilization).
- **Porter's Value Chain:** 5 Primary (Inbound Logistics, Operations, Outbound Logistics, Marketing & Sales, Service) + 4 Support (Procurement, HRM, Tech Development, Infrastructure).
- **Value = Performance + Capability ÷ Cost = Function ÷ Cost.**
- **VCA 5 Steps:** Identify activities → Calculate costs → Customer perception → Competitor chains → Decide advantage.
- **Value Innovation:** Simultaneous pursuit of differentiation and low cost (Blue Ocean).
- **Blue Ocean Examples:** Ford Model T, Apple iTunes, Netflix streaming.

1.2

Cost Control and Cost Reduction — Contemporary Techniques

DEFINITIONS · PROCESS · ADVANTAGES · FIVE STEPS TO STRATEGIC COST REDUCTION · TOOLS & TECHNIQUES · CASE STUDIES

Cost Control

DEFINITION

Cost Control is the regulation by executive action of the costs of operating an undertaking, particularly where such action is guided by cost accounting. It comprises all procedures and measures by which the cost of carrying out an activity is kept under check and aims at ensuring that costs do not go beyond the targeted level.

Standard costing and budgetary control are the conventional techniques adopted for cost control. It is exercised through setting standards or targets and comparing actual performance therewith, with a view to identifying deviations from standard norms and taking corrective actions. In other words, it is a **scientific management technique** to contain the costs of doing business.

Process of Cost Control

Setting the Targets → **Identifying the Deviations** → **Taking Corrective Actions**

Five Steps of Cost Control

a

Planning

Establish plans/targets in the form of budgets, standards, estimates — expressed in physical as well as monetary terms. These serve as yardsticks by which the planned objective can be assessed.

b

Communication

Plans and policies are made known to all those responsible. Directives issued by higher management; lower-level executives report performances upward. Communication is established in two directions.

c

Motivation

Performance starts; costs are ascertained; information about results achieved is collected and reported. The fact that costs are being compiled for measuring performance acts as a motivating force.

d

Appraisal & Reporting

Actual performance compared with predetermined plan; variances analysed as to their causes; reported to proper level of management.

e

Decision Making

Variances reviewed and decisions taken. Corrective actions, remedial measures or revision of target as required.

Key Points for Exercising Effective Cost Control

- **Quantity and price standards** should be set to, or estimated for, each physical unit. Factors influencing variances (inadequate facilities, poor organisation, poor materials) should not be ignored.
- To make standards realistic, **all concerned should be associated** in determining standard costs.
- Data collected should be kept to a **minimum**; proper collection and processing of cost control data are important.
- Different variances — price, usage, mix and efficiency — should be considered for materials, labour and overheads.
- No amount of detailed analysis can undo what has already been done; control measures should ensure mistakes are not repeated. **Prevention is possible only if the manager takes action before the event.**
- Essentials include realistic targets (based on work study data) and **flexible attitudes** regarding the standards set.

OBJECTIVE OF COST CONTROL

Cost control implies deriving maximum benefits for the costs incurred — the performance of the same job at a lower cost, or a better performance for the same cost.

Advantages of Cost Control

✓

Return on Capital

Achieving expected return on capital employed by maximising or optimising profit.

✓

Productivity

Increasing the productivity of available resources.

✓

Reasonable Price

Delivering product or service to customers at a reasonable price.

✓

Employment

Continued employment and job opportunity for workers.

✓

Economic Use

Economic use of limited resources of production.

✓

Credit Worthiness

Increased credit worthiness of the enterprise.

✓

Industry Prosperity

Prosperity and economic stability of the industry.

Cost Reduction

DEFINITION

Cost Reduction is the real and permanent reduction in the unit costs of goods manufactured or services rendered *without impairing their suitability for the use intended*.

Reductions due to windfalls, fortuitous receipts, changes in government policy (like reduction in taxes or duties), or temporary measures taken for tiding over financial difficulties do *not* fall under cost

reduction. A programme of cost reduction should in no way affect the quality of products nor lower the standards of performance.

PROFIT MAXIMISATION LOGIC

Profit is the result of two variable factors — **sales** and **cost**. The wider the gap between these two, the larger the profit. Profit can be maximised either by increasing sales or by reducing cost. In a competitive market, it may not be possible to increase sale price without adversely affecting sale volume. Hence avenues must be explored to cut down expenditure and reduce the cost of products.

Two Ways to Reduce Cost Per Unit

1

Reducing Expenditure

Volume of output remaining constant.

2

Increasing Productivity

Increasing volume of output and the level of expenditure remaining unchanged.

These aspects are closely linked and act together; there may be a reduction in expenditure and at the same time an increase in productivity.

Five Steps to Strategic Cost Reduction (PwC Research)

1. Start with Strategy

Have a clear view of the cost reduction strategy and ensure it is consistently understood across the organisation.

2. Align Costs to Strategy

Look across the whole organisation and differentiate the strategically-critical 'good-costs' from the non-essential 'bad-costs'.

3. Aim High

Be bold, be brave and be creative — use technology, innovation and new ways of working to radically optimise the cost base.

4. Set Direction and Show Leadership

Deliver cost optimisation as a strategic, business transformation programme.

5. Create a Culture of Cost Optimisation

Ensure a culture of ownership is embedded and continuous improvement is incentivised.

Tools & Techniques of Cost Reduction

Value Analysis | Business Process Re-engineering | Simplification & Standardisation | Benchmarking | Financial Restructuring | Work Study | Job Evaluation | Quality Control | Inventory Control | Credit Control | Lean Management | Target Costing

Cost Reduction Practices — Case Studies

CASE 1

Reputed Consulting Firm (RCF) Assists a Client in Cost Reduction

Situation: Client is a world leader in food & beverage industry. The departing executive committed the company to adding hundreds of basis points to the bottom line. CFO approached RCF in 2011 to establish a strategic cost reduction program.

Approach: Priority-based budgeting (not zero-based) where marginal services/costs need to be justified. RCF began by prioritizing savings in non-headcount areas — travel, entertainment, consultant fees. Goal: remove 10% of costs in year 1, then 5–10% annually for the next two years. Later renegotiated procurement costs and targeted emerging market locations for "efficient growth".

Cultural Lens: Every cost saving was put through the client's cultural lens to confirm fit for company and employees — the company had spent years cultivating a youthful, energetic culture.

Result: More than US$250 million in costs carved out over three years; another US$50 million expected from renegotiating contracts and procurement efficiencies.

CASE 2

A Four-Pronged Approach to Input Steel Prices and Commodity Purchases

1. **Capture the value of Scrap:** Up to 30% of input steel is unused and considered waste. Companies that account for scrap material value in component price can reduce material costs by **5–8%**.
2. **Increase Sourcing Power:** Purchase directly from large steel mills instead of intermediaries to gain negotiating leverage and reduce costs.
3. **Optimize Material Usage:** Reduce complexity, shrink part design costs, segment supplies. Can result in **5–8% cost reduction**.
4. **Include Scrap in Material Supply Chain:** Create a closed loop network — use scrap dealer for processing and transportation; sell scrap directly to a scrap-consuming supplier.

CASE 3

Ichalkaranji Power-Loomers Minimise Fixed Cost Burden

Context: Ichalkaranji — "Manchester of Maharashtra" — known for skilled weavers and power-loom industry. Average holding ~4 looms per micro-entrepreneur; ~80 metres of grey cloth/day from either cotton or polyester yarn.

Problem: Seasonal fluctuations — power-looms suffered from lack of buyers during slack season but had to pay wages to retain skilled labour (idle labour costs = fixed burden for entire slack period).

Solution: A renowned seed company (with international presence) needed grey cloth for packing agriculture seeds in cloth bags. Peak seed-packing demand coincided with the slack season of power-looms. The seed company offered variable cost-plus pricing during slack season.

Result: Loom owners reduced slack-season idle costs by **~80%**; seed company reduced packing material costs by **~20%** — a win-win cost reduction.

CASE 4

MEC Multiplies Profit Through Outsourcing

Context: Medium-scale electronic manufacturer in Central India; annual turnover ~₹5 crores. Components divided into A (10% quantity, 70% value), B (20% quantity, 20% value), C (70% quantity, 10% value).

Opportunity: Flooded with profitable orders worth ₹20 crores (4× existing turnover). Existing capacity could at best be stretched by 50% (₹7.5 crores).

Solution: Company could produce all A & B items; challenge was C items. Assembly labour was identified as the major limiting factor. Vendors agreed to assemble C items if given in SKD (Semi Knocked Down) form.

Result: MEC executed orders worth ₹20 crores successfully; productivity and profitability of MEC and its vendors multiplied through prudent deployment and optimum utilisation of labour resources.

CASE STUDY LEARNINGS

The tools and techniques of cost reduction could be many; the key is successful adaptation and implementation. Cost behaviour plays a significant role — variable costs may be prone to reduction on a short-term basis whereas fixed cost reduction may warrant long-term strategies. In the ultimate, it is the total cost that shall be reduced on a permanent basis. **Cost Reduction is the first step towards Cost Leadership.**

Difference Between Cost Control and Cost Reduction

SL	NOMENCLATURE	COST CONTROL	COST REDUCTION
1	**Objective**	Containing the cost in accordance with pre-set targets	Exploring ways and means of improving the targets
2	**Approach**	Attaining lowest possible costs under existing circumstances	A continuous process of analysis to find out new ways & means to achieve reduction in costs
3	**Nature**	Preventive function	Corrective function
4	**Emphasis**	On the past — on predetermined standards	On the present and the future — on feasible permanent reductions
5	**Assumptions**	Assumes existence of certain standards or norms	Assumes existence of concealed potential savings in the standards or norms

⚡ Quick Revision — Section 1.2

- **Cost Control:** Regulation by executive action; preventive; focuses on past (standards).
- **Cost Reduction:** Real and permanent reduction in unit costs; corrective; focuses on present & future.
- **Five steps of Cost Control:** Planning → Communication → Motivation → Appraisal & Reporting → Decision Making.
- **Five steps to Strategic Cost Reduction (PwC):** Start with strategy → Align costs to strategy → Aim high → Set direction & show leadership → Create a culture of cost optimisation.
- **Tools:** Value Analysis, BPR, Simplification & Standardisation, Benchmarking, Financial Restructuring, Work Study, Job Evaluation, Quality Control, Inventory Control, Credit Control, Lean Management, Target Costing.
- **Key learning:** Cost Reduction is the first step towards Cost Leadership.

1.3 Value Analysis, Value Engineering & Business Process Re-engineering

VALUE · VALUE EQUATION · PHASES OF VA · TYPES OF VA · VE · BPR

Value Analysis (VA) — Concept

SAVE DEFINITION

"Value analysis is the systematic application of recognised techniques which identify the function of a product or service to establish a monetary value for the function and to provide the necessary function reliability at the lowest overall cost." — Society of American Value Engineers (SAVE)

VA is a scientific approach that ensures all the functions of a product or service are carried out at the minimum cost without compromising quality, reliability, performance and appearance. It is a methodical approach to sharpening the efficiency and effectiveness of any process.

Four Types of Value

U

Use Value

Intrinsic value; measure of properties, qualities and features that make the product useful for the consumer.

C

Cost Value

Sum of all costs incurred in producing the product or rendering a service — raw material, labour, overheads.

E

Exchange Value

Measure of properties that make the product possible of being traded for another product or money.

★

Esteem Value

Measure of properties, features, attractiveness, graphic packaging that increases sales appeal — perception value.

VALUE EQUATION

$$\text{Value} = \{(\text{Performance} + \text{Capability}) \div \text{Cost}\} = \text{Function} \div \text{Cost}$$

Value addition is not just about minimizing cost. It is a **3D technique** — value can be increased by increasing function, reducing cost, or doing both simultaneously. The concept is adding value by enhancing functional worth.

PRICE DISCRIMINATION ILLUSTRATION

A luxury car and a basic small car both fulfil the same function — safe economical travel (use value). But the luxury car has greater **esteem value** and hence is priced at a phenomenal value. Similarly, a gold-plated ball pen commands a privileged pricing over a disposable pen even though use value is nearly the same.

Focus of Value Analysis

The key focus of the VA approach is the management of **'functionality'** to yield value for the customer. Example: Electric kettles were once metal-based; switching to plastic maintains the same functionality (boiling water) but results in cost savings for the manufacturer. If a company that traditionally made metal kettles did not review its design process, it would be severely disadvantaged when competing against the lower-cost plastic alternative.

RULES GOVERNING VA

- No cost can be removed if it compromises the quality or reliability of the product.
- Marketability / Saleability cannot be compromised — this gives the product appeal value.
- Any activity that reduces maintainability increases the cost of ownership to the customer and can lower the value attached to the product.

Seven Phases of Value Analysis

(i) Origination

Identification of a project; constitution of a project team with experts from various fields and departments.

(ii) Information

Collecting relevant facts relating to specifications, drawings, methods, materials, etc. Costs ascertained for each element studied.

(iii) Functional Analysis

Determine functions and uses of the product and its components. Cost and importance of each function identified. Value index computed on cost-benefit ratio; functions arranged in decreasing order of value.

(iv) Innovation

Creative phase — generation of new alternatives to replace or remove existing ones. Creative problem-solving techniques used to discover alternatives providing essential functions at lowest cost.

(v) Evaluation

Each alternative analysed; most promising selected; examined for economic and technical feasibility. Must meet standards of accuracy, reliability, safety, maintenance, environmental effects.

(vi) Choice

Decision makers choose the best alternative; programs and action plans developed for implementation.

(vii) Implementation

Chosen alternative put to actual use; progress continuously monitored and followed up to ensure desired results.

Types of Value Analysis Exercises

VA for Existing Products

Select an existing product sold in relatively large volumes. Has documented history; unites different managers with opinions and complaints. Issues include: inability to change existing designs due to tooling redesign expense; finite project team duration; high purchased costs implying supplier engagement.

VA for New Products (Value Engineering)

For new products, the team modifies the VA approach and operates in a less certain environment with poor available information. Requires much greater investment by the organisation in terms of skilled, experienced and proficient human resources.

VA for Product Families — Horizontal Deployment

When the VA team finds similarities with many products, benefits can be extended to all these products concurrently. Major commercial benefits; relatively short time to introduce changes.

Competitive VA (Strip Down)

VA used as a competitive weapon — applied to analysis of competitor products to calculate costs. Results: understand how competitor products are made, what weaknesses exist, production costs, and innovations incorporated.

RECOMMENDED INITIAL APPROACH

For companies with no real experience of VA, select a **single product** currently in production with a long life ahead. This offers the ability to gain experience, learn as a team, and test the tools and techniques with a product that has known characteristics and failings. The core advantage of VA is its potential for reducing costs — breaking down a product or service into components enables analysis of each component's importance and efficiency.

Value Engineering (VE)

VA VS VE

Value Analysis is the application of a set of techniques to an *existing* product with a view to improve its value — a **remedial** process. **Value Engineering** is the application of exactly the same set of techniques to a *new* product at the design stage to ensure bad features are not added — a **preventive** measure. VE is fundamental; VA is collateral — *prevention is better than cure*.

VE simply answers: *"What else will accomplish the purpose of the product, service, or process we are studying?"* Applicable to all sectors — manufacturing, construction, service, government, agriculture, education, healthcare.

ILLUSTRATIVE CASE

Aadarsh Instruments — Focus Adjustment Knob

Company: Aadarsh Instruments, Ambala — medical instrument manufacturer; ISO certified; exports to various countries. Products conform to international standards.

Application: Value Engineering applied to the Focus Adjustment Knob in model SL 250 for Slit Lamp in microscope — used in eye inspection.

Steps: (i) Selection of Product Plan, (ii) Gathering Product Information, (iii) Functional Analysis, (iv) Creativity Phase & Work-sheet, (v) Evaluation Sheet, (vi) Cost Analysis, (vii) Result & Conclusion, (viii) Implementation.

METRIC	VALUE
Cost before analysis	₹29.99
Total Cost of Nylon Knob	₹18.40
Saving per product	₹11.59
Percentage saving per product	38.64%
Annual Demand	8,000 units
Total Annual Saving	₹92,720
Value Improvement	62.98%

Advantages observed: Cost Reduction, Increase in overall production, Reduction in man-power, Reduction in scrap. The cost was brought down by a substantial margin and thereby the value of the product was increased.

Business Process Reengineering (BPR)

HAMMER & CHAMPY (1993)

"The fundamental rethinking and radical redesign of the business processes to achieve dramatic improvements in critical, contemporary measures of performance, such as cost, quality, service and speed."

BPR refers to a complete redesign of a process with an emphasis on finding creative new means to accomplish an objective. Companies start with a blank sheet of paper and rethink existing processes. They typically adopt a new value system that places increased emphasis on customer needs.

Companies reduce organisational layers and eliminate unproductive activities in two key areas: **First**, they redesign functional organisations into cross-functional teams. **Second**, they use technology to improve data dissemination and decision making.

Characteristics of BPR

- **Several jobs combined into one**
- **Workers make decisions**
- **Steps performed in logical order**
- **Work performed where it makes most sense**
- **Quality built in**
- **Manager provides single point of contact**
- **Centralized & decentralized operations combined**

Seven Principles of BPR

1. Processes should be designed to achieve a desired outcome rather than focusing on existing tasks.
2. Personnel who use the output from a process should perform the process.
3. Information processing should be included in the work which produces the information.
4. Geographically dispersed resources should be treated as if they are centralized.
5. Parallel activities should be linked rather than integrated.
6. Doers should be allowed to be self-managing.
7. Information should be captured once at source.

Key Benefits of BPR

↓

Reduction in Costs & Cycle Times

Eliminating unproductive activities and the employees who perform them.
Reorganisation by teams decreases need for management layers, accelerates information flows and eliminates errors/rework caused by multiple handoffs.

↑

Improvement in Quality

Reducing fragmentation of work and establishing clear ownership of processes.
Workers gain responsibility for their output and can measure performance based on prompt feedback.

Examples of BPR

Example 1: Credit Card Approval

Applicant submits → form reviewed → verification of information (credit report, references) → evaluation → decision → rejection letter OR account opening + card issuance. Process may take weeks due to workload and waiting time for verifications. BPR intervenes in the **processing part** to make it less time and money consuming. Business processes have three elements: inputs (data/materials), processing (multiple stages), and outcome (delivery of expected result).

Example 2: Ford Motors

Ford integrated and leveraged its supplier base by designing **Covisint** — an end-to-end infrastructure enabling an online, centralized marketplace connecting the automotive industry supply chain. Enhanced customer buying experience through redesigned Web sites. Result: increase in customer satisfaction, huge revenue opportunities, and both complexity and cost taken out of the supply chain.

BPR — RADICAL SHIFT

Rather than searching continually for minute improvement, re-engineering involves a **radical shift** in thinking about how an objective should be met. Re-engineering prescribes radical, quick and significant change. It can entail high risks, but it can also bring big rewards — most dramatic when new models are discovered for conducting business.

⚡ Quick Revision — Section 1.3

- **Value Analysis:** Systematic application of recognised techniques to identify product function, establish monetary value, provide function reliability at lowest overall cost (SAVE).
- **Four types of value:** Use, Cost, Exchange, Esteem.
- **Value Equation:** Value = (Performance + Capability) ÷ Cost = Function ÷ Cost.
- **Seven phases:** Origination → Information → Functional Analysis → Innovation → Evaluation → Choice → Implementation.
- **VA (remedial, existing product) vs VE (preventive, new product at design stage).**
- **BPR (Hammer & Champy):** Fundamental rethinking and radical redesign of business processes for dramatic improvements in cost, quality, service, speed.
- **BPR benefits:** Reduction in costs & cycle times; Improvement in quality.
- **BPR examples:** Credit card approval, Ford Motors (Covisint).

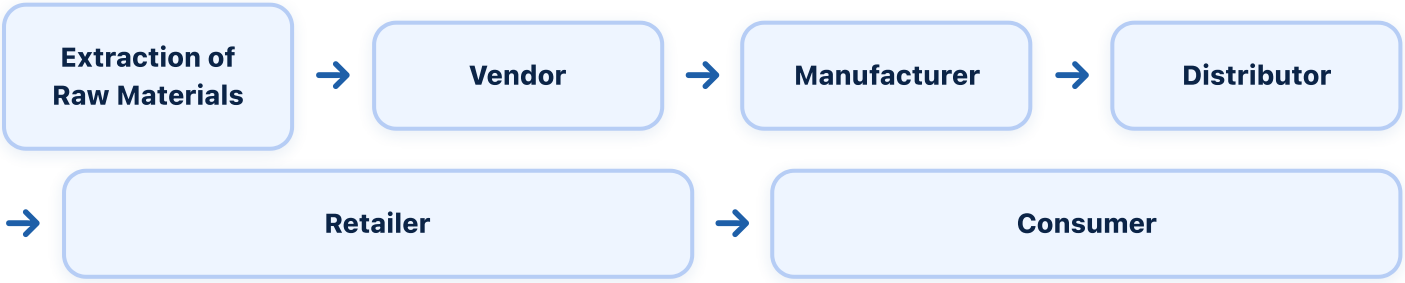
1.4 Supply Chain Management

SUPPLY CHAIN · GENERIC LINKS · SCM STRATEGY · IMPORTANCE · CASE STUDIES

Supply Chain — Concept

A **Supply Chain** refers to the entire gambit of linkages in manufacturing a product or rendering a service. For a manufacturing entity, it encompasses all activities from the extraction of raw materials till the delivery of the finished product to the ultimate consumer.

Generic Links of a Supply Chain



In its simplest form, a supply chain is the activities required by the organisation to deliver goods or services to the consumer. Supply chains are made up of all the links that participate in the design,

assembly, and delivery of a particular product.

IN SIMPLE TERMS

Vendors supply raw materials → Producers convert those raw materials into products → Warehouses store that product until it's needed → Distribution centers pick up and deliver that product → Retailers, online and in-store, bring that product to you.

Supply Chain Management (SCM)

Supply Chain Management encompasses every activity involved in maintaining the supply chain. The goal is to look holistically at the entire supply chain from supplier through to the consumer, and review three core areas — **people, process and systems** — in order to maximise value from all activities.

Each year, products get bigger and better, yet prices drop. It's the end result of SCM professionals working together — LCD glass panel fabricators in South Korea, semiconductor manufacturers in Taiwan, television assembly plants in Mexico. These global partners collaborate across time zones and oceans to decrease costs and increase performance in ways no single company ever could.

Three Core Areas of Supply Chain Strategy

P

People

Do you have the right number of staff with the right skill set?

P

Process

Are there waste activities within your current operating processes?

S

Systems

Are your systems enablers to the strategy or are legacy systems holding you back?

JFK & NASA — A LESSON IN COMMUNICATION

When in 1961 JFK visited NASA he asked a janitor what his job was. His reply: *"I'm sending a man to the moon."* This is a clear example of a well-communicated strategy and mission flowing throughout the whole organisation, with complete buy-in regardless of an individual's position.

Importance of Supply Chain Management

Boosts Customer Service

Right Location

Customers expect products to be available at the right location — customer satisfaction diminishes if an auto repair shop does not have necessary parts.

Right Delivery Time

Customers expect products to be delivered on time — customer satisfaction diminishes if pizza delivery is two hours late.

Right After-Sale Support

Customers expect products to be serviced quickly — satisfaction diminishes when a home furnace stops operating and repairs can't be made for days.

Reduces Operating Costs

Decreases Purchasing Cost

Retailers depend on supply chains to quickly deliver expensive products to avoid holding costly inventories — e.g., electronics stores require fast delivery of 60" flat-panel HDTVs.

Decreases Production Cost

Manufacturers depend on supply chains to reliably deliver materials — an unexpected parts shipment delay causing an auto assembly plant shutdown can cost ₹20,000 per minute.

Decreases Total Supply Chain Cost

Dell's revolutionary approach — making each computer based on a specific customer order, shipping directly — saved millions by avoiding large inventories.

Improves Financial Position

Increases Profit Leverage

Decreasing U.S. cereal supply chain costs just one cent per box would result in $13 million saved industry-wide over a five-year period.

Decreases Fixed Assets

Redesigning the network to serve customers from six warehouses rather than ten avoids building four very expensive buildings.

Increases Cash Flow

If a firm can make and deliver a product in 10 days rather than 70 days, it can invoice the customer 60 days sooner.

Societal Benefits

SCM knowledge and capabilities can be used to support medical missions, conduct disaster relief operations, and handle other types of emergencies. Supply chain experts diagnose problems, creatively work around disruptions, and figure out how to move essential products to people in need as efficiently as possible.

Case Study 1: Walmart's Inventory Innovations

SUPPLY CHAIN

Walmart — Five Strategic Pillars

1. **Fewer Links in the Supply Chain:** Founder Sam Walton selectively purchased bulk merchandise and transported it directly to stores. In the 1980s, Walmart began working directly with manufacturers to cut costs. Pioneered Vendor Managed Inventory (VMI).
2. **Strategic Vendor Partnerships:** Long practiced strategic sourcing; established partnerships offering potential for long-term and high-volume purchases in exchange for lowest possible prices. Network of global suppliers, warehouses, and retail stores behaves almost like a single firm.
3. **Cross-Docking:** Direct transfer of products from inbound to outbound truck trailers without extra storage. Goods cross from one loading dock to another, usually in 24 hours or less. Reduced Walmart's costs significantly.
4. **Advanced Inventory Technology:** In 2015, spent $10.5 billion on IT. First company-wide use of Universal Product Code (barcodes) in 1983. 'Savings Catcher' allows consumers to scan barcodes to compare prices. 'Retail Link' — mammoth database connected via global satellite system. RFID technology — 16% reduction in out-of-stocks; products with electronic product code replenished three times as fast.
5. **Top Stock:** Top shelves utilized for more storage, freeing up back rooms. Products on shelves sooner; more space for fulfilling online delivery orders.

Competitive Advantages: Lower product costs, reduced inventory carrying costs, improved in-store variety and selection, highly competitive pricing for the consumer. Helped Walmart become a dominant force in a competitive global market.

Case Study 2: Deere & Company

SUPPLY CHAIN

John Deere — Supply Chain Cost Reduction

Background: Deere & Company (brand name John Deere) — manufacture and supply of machinery used in agriculture, construction, and forestry. Despite pandemic challenges, delivered a year of solid performance in 2020 with Net sales and revenues of $35.54 billion and net income of $2.75 billion.

Challenge: Diverse product range; retail activity extremely seasonal (March–July). Replenishing dealers' inventory weekly using direct shipment and cross-docking from source warehouses near manufacturing facilities — too costly and too slow.

Path to Cost Reduction: Supply chain network-redesign program → intermediate "merge centers" → optimization of cross-dock terminal locations → consolidating shipments → break-bulk terminals during seasonal peak → increased use of third-party logistics providers → network optimizable tactically at any given point.

Results: Inventory decrease of $1 billion; customer delivery lead times reduced from 10 days to 5 or less; annual transportation cost savings of around 5%.

Case Study 3: Intel

SUPPLY CHAIN

Intel — Atom Chip Supply Chain Cost Reduction

Challenge: Supply chain costs of ~$5.50 per chip were affordable for units selling at $100; but the new "Atom" chip was priced at about $20. Only area of leverage — inventory. Nine-week order cycle. Intel could make no service trade-offs; no way to reduce duty payments; packaging already minimal.

Path to Cost Reduction: Decided to try "make to order" — an unlikely strategy for the semiconductor industry. Pilot with a manufacturer in Malaysia. Cutting chip assembly test window from 5 days to bi-weekly 2-day process. Introducing formal Standard Operating Planning Process (S&OP). Moving to vendor-managed inventory model.

Results: Order cycle time driven down from 9 weeks to just 2. Supply chain cost reduction of more than $4 per unit for the $20 Atom chip — far more palatable than the original $5.50.

CASE STUDY LEARNINGS

For an enterprise to make significant and sustainable cost improvements, substantial change must take place. None of the changes took place overnight — each company tackled issues in phases, learning more as they went along. To see sustainable cost reductions, one needs to view the big picture from a new perspective and be prepared to step out of the comfort zone and seek long-term distinct solutions.

⚡ Quick Revision — Section 1.4

- **Supply Chain:** Entire gambit of linkages from raw material extraction to ultimate consumer.
- **Generic links:** Raw Materials → Vendor → Manufacturer → Distributor → Retailer → Consumer.
- **SCM:** Holistic view of entire supply chain; three core areas — People, Process, Systems.
- **Importance:** Boosts customer service, reduces operating costs, improves financial position, societal benefits.
- **Walmart:** Fewer links, strategic vendor partnerships, cross-docking, advanced inventory technology, top stock.
- **Deere & Co.:** Merge centers, cross-dock optimization, 5% transportation cost savings, lead time 10→5 days.
- **Intel:** Make-to-order, order cycle 9 weeks → 2 weeks, cost reduction $5.50 → ~$1.50 per unit.

★ Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Strategic Cost Management

Cost management that specifically focuses on strategic issues — relative cost advantage or premium pricing.

Value Chain

A set of activities that an organisation carries out to create value for its customers.

Cost Control

The regulation by executive action of the costs of operating an undertaking, particularly where such action is guided by cost accounting.

Cost Reduction

The real and permanent reduction in the unit costs of goods manufactured or services rendered without impairing their suitability for the use intended.

Value Analysis

A function-oriented, structured, multi-disciplinary team approach to solving problems or identifying improvements.

Business Process Reengineering

A complete redesign of a process with an emphasis on finding creative new means to accomplish an objective.

Supply Chain

The entire gambit of linkages in manufacturing a product or rendering a service.

Value Engineering

Application of the same set of VA techniques to a new product at the design stage to ensure bad features are not added — a preventive measure.

? Multiple Choice Questions

WITH ANSWERS AND REASONING

1. Which of the following is not a primary activity of Value Chain?

A. Inbound Logistics B. Operations C. Service

D. Infrastructure

Answer: D

Infrastructure is a support (secondary) activity, not a primary activity.

2. Which of the following is not a secondary activity of Value Chain?

A. Procurement B. Human Resource Development C. Service

D. Technology Development

Answer: C

Service is a primary activity.

3. Which of the following is not a term normally used in value analysis?

A. Resale value B. Use value C. Esteem value

D. Cost value

Answer: A

The resale value is normally referred to as the exchange value.

4. A company has forecast sales and cost of goods sold for the coming year as ₹25 lakhs and ₹18 lakhs respectively. The inventory turnover has been taken as 9 times per year. If the inventory turnover increases to 12 times and the short-term interest rate on working capital is 10%, what will be the saving in cost?

A. ₹10,000 B. ₹20,000 C. ₹15,000

D. ₹5,000

Answer: D

Inventory at 9 times = ₹2,00,000; at 12 times = ₹1,50,000. Reduction = ₹50,000. Saving = ₹50,000 × 10% = ₹5,000.

Q1	Q2	Q3	Q4
D	C	A	D

State True or False

SL	STATEMENT	ANSWER
1	Strategic cost management refers to the cost management that specifically focuses on strategic issues.	**True**
2	'VA' and 'VE' are closely related terms so much so that they are frequently used interchangeably.	**True**
3	Cost reduction refers to the real and permanent reduction in the unit costs of goods manufactured or services rendered without considering their suitability for the use intended.	**False**
4	Supply Chain Management is not an avenue of cost reduction.	**False**

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

1

Define Strategic Cost Management (SCM) and discuss its relevance in the contemporary scenario.

2

What are the generic links of Porter's Value Chain?

3

Demonstrate the significance of Value Chain Analysis with an illustrative example.

4

List out the key points for exercising cost control.

5

Discuss the tools and techniques of cost reduction.

6

Differentiate Cost Control with Cost Reduction.

7

Narrate five steps to Strategic Cost Reduction.

8

Draft a report to your Managing Director emphasizing the importance of value analysis with reference to an existing problem in your organisation.

9

Explain the phases of Value Analysis.

10

Distinguish between Value Analysis and Value Engineering.

11

What is the need for Business Process Reengineering (BPR)?

12

Define Supply Chain. Discuss the generic links of supply chain.

13

What is Cross-Docking?

14

Explain Supply Chain Management with an illustrative case let.

Σ

Comprehensive Numerical Questions

PRACTICE PROBLEMS WITH WORKED-OUT SOLUTIONS

Problem 1 — Ever Forward Ltd.

PROBLEM

Ever Forward Ltd is manufacturing and selling two products: Splash and Flash, at selling prices of ₹3 and ₹4 respectively. The following sales strategy has been outlined for the year 2023:

(i) Sales planned for year will be ₹7.20 lakhs in the case of Splash and ₹3.50 lakhs in the case of Flash.

SOLUTION — COMPUTATION OF NUMBER OF UNITS, BREAK-EVEN & REDUCTION IN FC

ITEM	SPLASH	FLASH	TOTAL
No. of units to be sold	3,00,000	1,00,000	4,00,000
Break even units	1,80,000	60,000	—
Cost Reduction envisaged in FC (₹)	4,320	750	5,070

Problem 2 — The Forward Look Ltd.

PROBLEM

The profit works out to 12.5% of the capital employed. Relevant figures: Sales ₹5,00,000; Direct Materials ₹2,50,000; Direct Labour ₹1,00,000; Variable Overheads ₹40,000; Capital employed ₹4,00,000.

The new Sales Manager estimates a profit of about 23% on capital employed provided sales volume increases by 10%, selling price increases by 4%, and overall cost reduction in all elements of cost by 2%. Verify his contention.

SOLUTION

Percentage of Profit on Capital Employed = **23.195%** i.e. > 23%. **Recommendation:** The Sales Manager's proposal can be adopted.

Problem 3 — Electronic Corporation Ltd.

PROBLEM

Anticipated sales ₹40,00,000; unit selling price ₹20. Per unit cost: Direct material ₹9, labour ₹3, other variable expenses ₹3. Net profit 5%. Four proposals were discussed at the Executive Committee Meeting:

1. Centralization of administration — reduces fixed cost by ₹12,000.
2. Cost reduction programme — reduces cost by ₹1 per unit.
3. Increase units sold by 20% with 5% reduction in selling price.
4. Increase selling price by 10% with 5% reduction in sales units.

SOLUTION — EVALUATION OF ALTERNATIVES

ALTERNATIVE	(I)	(II)	(III)	(IV)
Proposal	Central administration (Reduction in F.C. by ₹12,000)	Variable cost Reduction by ₹1 per unit	20% increase in Sales units with 5% reduction in selling price	10% increase in Selling price and 5% reduction in sales units
Ranking	3	2	Reject	1
Suggestion	Accept combination of proposals (i) and (iii) which will generate an additional profit of ₹45,000 (12,000 + 33,000).			

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 1

ABBREVIATION	FULL FORM
BPR	Business Process Reengineering
RFID	Radio Frequency Identification Tag
SAVE	Society of American Value Engineers
SCM	Strategic Cost Management
SCM	Supply Chain Management
VA	Value Analysis
VCA	Value Chain Analysis
VE	Value Engineering
VMI	Vendor Managed Inventory
SKD	Semi Knocked Down
EDI	Electronic Data Interchange
S&OP	Standard Operating Planning Process

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Quality Cost Management

Module 2 · Managing Quality in a Competitive Environment

EDITION
December 2026

PAPER
Paper 16 · Strategic Cost Management

SECTION
Section A · Weightage 60%

MODULE WEIGHTAGE
5% of Section A

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading
✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

02

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 2 INCLUDES

2.1 Managing Quality in Competitive Environment · **2.2** Cost of Quality · **2.3** Total Quality Management · **2.4** Lean Accounting · **2.5** Six Sigma.

Module Learning Objectives

After studying this module, the students will be able to:

1

TQM, Lean & Six Sigma

Possess fair knowledge on Total Quality Management, Lean Accounting and Six Sigma.

2

Quality Cost Management

Effective Management and Control of Quality Costs.

5%	5	3
WEIGHTAGE IN SECTION A	CORE SUB-TOPICS	CASE STUDIES
Of 60% Section A total	2.1 to 2.5	AtlantiCare, RIMS, Ford

3
NUMERICAL PROBLEMS
Worked solutions

SYLLABUS STRUCTURE NOTE

Module 2 carries **5% weightage** within Section A. It introduces the quality dimension of cost management — a critical strategic concern in today's competitive marketplace. The module covers quality definitions, cost of quality, TQM, Lean Accounting, and Six Sigma.

2.1 Managing Quality in Competitive Environment

QUALITY DEFINITIONS · DIMENSIONS · QUALITY MANAGEMENT EVOLUTION · CASE STUDIES

What is Quality?

DEFINITION

Quality is that characteristic or a combination of characteristics that distinguishes one article from the other or from one service provider to another service provider or goods of one manufacturer from that of competitors or one grade of product from another when both are the outcome of the same factory.

The main characteristics that determine the quality of an article include **design, size, materials, chemical composition, mechanical functioning, electrical properties, workmanship, finish and appearance**. The quality of a product may be defined as the sum of a number of related characteristics such as shape, dimension, composition, strength, workmanship, adjustment, finish and colour.

Quality as Perception — Famous Quotes

"

"Quality is not an act. It is a habit."

— Aristotle. This is true and applicable to any act of a human being.

"

"Quality is conformance to requirements."

This is in line with the concept that quality is decided by the customer.

"

"Quality is zero defects."

No customer wants defects in the products or services he or she pays for.

"

"Quality is free."

— Phil Crosby. When there are no defects, there is no wastage and boosted sales — thus quality becomes free.

"

"Quality is the degree to which a set of inherent characteristics fulfils requirements."

— ISO 9000. An attempt to give universality to the term quality.

Common Definitions of Quality

DEFINITION	DESCRIPTION	EXAMPLE
Conformance to Specifications	Measures how well the product or service meets the targets and tolerances determined by its designers. Directly measurable.	Machine part dimensions specified as 3 ± 0.05 inches; hotel room service specified as 20 minutes with acceptable delay of 10 minutes.
Fitness for Use	Focuses on how well the product performs its intended function or use. User-based definition.	Mercedes Benz and Jeep Cherokee both meet fitness for use for transportation; but for mountain roads with fishing gear, Jeep has greater fitness.
Value for Price Paid	Combines economics with consumer criteria; assumes definition of quality is price sensitive.	Same personal finance seminar at two colleges at significantly different tuition rates — less expensive seminar gives greater value for price.
Support Services	Quality applies not only to product/service itself but also to people, processes, and organizational environment.	Quality of a university is judged not only by staff and courses but also by efficiency and accuracy of processing paperwork.
Psychological Criteria	Subjective definition focusing on judgmental evaluation — atmosphere, perceived prestige.	Hospital patient receiving average health care but very friendly staff may leave impression of high quality. Rolex watches and Mercedes-Benz automobiles.

Differences Between Manufacturing and Service Organizations

MANUFACTURING ORGANIZATIONS

- Produce a tangible product — can be seen, touched, directly measured
- Quality definitions focus on tangible product features
- Most common definition: **Conformance** — degree to which product characteristic meets preset standards
- Other definitions: Performance, Reliability, Features, Durability, Serviceability
- Relative importance based on individual customer preferences

VS

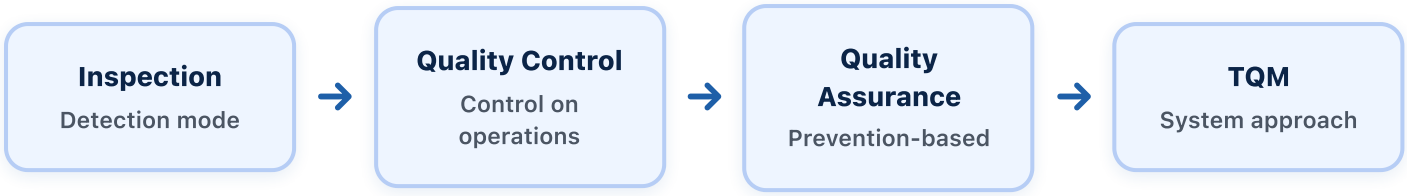
SERVICE ORGANIZATIONS

- Produce an intangible product — cannot be seen or touched, only experienced
- Intangible nature makes defining quality difficult
- Perceptions can be highly subjective
- Definitions: Responsiveness, Courtesy, Friendliness, Promptness, Atmosphere
- Time — amount of time customer waits for service
- Consistency — degree to which service is same each time

DIMENSIONS OF QUALITY — MANUFACTURING ORGANIZATIONS	DIMENSIONS OF QUALITY — SERVICE ORGANIZATIONS
Conformance to specifications	Tangible factors
Performance	Consistency
Reliability	Responsiveness to customer needs
Features	Courtesy/friendliness
Durability	Timeliness/promptness
Serviceability	Atmosphere

Quality Management — Evolution

Quality management is defined as **"coordinated activities to direct and control an organization with regard to quality"** (ISO 9000:2000). The practices of quality management have shown an evolution from **Inspection** → **Quality Control** → **Quality Assurance** → **Total Quality Management**.



1. Inspection

Defined as "Activities such as measuring, testing and gauging one or more characteristics of a product or service and comparing with specifications as in design to determine its conformity". This is the **"after the event" approach** — things which have happened are verified by measuring/testing and screening out those which do not meet specifications. Organisation is said to be working in a **"detection" mode**. Limitations: nonconforming products are a cost (waste of material, effort, rework, sold as "seconds" at lower price) — all resulting in a dent in profits.

2. Quality Control

Defined as "Operational techniques and activities that are used to fulfill requirements for quality". Organisations realised that "Inspection" alone was a costly affair. Quality control introduced the idea of **"control on operations"** — procedures, intermediate stage inspections, recording of performance of a process for giving feedback. Methods of inspection got sophisticated with tools like sample checks, lot size, etc. However, the intention and activity of preventing a non-conforming product reaching a customer depended solely on the screening inspection at the final stage of production. Result: lesser defects but still in **"detection mode"**.

3. Quality Assurance (QA)

Defined as "All those planned and systematic actions necessary to provide an adequate confidence that a product or service will satisfy the given requirements for quality". QA is a **prevention-based system**. The system improves product and service quality and increases productivity by placing emphasis on the design of product or service and relevant processes. The basis: the process that makes the product needs to be designed such that variation in process outcome is minimal in reference to design specifications, thus eliminating non-conformance. This is a **proactive approach** compared to the reactive one in the "detection mode".

4. Total Quality Management (TQM)

Integrating various processes of the business into "a whole" was at the basis — a true system approach to business. Such thinking resulted into TQM. No business process can work in isolation — interdependence and interaction between each of the business processes exist and must be addressed while operating a business.

QUALITY AS A KEY DIFFERENTIATOR

Quality is a key differentiator in a populated marketplace, driven by dynamic customer choices and competitive business offerings. Quality products make an important contribution to long-term revenue and profitability, building brand value by simply letting services and products speak for themselves.

WHY TQM?

In today's business environment, organizations face multiple challenges ranging from a global economic slowdown, challenging and agile competition, and technology moving at lightning pace. One of the ways in which an organization can build a strong, sustainable competitive advantage is via implementing **Total Quality Management (TQM)** practices.

Case Study 1: A Quality Education — Ramaiah Institute of

CASE STUDY

TQM in Higher Education — RIMS, Bangalore

Background: The global financial crisis hit higher education harder than many expected, especially in India. A report by one recruiter asserted just one in four Indian MBAs were adequately prepared for the business world. At RIMS, recruiters and accreditation bodies specifically called into question the quality of students' educations.

Challenge: RIMS struggled to compete with India's renowned Xavier Labour Research Institute. Faculty noticed clear hindrances in the success of graduates. The RIMS board decided it was time for a serious reassessment of quality management.

Approach: The school nominated Chief Academic Advisor Dr Krishnamurthy to head a volunteer team that would audit, analyse and implement process changes. The team was tasked with looking at three key dimensions: **assurance of learning, research and productivity, and quality of placements**. Each member underwent extensive training to learn about action plans, quality auditing skills and continuous improvement tools — such as the 'plan-do-study-act' cycle.

Diagnosis: Using a fishbone diagram, the team identified potential root causes. Control groups were administered domain-based knowledge tests:

METRIC	RIMS STUDENTS	COMPARISON
Knowledge base	36%	Harvard: 95%
Critical thinking abilities	9%	MIT: 93%
Mean salaries of graduates	$36,000	Kellogg: $150,000

Solution: Dr Krishnamurthy created an employability team, developed strategic architecture and designed pilot studies to improve the school's curriculum. Despite initial resistance, changes were extremely beneficial. Once students started seeing results, buy-in became complete and unconditional. The school provided shareholders with detailed plans and projections. New methods included incorporating case studies into the curriculum and a mandate requiring students to be certified in English by the British Council.

Result: General test scores increased by almost 10%; English certification scores increased from 42% to 51%. The number of top 100 businesses recruiting from the school shot up by 22%; average salary offers increased by $20,000; placement revenue rose by $50,000. RIMS skyrocketed up domestic and international education tables.

LEARNING FROM RIMS

No matter what the business is, total quality management can and will work. Yet this philosophical take on quality control will only impact firms that are in it for the long haul. Every employee must be in tune with the company's ideologies and desires to improve, and customer satisfaction must reign supreme.

⚡ Quick Revision — Section 2.1

- **Quality:** Characteristic(s) that distinguish one article/service from another; sum of shape, dimension, composition, strength, workmanship, finish, colour.
- **Five definitions:** Conformance to Specifications, Fitness for Use, Value for Price Paid, Support Services, Psychological Criteria.
- **Manufacturing vs Service:** Tangible (conformance, performance, reliability, features, durability, serviceability) vs Intangible (tangibles, consistency, responsiveness, courtesy, timeliness, atmosphere).
- **Evolution of Quality Management:** Inspection → Quality Control → Quality Assurance → TQM.
- **Inspection:** After-the-event, detection mode.
- **Quality Control:** Operational techniques, intermediate stage inspections, but still detection mode.
- **Quality Assurance:** Prevention-based system; design stage creates quality.
- **TQM:** System approach; interdependence of processes; continuous improvement.

2.2 Cost of Quality

PREVENTION · APPRAISAL · INTERNAL FAILURE · EXTERNAL FAILURE

The reason quality has gained such prominence is that organizations have gained an understanding of the **high cost of poor quality**. Quality affects all aspects of the organization and has dramatic cost implications. The most obvious consequence occurs when poor quality creates dissatisfied customers and eventually leads to loss of business.

Quality costs can be divided into two categories:

Category 1 — Quality Control Costs

Costs necessary for achieving high quality. These are of two types: **Prevention Costs** and **Appraisal Costs**.

Category 2 — Quality Failure Costs

Cost consequences of poor quality. These include **External Failure Costs** and **Internal Failure Costs**.

The first two costs (Prevention & Appraisal) are incurred in the hope of preventing the second two (Internal & External Failure).

The Four Cost of Quality Categories

P

Prevention Costs

All costs incurred in the process of preventing poor quality from occurring. Includes quality planning costs (developing and implementing a quality plan), product and process design costs (collecting customer information to designing processes that achieve conformance), employee training in quality measurement, and maintaining records of quality information.

A

Appraisal Costs

Costs incurred in the process of uncovering defects. Includes cost of quality inspections, product testing, and performing audits to make sure quality standards are being met. Also includes worker time spent measuring quality and the cost of equipment used for quality appraisal.

I

Internal Failure Costs

Associated with discovering poor product quality *before* the product reaches the customer site. One type is **rework** (cost of correcting the defective item). Sometimes the item is so defective it must be thrown away — this is called **scrap**, and its costs include all material, labour, and machine cost spent in producing the defective product.

E

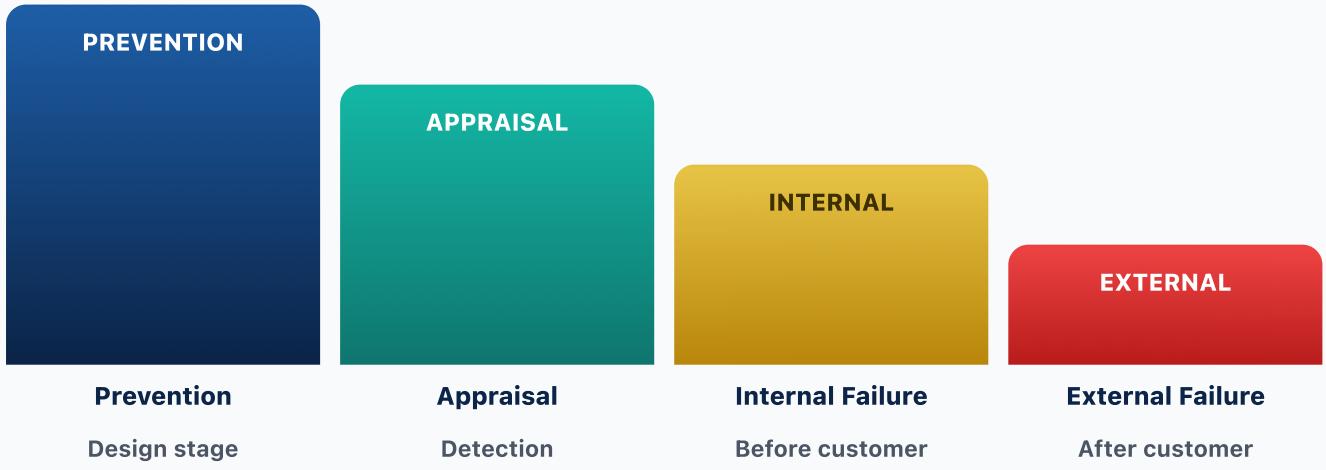
External Failure Costs

Incurred when inferior products are delivered to customers. Includes cost of handling customer complaints, warranty replacements, repairs of returned products and cost arising from a damaged company reputation.

Summary Table with Examples

COST CATEGORY	DEFINITION	EXAMPLES
Prevention Costs	Ensuring the failures do not happen	Quality training · Quality circles · Statistical process control activities · System development for prevention · Quality improvement
Appraisal Costs	Checking for failures	Testing and inspecting materials · Final product testing and inspecting · WIP testing and inspecting · Package inspection · Depreciation of testing equipment
Internal Failure Costs	Keeping defective products from falling into the hands of customers	Cost of Scrap (net of realization) · Cost of Spoilage · Cost of Rework · Down time due to defect in quality · Retesting
External Failure Costs	Costs of defects discovered by the customers	Cost of field servicing · Cost of handling complaints · Warranty repairs · Lost sales · Warranty replacements

Cost of Quality — A Continuum



CRITICAL INSIGHT

External Failure Costs are the most damaging — they not only cause immediate financial loss but also damage the company's reputation and future business prospects. Prevention Costs are the least expensive in the long run — investing in prevention reduces both appraisal and failure costs.

OPTIMAL QUALITY COST MODEL

As **Prevention and Appraisal Costs** increase, **Failure Costs** decrease. The **Total Quality Cost** is the sum of all four categories. There is an optimal point where total quality cost is minimum — beyond this point, additional prevention expenditure yields diminishing returns in reduced failure costs.

⚡ Quick Revision — Section 2.2

- **Cost of Quality Categories:** Prevention, Appraisal, Internal Failure, External Failure.
- **Prevention Costs:** Quality training, quality circles, statistical process control, system development, quality improvement.
- **Appraisal Costs:** Testing/inspecting materials, final product testing, WIP testing, package inspection, depreciation of testing equipment.
- **Internal Failure Costs:** Scrap (net of realization), spoilage, rework, down time, retesting.
- **External Failure Costs:** Field servicing, handling complaints, warranty repairs, lost sales, warranty replacements.
- **Golden rule:** Prevention & Appraisal costs are incurred to prevent Failure costs.

2.3 Total Quality Management (TQM)

PRINCIPLES · STEPS · PRAISE · 6C'S · 4P'S · PARETO ANALYSIS · CASE STUDY

Concept of TQM

DEFINITION

Total Quality Management is a philosophy of continuously improving the quality of all the products and processes in response to continuous feedback for meeting the customers' requirements. It aims to do things right the first time, rather than need to fix problems after they emerge.

T

Total

Quality involves everyone and all activities in the company. Mobilizing the whole organization to achieve quality continuously and economically.

Q

Quality

Understanding and meeting the customers' requirements. Satisfying the customers first time every time.

M

Management

Quality can and must be managed. Avoid defects rather than correct them.

TQM is a **vision based, customer focused, prevention oriented, continuous improvement strategy** based on scientific approach adopted by cost conscious people committed to satisfy the customers first time every time. It aims at managing an organization so that it excels in areas important to the customer.

Underlying Principles of TQM

1

Customer Focus

The first principle puts the focus back on the people buying your product or service. Your customers determine the quality of your product. When you understand what your customer wants or needs, you have a better chance of figuring out how to get the right materials, people, and processes in place to meet and exceed their expectations.

2

Total Employee Commitment

You can't increase productivity, processes, or sales without the total commitment of all employees. They need to understand the vision and goals that have been communicated. They must be sufficiently trained and given the proper resources to complete tasks.

3

Process Approach

Adhering to processes is critical in quality management. Processes ensure that the proper steps are taken at the right time to ensure consistency and speed up production.

4

Integrated System

Departments and functions should be interconnected with horizontal processes. In an integrated system, everybody in every department should have a thorough understanding of policies, standards, objectives, and processes.

5

Strategic and Systematic Approach

Multiple processes within a development or production cycle are managed as a system of processes in an effort to increase efficiency.

6

Continual Improvement

Optimal efficiency and complete customer satisfaction do not happen in a day. Your business should continually find ways to improve processes and adapt your products and services as customer needs shift.

7

Fact-based Decision-making

Analysis and data gathering lead to better decisions based on the available information. Making informed decisions leads to a better understanding of customers and your market.

8

Communications

Everybody in your organization needs to be aware of plans, strategies and methods that will be used to achieve goals. There is a greater risk of failure if you don't have a good communication plan.

Steps in Total Quality Management

Step 1 — Identification of Customers/Customer Groups

Through a team approach (a technique called Multi-Voting), the firm should identify major customer groups. This helps generate priorities in identification of customers and critical issues in the provision of decision-support information.

Step 2 — Identifying Customer Expectations

Once major customer groups are identified, their expectations are listed. The question to be answered: *What does the customer expect from the firm?*

Step 3 — Identifying Customer Decision-making Requirements and Product Utilities

By identifying the need to stay close to customers and following their suggestions, a decision-support system can be developed, incorporating both financial and non-financial information. The answer is sought by listing out managerial perceptions and not by actual interaction with customers.

Step 4 — Identifying Perceived Problems in Decision-making Process and Product Utilities

Using participative processes such as brainstorming and multi-voting, the firm seeks to list out its perception of problem areas and shortcomings in meeting customer requirements. This lists out areas of weakness where the greatest impact could be achieved through improvements.

Step 5 — Comparison with Other Firms and Benchmarking

Detailed and systematic internal deliberations allow the firm to develop a clear idea of their own strengths and weaknesses and of areas of most significant deficiency. Benchmarking allows the firm to see how other companies are coping with similar problems and opportunities.

Step 6 — Customer Feedback

Steps 1 to 5 provide an information base developed without reference to the customer. This is rectified at Step 6 with a survey of representative customers, which embraces their views on perceived problem areas. Interaction with customers helps correct the firm's own perceptions and refine its processes.

Steps 7 & 8 — Identification of Improvement Opportunities and Implementation of Quality Improvement Process

The outcomes of the customer survey, benchmarking and internal analysis provide inputs for Steps 7 and 8. This is done through a six-step process called **PRAISE**.

6C's of TQM

6 C'S	DESCRIPTION
Commitment	If a TQM culture is to be developed, total commitment must come from top management. It is not sufficient to delegate 'quality' issues to a single person. Quality expectations must be made clear by top management, together with the support and training required for its achievement.
Culture	Training lies at the centre of effecting a change in culture and attitudes. Negative perceptions must be changed to encourage individual contributions and to make 'quality' a normal part of everyone's job.
Continuous Improvement	TQM should be recognised as a 'continuous process'. It is not a 'one-time programme'. There will always be room for improvement, however small it may be.
Co-operation	TQM visualises Total Employee Involvement (TEI). Employee involvement and co-operation should be sought in the development of improvement strategies and associated performance measures.
Customer Focus	The needs of external customers (in receipt of the final product or service) and also the internal customers (colleagues who receive and supply goods, services or information), should be the prime focus.
Control	Documentation, procedures and awareness of current best practice are essential if TQM implementations are to function appropriately. Unless control procedures are in place, improvements cannot be monitored and measured nor deficiencies corrected.

4P's of TQM

AVOIDING TOTAL QUALITY PARALYSIS

It is possible that the organisation is led to Total Quality Paralysis, instead of improvement, by improper implementation of TQM. To avoid such disruption and paralysis, the following principles (called the four P's) should be followed:

P

People

To avoid misdirection, TQM teams should consist of team spirited individuals who have a flair for accepting and meeting challenges. Individuals not ideally suited to the participatory process of TQM should not be involved at all — e.g., lack of enthusiasm, non-attendance at TQM meetings, failure to complete delegated work, remaining a "Mute Spectator" at TQM meetings, etc.

P

Process

It is essential to approach problem-solving practically and to regard the formal process as a system designed to prevent participants from jumping to conclusions. As such, it will provide a means to facilitate the generation of alternatives while ensuring that important discussion stages are not omitted.

P

Problem

Problems need to be approached in a systematic manner, with teams tackling solvable problems with a direct economic impact, allowing for immediate feedback together with recognition of the contribution made by individual participants.

P

Preparation

Additional training on creative thinking and statistical processes is needed to give participants a greater appreciation of the diversity of the process. This training must quickly be extended beyond the immediate accounting circle to include employees at supervisory levels and also those who are involved at the data input stage.

PRAISE — Six-Step Quality Improvement Process

STEP	ACTIVITY	ELEMENTS
1	**Problem Identification**	• Areas of customer dissatisfaction • Absence of competitive advantage
2	**Ranking**	Prioritise problems and opportunities by — 1. Perceived importance, and 2. Ease of measurement and solution
3	**Analysis**	• Ask "Why?" to identify possible causes. Keep asking 'Why?' beyond the mere symptoms and avoid jumping to premature conclusion • Ask 'What?' to consider potential implications • Ask 'How much?' to quantify cause and effect
4	**Innovation**	• Use creative thinking to generate potential solutions • Operationalise these solutions by identifying: 1. Barriers to implementation 2. Available enablers 3. People whose co-operation must be sought
5	**Solution**	• Implement the preferred solution • Take appropriate action to bring about the required changes • Reinforce with training and documentation back-up
6	**Evaluation**	• Monitor the effectiveness of actions • Establish and interpret performance indicators to track progress towards objectives • Identify the potential for further improvements and return to Step 1

Difficulties in PRAISE Analysis

STEP	DIFFICULTIES	REMEDIES
1. Problem Identification	• Effects of a problem are apparent, but the problems themselves are difficult to identify • Problem may be identifiable, but it is difficult to identify a measurable improvement opportunity • Some problems are too vague to define e.g., morale, communication, productivity	• Participative approaches like brainstorming, multi-voting, panel discussion • Quantification and precise definition of problems
2. Ranking	• Difference in perception of individuals in ranking • Difference in preferences based on functions, e.g., production, finance, marketing • Lack of consensus between individuals	• Participative approach • Subordination of individual to group interest
3. Analysis	• Adoption of adhoc approaches and quick-fix solutions	• Lateral Thinking • Brainstorming
4. Innovation	• Lack of creativity or expertise • Inability to operationalise ideas, i.e., convert thoughts into action points	• Systematic evaluation of all aspects of each strategy
5. Solution	• Resistance from middle managers	• Effective internal communication • Training of personnel and managers • Participative approach
6. Evaluation	• Problems in implementation • Lack of measurable data for comparison of expectations with actuals	• Effective Control System to track actuals • Feedback system

CENTRAL PILLARS OF PRAISE

Central to the PRAISE system are — (a) **Quality Control** — the search for continuous improvements in quality — and (b) **Total Employee Involvement** — the co-operation and commitment of employees. This dual approach provides a single focus — **the customer** — whose increased satisfaction remains the primary goal of the procedure.

Three-Point Action Plan for PRAISE Implementation

1

Small to Big Issues

Big improvement opportunities are generally complex and require extensive interdepartmental co-operation. The choice of a relatively small problem in the first instance provides a greater chance of success. The TQM team has to proceed from small to big issues gradually.

2

Solvable Problem

The problem selected should not be trivial, but should be one with a potential impact and a clear improvement opportunity. Measurable progress towards implementation should be accomplished within a reasonable time to maintain the motivation of participants and advertise the success of the improvement itself.

3

Recognition of Participants

The successful projects and team members should receive appropriate recognition. Prominent individuals should be rewarded for their efforts through monetary/non-monetary prices as a measure of personal recognition and as encouragement to others.

Pareto Analysis — The 80:20 Rule

Pareto Analysis is a rule that recommends focus on the most important aspects of decision making in order to simplify the process. It is based on the **80:20 rule** first observed by Vilfredo Pareto, a nineteenth-century Italian economist. He noticed that 80% of the wealth of Milan was owned by 20% of its citizens.

USEFULNESS OF PARETO ANALYSIS

Pareto analysis provides the mechanism to control and direct effort by fact and not by emotions. It helps clearly establish top priorities and identify both profitable and unprofitable targets.

1. Prioritize problems, goals, and objectives to identify root causes
2. Select and define key quality improvement programs
3. Select key customer relations and service programs
4. Select key employee relations improvement programs
5. Select and define key performance improvement programs
6. Maximize research and product development time
7. Verify operating procedures and manufacturing processes
8. Boost/Assist product or services sales and distribution
9. Allocate physical, financial and human resources

Applications of Pareto Analysis

Pricing of a Product

In multi-product firms, approximately 20% of products may account for about 80% of total sales revenue. Pareto Analysis helps top management delegate pricing decisions for approximately 80% of products to lower management, concentrating on the 20% essential for survival.

Customer Profitability Analysis

Approximately 20% of customers generate 80% of the profit. Analysis is useful for evaluation of customer portfolio and decision making such as whether to continue serving a customer group.

ABC Analysis — Stock Control

Approximately 20% of total quantity of stock may account for about 80% of its value. By concentrating on small proportion of stock items, a firm can control most of the monetary investment in stocks.

Activity Based Costing

20% of an organization's cost drivers are responsible for 80% of the total cost. By analysing, monitoring and controlling those cost drivers that cause most cost, better control of overheads is obtained.

Quality Control

80% of reported problems can usually be traced to 20% of the various underlying causes. By concentrating efforts on rectifying the vital 20%, one can have the greatest immediate impact on product quality.

Pareto Analysis — Worked Example (Toy Company)

PROBLEM

A Toy company performs a Pareto analysis, given a set of defect types and frequencies of their occurrence. The sample data consists of information about 84 defective items. The items have been classified by their defect types as follows:

DEFECT TYPE	NO. OF ITEMS
Cracks (due to mishandling of raw material)	10
Improper shapes	8
Incomplete	8
Surface scratches	53
Others (due to bad quality raw material)	5

SOLUTION

DEFECT TYPE	NO. OF ITEMS	(%)	CUMULATIVE %
Surface scratches	53	63.10	63.10
Cracks	10	11.90	75.00
Improper shape	8	9.52	84.52
Incomplete	8	9.53	94.05
Others	5	5.95	100.00

Interpretation: The purpose of Pareto analysis in this example is to direct attention to the area where best returns can be achieved by solving most of the quality problems, perhaps just with a single action. In this case, use of good quality raw material (say plastic) may solve 63% of the problem; if raw material is handled properly, at least 75% of the problems may be overcome.

Case Study 2: The Customer Knows Best — AtlantiCare

CASE STUDY

TQM in Healthcare — AtlantiCare

Background: New Jersey-based healthcare provider AtlantiCare manages 5,000 employees at 25 locations. Patient satisfaction is the single-most important aspect of the healthcare industry. Managers wanted to implement improvements across the board.

Challenge: TQM isn't an easy management strategy to introduce; many attempts fall flat because firms maintain natural barriers to full involvement. Middle managers tend to complain their authority is being challenged when boots on the ground are encouraged to speak up.

Approach: The firm chose to adopt a **plan-do-check-act cycle (PDCA Cycle)**, revealing gaps in staff communication — which subsequently meant longer patient waiting times and more complaints. Instead of information trickling down from top-to-bottom, all employees were given freedom to provide vital feedback at each and every level.

Implementation: At orientation, staff receive a crash course in the company's **performance excellence framework** — a management system that organises the firm's processes into five key areas: quality, customer service, people and workplace, growth and financial performance. As employees rise through the ranks, this emphasis on improvement follows, so that managers can operate within the company's **tight-loose-tight process management style**.

Benchmark Goals: Better engagement at the point of delivery, increasing clinical communication, identifying and prioritising service opportunities.

METRIC	BEFORE	AFTER
Profits	$280 million	$650 million
Repeat customers	—	Tripled
Market share	—	Six-year high
Patients serviced	—	Dwarfed state numbers

⚡ Quick Revision — Section 2.3

- **TQM:** Philosophy of continuously improving quality of all products and processes; do things right the first time.
- **Three elements:** Total (everyone involved), Quality (meeting customer requirements), Management (quality can and must be managed).
- **Eight Principles:** Customer Focus, Total Employee Commitment, Process Approach, Integrated System, Strategic & Systematic Approach, Continual Improvement, Fact-based Decision-making, Communications.
- **Eight Steps:** Identify customers → expectations → decision-making requirements → perceived problems → benchmarking → customer feedback → identify improvement opportunities → implement improvement process.
- **6 C's:** Commitment, Culture, Continuous Improvement, Co-operation, Customer Focus, Control.
- **4 P's:** People, Process, Problem, Preparation.
- **PRAISE:** Problem Identification → Ranking → Analysis → Innovation → Solution → Evaluation.
- **Pareto Analysis:** 80:20 rule; focus on vital few.

2.4 Lean Accounting

LEAN MANUFACTURING · TOYOTA PRODUCTION SYSTEM · PRINCIPLES · PRACTICES · TOOLS

Lean Manufacturing

Idle resources have always remained the fiercest enemy of every cost manager. It could be idle labour, idle machines, idle facilities, idle stocks; anything and everything remaining idle tantamounts to an undue fixed burden which diminishes the impact of value chain. **Lean management** is one fascinating concept that supports the efforts of the Cost Manager in the elimination of waste of any kind.

FUNDAMENTAL PRINCIPLE OF LEAN

"Process, next in line, is the most important customer; Process, just before in line, is the most important vendor."

Historical Evolution of Lean

Benjamin Franklin

Contributed greatly to waste reduction philosophy/techniques. *"A penny saved is a penny gained."* Henry Ford cited Franklin as a major influence on his business practices.

Taiichi Ohno (1912–1990)

Symbol of Japan's manufacturing resurgence after World War II. Joined Toyota Automatic Loom Works. Felt there was no reason other than inefficiency and wastefulness why Toyota's productivity should be lower than Detroit's. Set out to eradicate inefficiency and eliminate waste. This became the core of the **Toyota Production System (TPS)** developed between mid-1940s and mid-1970s.

Key TPS Elements

Muda (elimination of waste), **Jidoka** (injection of quality), and **Kanban** (tags used as part of a system of just-in-time stock control).

Lean Manufacturing

Evolved from the Toyota Production System. Cornerstone is the elimination of waste from processes with a mindset of continuous improvement. Systematic elimination of waste by focusing on production costs, product quality and delivery, and worker involvement.

Paradigm Shift — Batch and Queue vs Single-Piece Flow

TRADITIONAL "BATCH AND QUEUE"

- Mass-production of large inventories of products in advance
- Based on potential or predicted customer demands
- Large production batches viewed as beneficial
- Inventories act as security blanket
- Setup costs amortized over large batches

→

LEAN "SINGLE-PIECE FLOW, PULL PRODUCTION"

- Production activities rearranged so processing steps are immediately adjacent
- Continuous and single piece flow
- Shift in demand accommodated immediately
- No loss of inventory stockpiles
- Elimination of waste inherent in long setups

Three Fundamental Reasons Companies Embrace Lean

1

Competitive Globalized Market

Requires companies to lower costs to increase margins and/or decrease prices through elimination of all non-value-added aspects of the enterprise.

2

Just-In-Time Demands

Meeting rapidly changing customer demands through rapid product mix changes and increases in manufacturing velocity is the key for survival.

3

High and Consistent Quality

Goods must be of high and consistent quality. Lean manufacturing facilitates this goal.

Major Elements of Lean

Lean is centered on preserving value with less work. It is a variation on the theme of efficiency based on optimizing flow. Major elements derive from the Toyota Production System (TPS). Lean methods and other improvement techniques such as Six Sigma, Total Quality Management, and Theory of Constraints have dominated manufacturing trends in the United States since the 1980s. Lean is the most commonly used approach.

LEAN IS A JOURNEY, NOT A DESTINATION

Lean is a process, a continuous journey, with renewable goals; it is not a destination. Once you achieve your current targets, it begins all over again with new ones. It never ends because you can always make it better. **Lean is doing more with less** — achieving more by using fewer resources (people, machine, material, capital, etc.) and doing only those activities that are essential to satisfy customer orders, and doing them well.

Benefits of Lean

- **Reduced lead times**
- **Improved delivery performance**
- **Shorter order-to-cash cycle**
- **Increased sales revenue**

- **Increased profits**
- **Lower operating costs**
- **Reduction in inventory (greater inventory turnover)**
- **Improved customer satisfaction**
- **Enhanced supplier relationships**
- **Greater employee morale and retention**
- **Improved product and service quality**
- **Reduced physical space requirement**
- **Availability of additional working capital**

Lean Accounting

DEFINITION

Lean Accounting is the application of lean thinking to all accounting and finance processes and systems. It is an essential component of a successful lean transformation for any organization.

Lean accounting uses a method that categorizes costs by **value stream** rather than by department. This approach "provides the basis for sound management decisions". Value stream accounting is "tracking revenue and the associated variable costs required to generate those sales."

Features of Value Stream Accounting

- **Costs calculated weekly**
- **No distinction between direct or indirect costs — all costs of value stream are direct costs**
- **Value stream costs include labour, materials, production support, machines and equipment, operation**

support, facilities and maintenance

More accurate picture by elimination of unnecessary costs outside control of value stream managers

Lean accounting groups together costs that fall outside of the value stream as **"business sustaining costs"** that do not get included in value stream costs. This helps businesses find better price points for products and do further research into high-cost areas.

Three Principles of Lean Accounting

1

Customer Value

Delivering the relevant and reliable information in a timely manner to all users of the information inside the organization.

2

Continuous Improvement

Improving accounting processes, cross-functional business processes and the information used inside the business for analysis and decision making.

3

Respect for People

Adopting a learning attitude by seeking to understand root causes of business problems and issues in a cross-functional, collaborative manner.

WHY LEAN ACCOUNTING?

Most companies embarking on lean manufacturing find that their accounting processes and management methods are at odds with the lean changes they are making. Traditional accounting and management methods were designed to support traditional manufacturing — they are based upon the mass production concept. Lean manufacturing breaks the rules of mass production, and so the traditional accounting and management methods warrant due modifications.

Purpose & Benefits of Lean Accounting

PURPOSE	BENEFITS
Tell managers about the flow through the Value Stream	Creating capacity in accounting by eliminating waste in accounting processes
Tell managers about the capacity for extra work in the Value Stream	Accounting fully participating in cross-functional continuous improvement
Tell managers about the incremental costs of alternative decisions and actions	Flowing relevant and reliable information to all internal stakeholders for effective decision making
—	Leveraging accounting's analytical skills as lean financial coaches throughout the organization

Principles, Practices and Tools of Lean Accounting

SL	PRINCIPLES	PRACTICES	TOOLS OF LEAN ACCOUNTING
1	**Lean & simple business accounting**	Continuously eliminates waste from the transactions, processes, reports, and other accounting methods	• Value stream mapping (current & future state) • Kaizen (lean continuous improvement) • PDCA (Planning, Doing, Checking and Acting) problem solving
2	**Accounting processes that support lean transformation**	Management control & continuous improvement	• Performance Measurement Linkage Chart; linking metrics for cell/process, value streams, plant & corporate reporting to the business strategy, target costs, and lean improvement • Value stream performance boards containing break-through and continuous improvement projects • Box scores showing value stream performance
3	**Inventory valuation**	Compliance with Sarbanes-Oxley Act of 2002 (SOX). SOX is a U.S. federal law requiring all public companies doing business in the United States to comply with the regulation.	• Simple methods to value inventory without the requirement for perpetual inventory records and product costs can be used when the inventory is low and under visual control.

LEAN ACCOUNTING IS ITSELF LEAN

While Lean Accounting is still a work-in-process, there is now an agreed body of knowledge that is becoming the standard approach to accounting, control, and measurement. These principles, practices, and tools have been implemented in a wide range of companies at various stages on the journey to lean transformation. **Lean Accounting is itself lean, low-waste, and visual**, and frees up finance and accounting personnel time so they can become actively involved in lean change instead of being merely "bean counters."

⚡ Quick Revision — Section 2.4

- **Lean Manufacturing:** Systematic elimination of waste by focusing on production costs, product quality and delivery, and worker involvement.
- **Toyota Production System (TPS):** Taiichi Ohno's contribution — Muda (waste elimination), Jidoka (quality injection), Kanban (JIT stock control).
- **Paradigm Shift:** Batch and Queue → Single-Piece Flow, Pull Production.
- **Three reasons for Lean:** Competitive globalized market, JIT demands, high and consistent quality.
- **Lean is doing more with less.**
- **Lean Accounting:** Application of lean thinking to all accounting and finance processes and systems.
- **Value Stream Accounting:** Costs categorized by value stream rather than department; all costs of value stream are direct costs.
- **Three Principles:** Customer Value, Continuous Improvement, Respect for People.
- **Business Sustaining Costs:** Costs outside the value stream, not included in value stream costs.

2.5 Six Sigma

CONCEPT · DMAIC · DMADV · SIGMA LEVELS · ROLES · CASE STUDY

Concept of Six Sigma

DEFINITION

Six Sigma is a set of practices originally developed by Motorola to systematically improve processes by eliminating defects. A defect is defined as non-conformity of a product or service to its specifications.

While the particulars of the methodology were originally formulated by **Bill Smith at Motorola in 1986**, Six Sigma was heavily inspired by six preceding decades of quality improvement methodologies such as quality control, TQM, and Zero Defects.

Six Sigma Asserts

a

Reduce Variation

Continuous efforts to reduce variation in process outputs is key to business success.

b

Measurable Processes

Manufacturing and business processes can be measured, analyzed, improved and controlled.

c

Total Commitment

Succeeding at achieving sustained quality improvement requires commitment from the entire organization, particularly from top-level management.

SIX SIGMA QUALITY LEVEL

```
6σ = 99.99966% defect-free = 3.4 defects per million opportunities (DPMO)
```

A six-sigma process is one in which **99.99966%** of all opportunities to produce some features of a part are statistically expected to be free of defects. It is a disciplined, data-driven approach and methodology for eliminating defects (driving toward six standard deviations between the mean and the nearest specification limit) in any process — from manufacturing to transactional and from product to service.

Sigma Level Table

SIGMA LEVEL	DEFECTS PER MILLION OPPORTUNITIES (DPMO)	PERCENTAGE DEFECTS	YIELD
1σ	691,462	69%	31%
2σ	308,538	31%	69%
3σ	66,807	6.7%	93.3%
4σ	6,210	0.62%	99.38%
5σ	233	0.023%	99.977%
6σ	3.4	0.00034%	99.99966%

Two Key Methodologies

Six Sigma has two key methodologies: **DMAIC** and **DMADV**, both inspired by W. Edwards Deming's Plan-Do-Check-Act Cycle. DMAIC is used to improve an existing business process, and DMADV is used to create new product or process designs for predictable, defect-free performance.

DMAIC — EXISTING PROCESS

- **D**efine the process improvement goals that are consistent with customer demands and enterprise strategy
- **M**easure the current process and collect relevant data for future comparison
- **A**nalyze to verify relationship and causality of factors. Determine what the relationship is, and attempt to ensure that all factors have been considered
- **I**mprove or optimize the process based upon the analysis using techniques like Design of Experiments
- **C**ontrol to ensure that any variances are corrected before they result in defects.

VS

DMADV — NEW PRODUCT/PROCESS

- **D**efine the goals of the design activity that are consistent with customer demands and enterprise strategy
- **M**easure and **I**dentify CTQs (critical to qualities), product capabilities, production process capability, and risk assessments
- **A**nalyze to develop and design alternatives, create high-level design and evaluate design capability to select the best design
- **D**esign details, optimize the design, and plan for design verification. This phase may require simulations

Set up pilot runs to establish process capability, transition to production and thereafter continuously measure the process and institute control mechanisms

- **V**erify the design, set up pilot runs, implement production process and handover to process owners

Six Sigma Roles

ROLE	RESPONSIBILITIES
Executive Leadership	Includes CEO and other key top management team members. Responsible for setting up a vision for Six Sigma implementation. Empower other role holders with the freedom and resources to explore new ideas for breakthrough improvements.
Champions	Responsible for Six Sigma implementation across the organization in an integrated manner. Drawn from upper management. Act as mentors to Black Belts. At GE, this level of certification is now called "Quality Leader".
Master Black Belts	Identified by champions; act as in-house expert coaches for the organization on Six Sigma. Devote 100% of their time to Six Sigma. Assist champions and guide Black Belts and Green Belts. Ensure integrated deployment of Six Sigma across various functions and departments.
Experts	Primarily used within Aerospace and Defense Business Sectors. Work across company boundaries, improving services, processes, and products for their suppliers, their entire campuses, and for their customers. Raytheon Incorporated was one of the first companies to introduce Experts.
Black Belts	Operate under Master Black Belts to apply Six Sigma methodology to specific projects. Devote 100% of their time to Six Sigma. Primarily focus on Six Sigma project execution, whereas Champions and Master Black Belts focus on identifying projects/functions for Six Sigma.
Green Belts	Employees who take up Six Sigma implementation along with their other job responsibilities. Operate under the guidance of Black Belts and support them in achieving the overall results.
Yellow Belts	Employees who have been trained in Six Sigma techniques as part of a corporate-wide initiative, but have not completed a Six Sigma project and are not expected to actively engage in quality improvement activities.

Case Study 3: Six Sigma Reduces Costs & Improves

CASE STUDY

Ford Motors — Six Sigma Black Belt Project

Background: Ford's balanced scorecard system provides reporting tools with monthly values versus target figures. Using this evaluation system, the auto-maker classifies data as:

- **Green:** measures are on or over target
- **Yellow:** metrics are under target, but better than last year
- **Red:** results are under target

The Red Flag: In fall 2009, data for body paint consumption for the Focus and Kuga were classified as red. Basecoat paint consumption stood at 3.74 kg/unit in 2007, while current consumption was 4.18 kg/unit. Ford officials selected this improvement as a Six Sigma Black Belt project.

Project Team: Of the plant's 7,000 employees, more than 50 are Six Sigma Black Belts and another 400 are trained as Green Belts. Team leader and Six Sigma Black Belt Martin Fischer based selections on candidate's responsibilities, subject-matter expertise and process ownership.

Define: The goals of the project were threefold:

1. Reduce costs: Reduce paint consumption to lower production costs (predicted $1.5 million annually)
2. Improve customer satisfaction: Improve process capability to better meet customer needs (target: 127,000 ppm reduction)
3. Lower environmental impact: Reduce solvent consumption to achieve a better VOC balance (projected 50,000 kg annually in VOC savings)

Measure: Value-stream mapping served as a visual tool to understand the flow of material and paint application process. Cause-and-effect diagrams were useful for identifying root causes. Data collection plan focused on six critical Xs: daily basecoat consumption, paint film thickness, consumption per robot, consumption per manual painter, first-time through rate versus consumption, and application equipment.

Analyze: The Six Sigma team conducted a '5 Why analysis' and test trials. Results showed factors one, two, four, and five were not significant. Factor three (consumption per robot) showed an increase for the liftgate robot. Factor six (application equipment) — a damaged solvent recovery valve was causing direct paint flow from the color changer to the recycling tank.

Improve: Four potential improvement actions were identified for the defective solvent recovery valve:

1. Replace plastic valves with stainless steel valves (45% performance improvement)
2. Create an automatic recovery valve check system
3. Check the valves weekly
4. Eliminate the solvent recovery process

For the liftgate robot factor, three potential solutions were identified. Testing showed changing to manual only painting processes for interior painting would save 0.28 kg/unit.

Control: The new monitoring system and standard operating procedures help the Saarlouis plant sustain the results. All standard operating procedures are part of the plant's ISO 9001 compliant quality management system and are included in routine audits.

Result: Basecoat paint consumption dropped from 4.18 kg/unit to a mean consumption of 3.3 kg/unit. The team achieved every project goal and even exceeded the expected cost reduction by a half million dollars annually. The project earned finalist honours in ASQ's International Team Excellence Awards (ITEA) and presented at the 2011 World Conference on Quality and Improvement.

KEY LEARNING FROM FORD SIX SIGMA

"They worked together as a team in an excellent way, proving the power of a team and the sum of competencies in a team." The cross-functional team included members from production, maintenance, quality, manufacturing engineering, and the supplier.

⚡ Quick Revision — Section 2.5

- **Six Sigma:** Set of practices originally developed by Motorola to systematically improve processes by eliminating defects.
- **Defect:** Non-conformity of a product or service to its specifications.
- **6σ quality:** 99.99966% defect-free = 3.4 DPMO.
- **Two methodologies:** DMAIC (improve existing) and DMADV (create new designs).
- **DMAIC:** Define → Measure → Analyze → Improve → Control.
- **DMADV:** Define → Measure & Identify → Analyze → Design → Verify.
- **Roles:** Executive Leadership, Champions, Master Black Belts, Experts, Black Belts, Green Belts, Yellow Belts.
- **Ford Six Sigma Project:** Reduced basecoat paint consumption from 4.18 to 3.3 kg/unit; exceeded expected cost reduction by $0.5 million annually.

★ Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Quality

Characteristic or combination of characteristics that distinguishes one article from the other or goods of one manufacturer from that of competitors or one grade of product or service from another when both are the outcome of the same organisation.

Quality Management

"Coordinated activities to direct and control an organization with regard to quality" (ISO 9000:2000).

Prevention Costs

All costs incurred in the process of preventing poor quality from occurring.

Appraisal Costs

Costs incurred in the process of uncovering defects.

Internal Failure Costs

Costs associated with discovering poor product quality before the product reaches the customer site.

External Failure Costs

Costs incurred when inferior products are delivered to customers.

Total Quality Management

A philosophy of continuously improving the quality of all the products and processes in response to continuous feedback for meeting the customers' requirements.

Lean Accounting

The application of lean philosophy to all accounting and finance processes and systems.

Six Sigma

A set of techniques and tools for process improvement.

Multiple Choice Questions

WITH ANSWERS

1. TQM stands for

A. Technical Quantitative Management

B. Total Quality Management

C. Theory of Queuing Management

D. None of the Above

Answer: B

2. Four Ps of Total Quality Management

A. Principles, Project, Problem, & Process

B. People, Process, Problem & Preparation

C. Product identification, Product quality, Product utility & Product expectation

D. None of the above

Answer: B

3. PRAISE stands for

A. Appreciating someone

B. Product, Recognition, Adoption, Invention, Solution & Evaporation

C. Problem Identification, Ranking, Analysis, Innovation, Solution & Evaluation

D. None of the above

Answer: C

4. Six Sigma is about

A. Quality systems

B. Quality control process

C. Statistical technique

D. None of the above

Answer: A

5. DMADV is a methodology associated with

A. Pareto Analysis

B. PRAISE

C. Six Sigma

D. None of the above

Answer: C

6. Pareto analysis recognizes

A. 80:20 Rule

B. 50:50 Rule

C. 20:80 Rule

D. None of the above

Answer: A

7. Cost of Rework is a cost related to

A. Internal failure

B. Appraisal

C. Prevention

D. None of the above

Answer: A

8. The cost incurred to ensure that failures do not happen

A. External failure cost

B. Internal failure cost

C. Prevention cost

D. None of the above

Answer: C

9. Which of the following is not the quality parameter for service organizations?

A. Consistency B. Friendliness C. Durability

D. Promptness

Answer: C

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
B	B	C	A	C	A	A	C	C

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

1

Write an elaborative note on managing quality in a competitive environment.

2

Detail and discuss the costs of quality.

3

"Total Quality Management is vital for growth" — Justify.

4

Narrate the steps for implementing the Total Quality Management.

5

What do you understand by 6C's?

6

State the relevance of PRAISE analysis.

7

Comment about the utility of Pareto Analysis.

8

Discuss the significance of lean accounting.

9

Narrate the principles, practices, and tools of lean accounting.

10

Define and discuss DMAIC.

11

Define and discuss DMADV.

Σ Comprehensive Numerical Questions

PRACTICE PROBLEMS WITH WORKED-OUT SOLUTIONS

Problem 1 — Zebra Limited

PROBLEM

Zebra Limited introduced a quality improvement program and the following results are observed:

PARTICULARS	2022-23 (₹ LAKHS)	2023-24 (₹ LAKHS)
Sales	10,000	10,000
Scrap	100	50
Rework	650	550
Production inspection	250	325
Product Warranty	500	250
Quality Training	125	250
Materials inspection	120	90

Required: (a) Classify the quality costs and express each class as a percentage of sales. (b) Compute the increase in the amount of profit due to quality improvement.

SOLUTION (A)

COST CLASSIFICATION	2022-23 (₹ LAKHS)	2023-24 (₹ LAKHS)	AS % TO SALES 2022-23	AS % TO SALES 2023-24
Prevention Costs	125	250	1.25%	2.50%
Appraisal Costs	370	415	3.70%	4.15%
Internal Failure Costs	750	600	7.50%	6.00%
External Failure Costs	500	250	5.00%	2.50%
Total Quality Costs	**1,745**	**1,515**	**17.45%**	**15.15%**

Solution (b): Increase in profits during 2023-24 due to quality improvement = **₹230 lakhs**

Problem 2 — Component Supplier Selection

PROBLEM

A Company manufactures a single product, which requires two components. The Company purchases one of the components from two suppliers: X Ltd and Y Ltd. The price quoted by X Ltd is ₹180 per 100 units and it is found that on an average 3% of the total receipt from this supplier is defective. The corresponding quotation from Y Ltd is ₹174 per 100 units, with a defect rate of 5%. If the defectives are not detected, they are utilized in production causing a damage of ₹180 per 100 units of the defective component.

The Company intends to introduce a system of inspection for the components on receipt. The Inspection cost is estimated at ₹24 per 100 units of the component. Such an inspection will be able to detect only 90% of the defective components received. No payment will be made for components found to be defective in Inspection.

Required: (a) Justify the Inspection at the point of receipt. (b) Assuming a total requirement of 10,000 units, ascertain the lowest supplier.

SOLUTION

Recommendation:

a. Inspection at the point of receipt is **not advantageous**, due to additional cost per 100 good components, i.e., (205.22 – 191.13) = 14.09 in case of X Ltd, and (200.07 – 192.63) = 7.44 in case of Y Ltd.
b. Purchase from X Ltd. without inspection is cheaper, as there is a cost saving of ₹1.50, i.e., (192.63 – 191.13) per 100 good components.

Problem 3 — Rags Ltd. (Sports Shoes)

PROBLEM

Rags Ltd. manufactures and sells premium quality sports shoes in India. Finished products show some rectifiable defects. These problems can be detected and rectified during internal inspection. Inspection cost is ₹30 per unit. Rectification/Rework cost is ₹18 per unit.

During 2023, 60,000 pairs of shoes were manufactured and sold. After inspection, defect was detected in respect of 5% of output. After sales, customers reported defects in respect of 6% of output. These shoes were received back from customers at a transportation cost of ₹10 per pair. Due to negative publicity arising out of sale of defective materials, loss in sales is expected in next year to the extent of 5% of external failures.

Required: (a) Analyze the cost of quality showing its elements separately with working. (b) If the selling price per pair of shoes is ₹600 and variable cost is 60% of sales, fixed cost is ₹5,50,000 p.a., prepare the profitability statement for the product during 2023.

SOLUTION

PARTICULARS	AMOUNT (₹)
Total Quality Cost	₹19,98,000

PARTICULARS	AMOUNT (₹)
Profit	₹1,18,52,000

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 2

ABBREVIATION	FULL FORM
DMAIC	Define – Measure – Analyse – Improve – Control
DMADV	Define – Measure – Identify – Analyse – Design – Verify
DPMO	Defects Per Million Opportunities
ISO	International Organization for Standardization
PRAISE	Problem Identification – Ranking – Analysis – Innovation – Solution – Evaluation
PDCA	Plan-Do-Check-Act
PDSA	Plan-Do-Study-Act
QA	Quality Assurance
QC	Quality Control
SIPOC	Suppliers, Inputs, Process, Outputs, Customers
SOFP	Sales Operation and Financial Planning
TEI	Total Employee Involvement
TQM	Total Quality Management
4P's	People-Process-Problem-Preparation
TPS	Toyota Production System
VOC	Volatile Organic Compounds
CTQ	Critical to Qualities
S&OP	Standard Operating Planning Process

Strategic Cost Management — Master Revision Handbook

Module 2 · December 2026 Edition

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2.1 Managing Quality in Competitive Environment

2.2 Cost of Quality

2.3 Total Quality Management

2.4 Lean Accounting

2.5 Six Sigma

STUDY TOOLS

Case Studies (AtlantiCare, RIMS, Ford)

Formula Boxes

Quick Revision Sheets

MCQs with Reasoning

Numerical Problems

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Decision Making Techniques

Module 3 · Case Study-Based Approach

EDITION
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PAPER
Paper 16 · Strategic Cost Management

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MODULE WEIGHTAGE
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✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

03

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 3 INCLUDES

3.1 Decisions Involving Alternative Choices · **3.2** Pricing Decisions and Strategies · **3.3** Transfer Pricing · **3.4** Relevant Cost Analysis · **3.5** Target Costing · **3.6** Product Life Cycle Costing · **3.7** Asset Life Cycle Costing · **3.8** Decision Making using Probability.

Module Learning Objectives

After studying this module, the students will be able to:

1

Proficient Knowledge

Proficient knowledge on tools and techniques — Marginal Costing, Transfer Pricing, Target Costing, Life Cycle Costing, etc.

2

Application of Techniques

Application of various techniques of Strategic Cost Management across manifold enterprises.

3

Business-Level Strategies

Developing business level Cost Management Strategies for gaining Competitive Advantage.

25%	8	25+
WEIGHTAGE IN SECTION A	CORE SUB-TOPICS	ILLUSTRATIONS
Highest weightage module	3.1 to 3.8	Worked examples

8
CASE STUDIES
Real-world applications

SYLLABUS STRUCTURE NOTE

Module 3 carries the **highest weightage (25%)** within Section A. It is the core decision-making module covering marginal costing, pricing strategies, transfer pricing, relevant costs, target costing, life cycle costing, and probability-based decision making. This module is case-study based and requires strong conceptual understanding plus numerical application.

3.1

Decisions Involving Alternative Choices

DECISION MAKING PROCESS · COST BEHAVIOUR · CONTRIBUTION · BEP · PROFIT PLANNING · KEY FACTOR ANALYSIS

Decision Making — Concept

Decision making is the outcome resulting from the process of evaluation of the available alternatives and choosing the best. Business decisions involve selecting between alternatives such as make or buy, take or refuse orders, place special orders, export versus local, shut down or continue, expand or contract, replace present equipment with new machinery, sell at split-off point or process further, etc.

ROBERT FROST — THE ROAD NOT TAKEN

"Two roads diverged in a wood, and I took the one less travelled by, and that has made all the difference." But unfortunately, not every decision is as simple as "Let's just take this path and see where it goes" — especially when a decision is related to business.

Seven Steps of Decision Making

1. Identify the Problem

Clearly define the problem you need to solve or the question you need to answer. If you mis-identify the problem or if the problem is too broad, you'll knock the decision train off the track before it even leaves the station. Make the goal measurable and timely.

2. Gather Relevant Information

Do an internal assessment seeing where your organization has succeeded and failed in areas related to your decision. Also seek information from external sources.

3. Identify the Alternatives

With relevant information at your fingertips, identify possible solutions. There is usually more than one option to consider when trying to meet a goal.

4. Evaluate the Evidence

Evaluate the evidence for or against said alternatives. See what companies have done in the past to succeed in these areas. Identify potential pitfalls for each alternative and weigh against possible rewards.

5. Choose from Alternatives

This is the part where you actually make the decision. Hopefully you've identified and clarified what decision needs to be made, gathered all relevant information, and developed and considered the potential paths to take.

6. Implement the Choice

Once you've made your decision, act on it! Develop a plan to make your decision tangible and achievable, and then assign tasks to your team.

7. Review the Results

After a predetermined amount of time, take an honest look back at your decision. Did you solve the problem? Did you meet your goals? If so, take note of what worked. If not, learn from mistakes as you begin the decision-making process again.

Cost Behaviour

DEFINITION

Cost Behaviour refers to the changes in input costs in relation to the level of production. Costs may increase or decrease proportionately with the level of production (variable costs), or they may not change at all (fixed costs). Some costs, semi-variable in nature, have both variable and fixed elements.

General Behaviour Pattern of Elements of Cost

ITEM	TRACEABILITY	BEHAVIOUR
Raw Materials	Direct	Variable
Process Materials and Chemicals	Direct	Variable
Utilities (Power, Fuel, etc.)	Direct	Variable
Direct Employee Cost	Direct	Variable
Direct Expenses	Direct	Variable
Consumables, Stores & Spares	Indirect	Semi Variable
Repairs & Maintenance	Indirect	Semi Variable
Quality Control Expenses	Indirect	Semi Variable
Research & Development Expenses	Indirect	Discretionary
Technical Know-how Fee / Royalty	Indirect	Contractual
Depreciation / Amortization	Indirect	Fixed
Primary Packing Cost	Direct	Variable
Administrative Overheads	Indirect	Semi Fixed
Selling and Distribution Overheads	Direct / Indirect	Variable / Semi Variable
Interest and Financing Charges	Indirect	Semi Variable / Fixed

KEY INSIGHT

Variable Costs are controlled by means of **standards** at the operational level whereas Fixed Costs are controlled by means of **budgets** at the strategic level. Variable Costs are prone to control even at the lower level; whereas Fixed Costs are prone to better controls from the higher level.

The Concept of Contribution

CONTRIBUTION FORMULA

```
Contribution = Sales − Variable Costs
```

```
Profit = Contribution − Fixed Costs
```

```
PV Ratio = (Contribution ÷ Sales) × 100
```

```
Sales = Contribution ÷ PV Ratio
```

Contribution is the excess of Sales Value over Variable Cost. It represents the margin available to meet the Fixed Costs. Excess of Contribution over Fixed Cost denotes the Profit.

Illustration — ABL Model T Contribution Analysis

SL	PARTICULARS	DATA
1	Sales (Number)	900 units
2	Selling Price (₹/Piece)	7,00,000
3	Sales (₹ Lakhs)	6300.00
4	Variable Costs (₹ Per Piece) — Raw Material	4,34,000
5	Variable Expenses	1,26,000
6	Sub Total Variable Cost	5,60,000
7	Variable Cost as % to Selling Price	80.00%
8	Contribution per piece	1,40,000
9	Total Contribution (₹ Lakhs)	1260.00
10	Profit Volume Ratio	20.00%
11	Fixed Costs (₹ Lakhs)	882.00
12	Profit (₹ Lakhs)	378.00

Break Even Analysis

BEP FORMULAE

```
Break Even Point (Units) = Fixed Costs ÷ Contribution per unit
```

```
Break Even Point (₹) = Fixed Costs ÷ PV Ratio
```

```
Margin of Safety = Total Sales – Breakeven Sales
```

```
Profit = Margin of Safety × Profit Volume Ratio
```

BEP signifies the level of activity at which there is neither profit nor loss — where Total Revenues equal Total Costs and Contribution equals Fixed Costs.

ABL BEP Analysis

SL	PARTICULARS	DATA
1	Sales (₹ Lakhs)	6300.00
2	Contribution (₹ Lakhs)	1260.00
3	Profit Volume Ratio	20%
4	Fixed Costs (₹ Lakhs)	882.00
5	BEP Sales (₹ Lakhs) = (4/3)	4410.00
6	BEP Sales (Units) = (4410.00/7.00)	630 units

INTERPRETATION

ABL breaks even at a sale level of ₹4,410 lakhs (630 units, 70% of total sales). At this level, a contribution of ₹882 lakhs is generated which equals the Fixed Costs. The contribution accruing from margin of safety equals the profit which is ₹378 lakhs being 20% of ₹1,890 lakhs (₹6,300 – ₹4,410).

Profit Planning

PROFIT PLANNING FORMULAE

```
Contribution Target = Fixed Costs + Target Profit

Sales Target (₹) = Contribution Target ÷ PV Ratio

Sales Target (Units) = Sales Target (₹) ÷ Selling Price
```

ABL Profit Plan — Target Profit ₹400 Lakhs

SL	PARTICULARS	FORMULA / WORKINGS	RESULT
1	Profit Target (₹ Lakhs)	Intention	400.00
2	Contribution Target (₹ Lakhs)	Fixed Costs + Target Profit = 882 + 400	1282.00
3	Sales Target (₹ Lakhs)	Contribution ÷ PV Ratio = 1282 ÷ 20%	6410.00
4	Sales Target (Units)	64,10,00,000 ÷ 7,00,000	915.71 (≈916 units)

Key Factor Analysis (Limiting Factor Analysis)

DEFINITION

Key Factor Analysis enables allocation of available resources towards achieving maximum contribution and thereby maximum profits. When there is any limitation in relation to any of the input factors, the choice can be made by ascertaining the **contribution per unit of that factor of production which is limited** in the given situation.

The limiting factors could be any of the critical input resources such as scarce raw materials, skilled labour hours, or special machine hours. The product with the highest contribution per unit of the limiting factor is given the first preference followed by others.

Impact Analysis — Change in Production Levels

SL	PARTICULARS	EXISTING LEVEL	INCREASED LEVEL	DECREASED LEVEL
1	Number of Units	900	950	850
2	Sales @ ₹7,00,000 per unit (₹ Lakhs)	6300.00	6650.00	5950.00
3	Contribution @ 20% PV Ratio	1260.00	1330.00	1190.00
4	Fixed Costs (₹ Lakhs)	882.00	882.00	882.00
5	Profit (₹ Lakhs)	378.00	448.00	308.00
6	Profitability (%)	6.00	6.74	5.18
7	Impact on Profit (₹ Lakhs)	—	70.00	(70.00)
8	Impact on Profitability (% Points)	—	0.74	(0.82)

Evaluation of Alternatives — Model T vs Model P

SL	PARTICULARS	MODEL T	MODEL P
1	Number of Tractors	50	50
2	Selling Price (₹/Piece)	7,00,000	8,00,000
3	Variable Costs (₹ Per Piece)	5,60,000	6,40,000
4	Contribution per piece	1,40,000	1,60,000
5	Total Contribution (₹ Lakhs)	70.00	80.00
6	Differential Contribution (₹ Lakhs)	10.00	
7	Preferred Choice	**Model P**	

DECISION RULE

Between Model T and Model P, per unit contribution of Model P is higher at ₹1,60,000 in comparison to ₹1,40,000 of Model T. Hence the obvious choice for utilising the enhanced capacity falls on **Model P** whereby differential contribution of ₹10 lakhs is generated in comparison.

Illustration — Ashok Gears Ltd. (Computation of BEP & Profit Planning)

PROBLEM

Net Revenue ₹80,00,000; Expenses (including ₹40,00,000 Fixed Cost) ₹88,00,000; Net Loss ₹8,00,000. The manager believes an increase of ₹20,00,000 in fixed expenditure on advertising will increase sales substantially.

Required: (i) At what sales volume will the Company break even? (ii) What sales volume will result in a Net Profit of ₹4,00,000?

SOLUTION

PARTICULARS	WORKINGS	RESULT
Net Revenue	Given	₹80,00,000
Variable Expenses	₹88,00,000 – ₹40,00,000	₹48,00,000
Contribution	₹80,00,000 – ₹48,00,000	₹32,00,000
PV Ratio	₹32,00,000 ÷ ₹80,00,000	40%
Revised Fixed Cost	₹40,00,000 + ₹20,00,000	₹60,00,000
Break Even Sales	₹60,00,000 ÷ 40%	₹1,50,00,000
Required Sales for ₹4,00,000 profit	(₹60,00,000 + ₹4,00,000) ÷ 40%	₹1,60,00,000

Illustration — Accept or Reject (Export Order)

PROBLEM

A manufacturing company currently operating at 80% capacity has received an export order from Middle East, which will utilise 40% of the capacity of the factory. The order has to be either taken in full and executed at 10% below the current domestic prices or rejected totally. Current sales and cost data: Sales ₹16.00 lakhs; Direct Material ₹5.80 lakhs; Direct Labour ₹2.40 lakhs; Variable Overheads ₹0.60 lakhs; Fixed Overheads ₹5.20 lakhs.

Required: Evaluate three alternatives.

SOLUTION

PARTICULARS	ALTERNATIVE (A) DOMESTIC ONLY	ALTERNATIVE (B) EXPORT + DOMESTIC	ALTERNATIVE (C) INCREASE CAPACITY
Capacity	80%	Export 40% + Domestic 60%	Export 40% + Domestic 80%
Sales (₹ Lakhs)	16.00	19.20	23.20
Variable Costs (₹ Lakhs)	8.80	11.00	13.35
Contribution (₹ Lakhs)	7.20	8.20	9.85
Fixed Costs (₹ Lakhs)	5.20	5.20	5.85
Profit (₹ Lakhs)	**2.00**	**3.00**	**4.00**

Suggestion: Alternative (C) with the highest profit of ₹4.00 lakhs works out to be the best.

Illustration — Limiting Factor Analysis & Optimum Mix

PROBLEM

A manufacturer has three products A, B, and C. Annual sales (units): A 6,000; B 6,000; C 750. Selling Price: A ₹20; B ₹31; C ₹39. Unit Cost: A ₹18; B ₹24; C ₹30. Processing time required per unit: A 1 hour; B 1 hour; C 2 hours. The firm is working at full capacity (13,500 processing hours per year). Fixed manufacturing overheads are absorbed into unit costs by a charge of 200% of variable costs.

Required: Calculate the best production programme and increase in net profit. Identify the shadow price of processing hour.

SOLUTION

PARTICULARS	A	B	C
Selling Price	₹20	₹31	₹39
Variable Cost (1/3 of Unit Cost)	₹6	₹8	₹10
Contribution per unit	₹14	₹23	₹29
Processing hours per unit	1	1	2
Contribution per hour	₹14	₹23	₹14.50
Preference	II	I	III

Optimum Mix: B = 8,000 units; C = 2,000 units; A = 1,500 units

Increase in Profit: ₹19,250. **Shadow Price:** ₹14 per processing hour.

Illustration — Optimum Product Mix (Multiple Limiting Factors)

PROBLEM

A farm of 96 hectares to grow tomatoes of four varieties: Royal Red, Golden Yellow, Juicy Crimson and Sunny Scarlet. 68 hectares suitable for all four; 28 hectares suitable only for Golden Yellow and Juicy Crimson. Market requires minimum 1,000 boxes of any one variety; not more than 22,750 boxes of any one variety. Area in complete hectares.

SOLUTION — PRODUCT PREFERENCE BASED ON CONTRIBUTION PER HECTARE

PRODUCT	CONTRIBUTION PER HECTARE (₹)	RANK
Royal Red	9,310	I
Golden Yellow	(770)	IV
Juicy Crimson	1,316	III
Sunny Scarlet	5,046	II

Optimum Mix: Royal Red 62 ha; Golden Yellow 10 ha; Juicy Crimson 18 ha; Sunny Scarlet 6 ha.

Maximum Profit: ₹2,63,484

⚡ Quick Revision — Section 3.1

- **Decision Making Process:** Identify problem → Gather information → Identify alternatives → Evaluate evidence → Choose → Implement → Review results.
- **Cost Behaviour:** Variable, Fixed, Semi-variable, Discretionary, Contractual.
- **Contribution = Sales – Variable Costs.** Profit = Contribution – Fixed Costs.
- **PV Ratio = (Contribution ÷ Sales) × 100.**
- **BEP (Units) = Fixed Costs ÷ Contribution per unit.** BEP (₹) = Fixed Costs ÷ PV Ratio.
- **Margin of Safety = Total Sales – Breakeven Sales.**
- **Key Factor Analysis:** Allocate resources based on contribution per unit of limiting factor.
- **Impact Analysis:** Study effect of changes in production, price, or cost on profit.

3.2 Pricing Decisions and Strategies

KEY PRICING STRATEGIES · PRODUCT DIFFERENTIATION · COST LEADERSHIP · YIELD MANAGEMENT · CASE STUDIES

Concept of Pricing

Pricing can make or break a business. Setting prices for products or services does not simply come down to costs — it is worthwhile to examine the idea of what the business wants its pricing strategy to achieve. Product prices determine the revenue stream of a business. Prices must be sufficient to cover the costs and profit. Before lowering prices, it is preferable to lower costs to maintain a stable profit margin and a stable cash flow into the business.

KEY INSIGHT

Any pricing strategy must be chosen to ensure maximum profit. Knowing the market and customer base are key elements to choosing the right pricing strategy.

Key Pricing Strategies

1

Profit Orientation

Profit-oriented pricing makes profit the top priority when figuring out the ideal price to set. Looks for the sweet spot that allows the entity to charge as much as possible without alienating potential customers. Can aim to maximize profit per unit or maximize overall profit by setting a competitive price that increases total units sold.

2

Competitor-Based Pricing

Uses the price that is set to appeal to customers and define the niche relative to competitors. Competitive pricing is charging a price comparable to other vendors selling the same item. Does not necessarily rely on setting a lower price. Can set a price in the same ballpark, or choose a higher price to send the message that the product is superior.

3

Demand Based Pricing

Based on the customer's demand and the perceived value of the product. The customer's responsiveness to purchase the product at different prices is compared and then an acceptable price is set. Determined by establishing the optimal relationship between profit and volume. Includes Price Skimming, Price Discrimination and Price Penetration.

4

Cost Plus Pricing

A set profit margin is added to the total cost of a product — including materials, labour and overhead. Company first determines its break-even price. Then a markup is set for each unit, based on the profit the company needs to make, its sales objectives and the price it believes customers will pay. Used in large development projects and government contracts.

5

Mark-up Pricing

A percentage is added to the wholesale cost of a product. Mark up refers to the value that a firm adds to the cost price of a product. The value added is called the mark-up. The mark-up added to the cost price usually equals retail price. FMCG typically allows lower markup (5%–8%) due to high volume.

Demand Based Pricing — Three Types

↑

Price Skimming

Initial price set very high so that only customers with more purchasing power can buy. Price reduced gradually so that price-sensitive customers can now buy. Finally, price at which company can operate in profit is set up. Company gets ahead of competition. **Example:** Apple follows Price Skimming.

≠

Price Discrimination

Customers charged differently based on different demand levels. Airline ticket prices increase as travel date gets closer. Inelastic demand during the end makes price very high. Another type: customers in different markets/areas charged differently for the same product or service.

↓

Price Penetration

Exact opposite of price skimming. Initial price kept really low to attract more customers and increase market share. Discounts, inaugural price, first 100 buyers, etc. Can be risky because customers do not like growing prices once accustomed to a low price. **Example:** Initially during launch, Jio used Price Penetration.

Effective Pricing

Effective Pricing is the one that satisfies all the stakeholders — the producer, distributor and the consumer. It fits into the criteria of profit orientation, competition, demand base as also the cost plus and markup. In a way it may be called a long run calibrator of price equilibrium over a business cycle.

Steps Involved in Determining an Effective Price

1. Analysis of Financial Statements

2. Analysis of Cost Behaviour

3. Analysis of the Profit Gap

4. Evolving Cost Reduction Strategies

5. Determination of Feasible Prices for Different Capacity Levels

6. Determination of Effective Price

7. Establishing Cost Controls

8. Review, Revise and Reset

Strategic Pricing Approaches

(i) Product Differentiation

DEFINITION

Product Differentiation is the process of distinguishing a product or service from others, to make it more attractive to a particular target market. This involves differentiating it from competitors' products as well as the firm's own products.

The strategy is adopted to build up specific competitive advantages over competitors by tapping the unique resource endowments exclusive to an entity. The major sources of product differentiation may be traced to differences in **quality, functional features, and availability** (timing and location).

OBJECTIVE OF DIFFERENTIATION

To develop a position that potential customers see as unique. The unique features create a perception of esteem value for the product, which goes beyond pricing considerations. Well-established differentiation makes customers in a given segment develop a lower sensitivity to the non-price features of the product. Many a time product differentiation is driven by the factors of esteem value, enabling the possibility of charging a price premium.

(ii) Cost Leadership

DEFINITION

Cost Leadership is a generic strategy adopted to gain competitive advantage. The strategy aims at the firm winning market share by appealing to cost-conscious or price-sensitive customers. This is achieved by having the lowest prices in the target market segment, or at least the lowest price to value ratio.

Three Dimensions of Cost Leadership

1

Optimum Utilisation of Assets

Achieving a high asset utilization. In manufacturing, it involves production of high volumes of output. In service industries, a restaurant that turns tables quickly or an airline that turns around flights rapidly. Fixed costs are spread over a larger number of units resulting in a lower unit cost.

2

Cost Conscious Culture

Inculcating a cost conscious culture across the organisation and achieving low direct and indirect operating costs. Achieved by offering high volumes of standardized products, offering basic no-frills products and limiting customization. Maintaining this strategy requires a continuous search for cost reductions in all aspects of the business.

3

Concurrent Value Chain Control

Control over the value chain encompassing all functional groups (finance, supply/procurement, marketing, inventory, information technology etc.) to ensure low costs. For supply/procurement chain: bulk buying to enjoy quantity discounts, squeezing suppliers on price, instituting competitive bidding for contracts, working with vendors to keep inventories low using JIT purchasing or Vendor-Managed Inventory.

COST LEADERSHIP & CONTINUOUS IMPROVEMENT

The low-cost leadership strategies are prone to be imitated by competitors, and thus low-cost leadership is not a one-time process. A successful way of adopting this strategy can be by using the Japanese mantra of **"Kaizen"** that focuses on continuous improvement. **Target Costing** is perceived as the most effective means in this direction. Global giants Wal-Mart and McDonald's are cited as interesting examples of being cost effective leaders in their respective fields.

(iii) Yield Management

DEFINITION

Yield Management (also known as Revenue Management) is a set of revenue maximization strategies and tactics meant to improve business profitability. It is a technique that determines the best pricing policy for optimising profits. It adopts the principle of pricing the products and services at what the market can bear.

The core strength of Cost Management is **Prudent Deployment and Optimum Utilisation of Available Resources**. It could be an unfilled seat in a flight, a vacant berth in a train, an unoccupied hotel room, or an empty bed in a hospital. In all such eventualities, capacity unutilised is revenue lost. Yield Management addresses the ticklish issue of underutilisation of available capacity.

Historical Origin — American Airlines (1985)

THE PEOPLEEXPRESS CHALLENGE

Deregulation is generally regarded as the catalyst for yield management in the airline industry. The Airline Deregulation Act in 1978 in USA paved the way for deregulation and facilitated entry of new players. PeopleExpress offered very low competitive fares, denting bigger airlines severely.

On January 17, 1985, American Airlines launched its **"Ultimate Super Saver Fares"**. There were two differences from PeopleExpress: (a) Booking at least two weeks prior to departure and staying over a Saturday night; (b) The number of seats that could be sold for the discounted price was restricted. This segmented the market between leisure travelers and business travelers. Eventually PeopleExpress was pushed to the edge.

Features of Yield Management

1

Fixed Capacity

The quantum of products or services available for sale is governed by the principle of Fixed Capacity. Airline seats, hotel rooms, hospital beds — capacity remains constant for the time being.

2

Perishable Products

The life span of products/services is very limited. An airline seat that remains vacant or a hotel room that remains unoccupied adds no value. If not sold in time, the value is lost forever.

3

Low Marginal Costs

Fixed costs are relatively high compared to variable costs. Once capacity is created, the cost of serving an additional customer is quite meagre. High fixed cost and low marginal cost enable price differentiation.

4

Price Elastic Demand

Lower prices draw more customers; higher prices wean away unwilling customers. Seasonal fluctuations in demand. Peak season: higher prices; lean season: lower prices.

5

Segmented Market

Different classes of customers are willing to pay different prices for using the same amount of products or services. Business traveler (time-conscious, higher price) vs vacation customer (price-sensitive, lower price).

6

Advance Booking

Requests for bookings start early. Suppliers have enough leeway to adjust prices based on variation between advance bookings and expected demand.

Process of Yield Management

Identification of Market Segments → **Forecasting and Pricing** → **Segment-wise Allocation of Inventory** → **Overbooking**

Complexities of Yield Management

Performance Measurement

Measuring performance of any Yield Management system is a major issue. Occupancy rates and yield are measures affected by external competition.

Customer Loyalty

Differential pricing is here to stay and customers seem resigned to the fact. However, some customers do not like this practice and penalize the service provider by not becoming a patron.

Employee Motivations

Yield Management can impact the motivational level of employees. Sometimes employees are paid a percentage of sales they make, motivating them to make group bookings which may be contradictory with the objectives.

Customer Response

Introduction of yield management can sometimes fail to achieve optimised revenue in practice because of corporate image problems. Deutsche Bahn (2002) experiment proved highly unpopular with passengers.

YIELD COMPETENCY MATRIX — SIX QUESTIONS

1. Is the unit of product or service governed by the principle of Fixed Capacity?
2. Is the unit of product or service Perishable?
3. Does the product or service warrant Low Marginal Costs?
4. Is the unit of product or service prone to Price Elastic Demand?
5. Is the unit of product or service poised for a Segmented Market?
6. Can the unit of product or service be offered by means of Advance Booking?

If the answers are "Yes", it is time to tread towards the path of Yield Management and catch the train of Revenue Maximisation.

Case Study 1: Wal-Mart — Price Perfection Enables Perpetual Growth

CASE STUDY

Wal-Mart — Everyday Low Prices

Many retailers are trying to replicate the success of Wal-Mart Stores Inc., the world's largest retailer founded over 60 years ago, on "Everyday Low" prices. The central goal of Wal-Mart is to keep retail prices low — and the company has been very successful at this. Experts estimate that Wal-Mart saves shoppers at least 15 percent on a typical cart of groceries.

Founder Sam Walton's Philosophy: "Offer shoppers lower prices than they get anywhere else". Even if his margins weren't as fat as competitors, he figured he could make up for that in volume.

Key Strategies:

- **Cutting-edge technology:** Pushed retail industry to establish universal bar code; first major retailer to demand manufacturers use RFID.
- **Pushing suppliers:** Legendary stories of how Wal-Mart pushes manufacturers into selling the same product at lower and lower prices.
- **Frugal corporate culture:** Walton continued to drive an old pickup truck and share budget-hotel rooms with colleagues.

Example — Lakewood Engineering: In early 1990s, a 20-inch box fan cost $20. Wal-Mart pushed the manufacturer to lower the price. Lakewood automated production, badgered its own suppliers to knock down prices, and in 2000 opened a factory in China. By 2003, the price on the fan in a Wal-Mart store had dropped to about $10.

Example — Coca-Cola: At Wal-Mart's request, Coke and its largest bottler changed the way they deliver Powerade in the United States. Coke also now allows Wal-Mart in on the research-and-development process. In 2005, Coke planned to launch one new diet cola called Coke Zero. At Wal-Mart's request, it changed the name to Diet Coke with Splenda and launched a separate product called Coke Zero.

Key Learning: Wal-Mart considers 'Everyday Low Prices' are the prices that would be afforded by its customers which in turn would enable customer retention and customer attraction whereby the revenues are propped up. In the process Wal-Mart pulls off its voluminous growth through perfecting 'Everyday Low Prices' revealing that **Price Perfection enables Perpetual Growth**.

Case Study 2: McDonald's Cost Leadership Strategy

CASE STUDY

McDonald's — Cost Leadership

The restaurant industry is known for yielding low margins that can make it difficult to compete with a cost leadership marketing strategy. However, McDonald's — the near-eighty-year-old restaurant chain — has been extremely successful with Cost Leadership Strategy by offering basic fast-food meals at low prices.

Two Competitive Advantages: (a) Cheap and (b) Fast — and everything they do within the organization works towards these specific goals.

McDonald's Vision: "To be the world's best quick service restaurant experience. Being the best means providing outstanding quality, service, cleanliness, and value, so that we make every customer in every restaurant smile."

Systematised Cost Management Activities:

- **Process Automation & Simplification:** Computer operated machinery; dual drive-through to decrease wait time.
- **Standardisation of Ingredients:** Standard grade meat that fits exactly into their needs.
- **Lower Employee Costs:** Division of labour allows hiring and training inexperienced employees; low wages; few managers who earn higher wages.
- **Chain of Convenient Stores:** Building a McDonald's store everywhere.
- **Low price margins driving High Volumes.**

Result: The synergic impact of McDonald's Cost Leadership is that of maintaining its position as one of the Multinational Industry Leaders.

Case Study 3: Apple — Product Differentiation

CASE STUDY

Apple — Product Differentiation

Apple Inc. is an American multinational corporation headquartered in Cupertino, California. The company designs, develops, and sells consumer electronics, computer software, online services, and personal computers.

Competitive Factors: Price, product features, relative performance, product quality and reliability, design innovation, software and peripherals, marketing and distribution capability, service and support, and corporate reputation.

Business Strategy: Leverages on its unique ability to design and develop its own operating systems, hardware, application software, and services to provide customers new products and solutions with superior ease-of-use, seamless integration, and innovative design.

Key Achievements:

- June 2011: Most valuable consumer-facing brand in the world.
- Q1 2014: Sold 51 million iPhones (all-time quarterly record), 26 million iPads, 4.8 million Macs.
- Fortune magazine named Apple the most admired company in the United States in 2008, and in the world from 2008 to 2012.
- Boston Consulting Group ranked Apple as the world's most innovative brand for a number of years since 2005.

Steve Jobs' Four Pillars:

1. Offer a small number of products
2. Focus on the high-end
3. Give priority to profits over market share
4. Create a halo effect that makes people starve for new Apple products

Premium Example: In May 2015, Apple charged ₹46,000 for its iPhone 6 whereas Galaxy S5 mini (a compatible product of Samsung) was priced at ₹21,000. Apple was able to collect a premium of **119%** for its carefully carved out characteristic of unique Product Differentiation.

Case Study 4: Yield Management in Indian Railways

CASE STUDY

Indian Railways — Yield Management

Rail vs Air: Captain Gopinath dreamt of bringing air travel within the reach of the common man and conceived Air Deccan — the first low-cost airline to fly pan-India in 2003. Air Deccan took to the strategy of Yield Management and pioneered the concept of low-cost travel. By 2006, Air Deccan was perceived to have converted air travel into a mass commodity. For the first time, train travellers started shifting from rail journey to flight journey.

Indian Railways' Response — Dynamic Pricing Policy (2006): Introduced for freight as well as passenger, for peak and non-peak seasons, premium and non-premium services, and for busy and non-busy routes. For freight: non-peak season is 1st July to 31st October. For passenger segment: 15th January to 15th April and 15th July to 15th September.

Key Initiatives:

- **Reservation Against Cancellation (RAC):** Fill up vacant berths and seats that go idle due to passenger cancellations, quota vacancies, no shows, etc. Each train is given a fixed number of RAC allocations by splitting some berths into seats. A practical example of programmed overbooking.
- **Auto Up-gradation:** In case of upper-class seats remaining vacant, lower-class passengers are upgraded to higher class travel. On average there are two upgrades for every wait-listed upgraded confirmation. Maximises passenger capacity utilisation and builds goodwill.
- **Tatkal:** Demand-driven differential pricing wherein last-minute passengers are offered tickets at a premium. Tatkal charges: 10% for second class and 30% for all other classes. Premium Tatkal: dynamic pricing by increasing fare for subsequent bookings.
- **Slack Season:** Differential pricing by offering lower train fares during non-peak travel periods. Attracts more customers; minimises capacity losses.
- **Clone Trains:** Run on high-demand routes within an hour of a scheduled train's departure to accommodate those on its waiting list. Premium train automatically announced whenever waitlisted passengers went up beyond a point.
- **Add On Revenue (AOR):** Monetization of land and buildings, leasing out land for horticulture and solar energy, monetizing data and free services, exploiting advertising potential, liberalizing parcel policies, augmentation of revenues from manufacturing activity.
- **Surge Pricing:** Effective from 9th September 2016, IR has gone for Surge Pricing on an experimental basis. Ticket prices of Rajdhani, Duronto and Shatabdi trains keep increasing by ten percent of basic fare with every ten percent of tickets sold.

Learning Track: The capacity for every train trip remains fixed; once the train is scheduled, the seats and berths acquire the nature of perishability; trip-based marginal costs are practically nil; demand for travel can be influenced by pricing mechanism; passenger market is segmented; and

the facility of advance booking exists. All major features of yield management are visible in the Indian Railway Model.

⚡ Quick Revision — Section 3.2

- **Five Key Pricing Strategies:** Profit Orientation, Competitor-Based, Demand Based, Cost Plus, Mark-up.
- **Demand Based Pricing — Three Types:** Price Skimming (high initial, gradually lower), Price Discrimination (different prices based on demand levels), Price Penetration (low initial to capture market).
- **Product Differentiation:** Distinguishing a product/service to make it more attractive to a particular target market; driven by esteem value; enables price premium.
- **Cost Leadership:** Winning market share by appealing to cost-conscious customers; three dimensions — Optimum Utilisation of Assets, Cost Conscious Culture, Concurrent Value Chain Control.
- **Yield Management:** Revenue maximization strategies; pricing at what the market can bear; features — Fixed Capacity, Perishable Products, Low Marginal Costs, Price Elastic Demand, Segmented Market, Advance Booking.
- **Yield Management Process:** Identification of Market Segments → Forecasting and Pricing → Segment-wise Allocation → Overbooking.
- **Examples:** Wal-Mart (Everyday Low Prices), McDonald's (Cost Leadership), Apple (Product Differentiation), Indian Railways (Yield Management).

3.3 Transfer Pricing

CONCEPT · METHODS · BENEFITS · ILLUSTRATIONS

Concept of Transfer Pricing

DEFINITION

Transfer Price (TP) is the notional value of goods and services transferred from one division to the other division of an organisation. In other words, when internal exchange of goods and services takes place between the different divisions of a firm, they have to be expressed in monetary terms. The monetary amount for those interdivisional exchanges is called 'Transfer Price'.

Transfer price is the price that one segment (sub unit, department, division etc.) of an organization charges for a product or services supplied to another segment of the same organization. Transfer prices are used when individual entities of a larger multi-entity firm are treated and measured as separately run entities.

WHY TRANSFER PRICING?

Transfer Pricing is needed to monitor the flow of goods and services among the divisions of a company and to facilitate the divisional performance measurement. The primary utility is to measure the notional sales of one division to another division. The system adopted has a significant effect on the performance evaluation of various divisions.

Impact of Transfer Price on Divisional Performance

IF TRANSFER PRICES ARE SET TOO HIGH

- The **'selling center'** will be favoured
- Buying division's costs increase
- Buying division's profit decreases
- May lead to sub-optimal decisions

VS

IF TRANSFER PRICES ARE SET TOO LOW

- The **'buying center'** will receive an unwarranted proportion of profits
- Selling division's revenues decrease
- Selling division's profit decreases
- May demotivate selling division

Transfer Pricing in Global Context

In the current era of globalization, transfer pricing practice extends to cross-border transactions as well as domestic ones. Multi-National Corporations (MNCs) are legally allowed to use the transfer pricing

method for allocating earnings among their various subsidiary and affiliate companies. However, companies at times can also use this practice for planning their taxable income and reducing their overall taxes.

TAX IMPACT ILLUSTRATION

Let's say an automobile manufacturer has two divisions: Division A (manufactures software) and Division B (manufactures cars). Division A sells software to other carmakers as well as its parent company. Division B pays Division A for the software. If Division A decides to charge a lower price to Division B instead of using the market price, Division A's revenues get reduced and profits dip. Division B's costs of goods sold become lower and increase its profits. If Division A is in a high-tax country and Division B in a low-tax country, the company can save on taxes by making Division A less profitable and Division B more profitable. By making Division A charge lower prices and passing on savings to Division B, the tax liability of A is reduced. As Division B will be taxed at a lower rate, there would be consequential reduction in overall tax-outflow.

Methods of Transfer Pricing

i. Pricing Based on Cost

In these methods, "cost" is the base. Following methods fall under this category:

- **Actual cost** — based on actual costs incurred
- **Cost Plus** — actual cost plus a markup
- **Standard Cost** — based on predetermined standard costs
- **Marginal Cost** — only variable costs considered

ii. Market Price as Transfer Price

Under this method, transfer price will be determined according to the prevailing market price. This is appropriate when there is a perfectly competitive market for the intermediate product.

iii. Negotiated Pricing

Under this method, the transfer prices are fixed through negotiations between the selling and buying divisions. Both divisions have autonomy to arrive at a mutually agreeable price.

iv. Pricing Based on Opportunity Cost

This pricing recognizes the minimum price that the selling division is ready to accept and the maximum price that the buying division is ready to pay. The transfer price lies between these two limits.

Benefits of Transfer Pricing Policy

i

Divisional Performance Evaluation

Divisional performance evaluation is made easier.

ii

Healthy Competition

It will develop healthy inter-divisional competitive spirit.

iii

Management by Exception

Management by exception is possible.

iv

Co-ordination

It helps in co-ordination of divisional objectives in achieving organizational goals.

v

Policy Decisions

It provides useful information to top management in making policy decisions like expansion, subcontracting, closing down of a division, make or buy decisions, etc.

vi

Check on Supplier's Prices

Transfer Price will act as a check on supplier's prices.

vii

Economic Entity

It fosters economic entity and free enterprise system.

viii

Optimal Allocation

It optimizes the allocation of company's financial resources based on relative performance of various profit centres.

ix

Strategic Tax Planning

Transfer pricing plays a vital role in strategic tax planning. Companies can shift tax liabilities to low-cost tax jurisdictions and gain cost advantages.

Illustration — Division A Transfer Price (Cost Plus ROI)

PROBLEM

Your company fixes inter-divisional transfer prices for its products on the basis of cost, plus a return on investment in the division. The Budget for Division A for 2023-24:

PARTICULARS	AMOUNT (₹)
Fixed Assets	5,00,000
Current assets	3,00,000
Debtors	2,00,000
Annual Fixed Cost of the Division	8,00,000
Variable Cost per unit of Product	10
Budgeted Volume	4,00,000 units per year
Desired ROI	28%

Required: Determine the transfer price for Division A.

SOLUTION

PARTICULARS	WORKINGS	AMOUNT (₹)
Variable Cost per Unit	Given	10.00
Fixed Cost per Unit	₹8,00,000 ÷ 4,00,000	2.00
Total Assets	₹5,00,000 + ₹3,00,000 + ₹2,00,000	₹10,00,000
Desired ROI per Unit	(₹10,00,000 × 28%) ÷ 4,00,000	0.70
Transfer Price per Unit	VC + FC + ROI = 10.00 + 2.00 + 0.70	**₹12.70**

Illustration — Transferor Ltd. (Alternative Transfer Prices)

PROBLEM

Transferor Ltd. has two processes, Preparing and Finishing. Normal output per week is 7,500 units at 75% capacity. Transferee Ltd. requires 2,000 units per week of prepared material. Existing cost structure of one prepared unit: Material ₹2.00 (variable 100%); Labour ₹2.00 (Variable 50%); Overhead ₹4.00 (variable 25%). Sale price of completed unit is ₹16 with profit of ₹4 per unit. Capital employed ₹20 lakhs.

Required: Construct the effect on profits for six months (25 weeks) with alternative transfer prices: (i) Marginal Cost, (ii) Marginal Cost + 25%, (iii) Marginal Cost + 15% ROI, (iv) Existing Cost, (v) Existing Cost + Proportionate Profit, (vi) Agreed market price of ₹8.50.

SOLUTION — EFFECT ON PROFIT (₹)

ALTERNATIVE	TRANSFER PRICE (₹)	EFFECT ON PROFIT PER UNIT (₹)	FOR 50,000 UNITS (₹)
(i) Marginal Cost	4.00	0	Nil
(ii) Marginal Cost + 25%	5.00	1.00	50,000
(iii) Marginal Cost + 15% ROI	7.00	3.00	1,50,000
(iv) Existing Cost	8.00	4.00	2,00,000
(v) Existing Cost + Proportionate Profit	10.67	6.67	3,33,500
(vi) Agreed Market Price	8.50	4.50	2,25,000

Illustration — P.H. Ltd. (Profit Centre Transfer Pricing)

PROBLEM

P.H. Ltd. has two manufacturing departments organised into separate profit centres — Basic Unit and Processing Unit. Basic Unit has a production capacity of 4,000 tonnes per month of Chemvax but at present sales are limited to 2,000 tonnes to outside market and 1,200 tonnes to Processing Unit. Transfer price for 2023 was agreed at ₹400 per tonne. Due to heavy competition, Basic Unit reduced wholesale trade price to ₹360 per tonne with effect from 1st June 2023. This price was not made applicable to sales to Processing Unit.

Required: Prepare statement showing estimated profitability for June 2023 for each unit and the company as a whole on different bases.

SOLUTION — KEY RESULTS

BASIS	BASIC UNIT PROFIT (₹)	PROCESSING UNIT PROFIT (₹)	TOTAL PROFIT (₹)
80% capacity, TP ₹400/tonne	3,08,000	1,56,000	4,64,000
100% capacity, TP ₹400/tonne	4,60,000	20,000	4,80,000
80% capacity, market price ₹360, TP ₹400	2,28,000	1,56,000	3,84,000
100% capacity, market price & TP ₹360	3,00,000	1,00,000	4,00,000

Comment: Overall profit is more (₹4,80,000) at 100% capacity of Basic Unit with a transfer price of ₹400 per tonne being the market price. However, from the viewpoint of the Processing Unit, it will not be interested to buy more than 1,200 tonnes from the Basic Unit because its profit gets reduced when it takes additional units.

⚡ Quick Revision — Section 3.3

- **Transfer Price:** Notional value of goods and services transferred from one division to another division of an organisation.
- **Four Methods:** Cost-based (Actual, Cost Plus, Standard, Marginal), Market Price, Negotiated, Opportunity Cost.
- **Benefits:** Divisional performance evaluation, healthy competition, management by exception, co-ordination, policy decisions, check on supplier's prices, economic entity, optimal allocation, strategic tax planning.
- **Rule:** If TP is set too high, selling center is favoured; if too low, buying center gets unwarranted profit.
- **Global Context:** MNCs use transfer pricing for tax planning — shifting profits to low-tax jurisdictions.

3.4 Relevant Cost Analysis

RELEVANT COSTS · IRRELEVANT COSTS · DECISION APPLICATIONS · ILLUSTRATIONS

Concept of Relevant Costs

DEFINITION

Relevant Costs are costs appropriate to aiding the making of specific management decisions. In the context of decision making, only those costs are relevant which are pertinent to the decision at hand. Relevant costs are those expected future costs that are essential but differ for alternative courses of action.

Relevant cost analysis is an incremental analysis which considers only relevant costs (costs that differ between alternatives) and ignores sunk costs (costs which have been incurred, cannot be changed, and hence are irrelevant to the situation).

Three Criteria for a Cost to be Relevant

1

Futuristic

Past costs are irrelevant, as we cannot change them by current decisions and they are common to all alternatives that we may choose.

2

Incremental

Expenditure which will be incurred or avoided as a result of making a decision. Any costs which would be incurred whether or not the decision is made are not said to be incremental to the decision.

3

Cash Flow

Expenses such as depreciation are not cash flows and are therefore not relevant. Similarly, the book value of existing equipment is irrelevant, but the disposal value is relevant.

Categories of Relevant Costs

(i) Differential Costs

The difference in costs under two or more decision alternatives. Where the same item with the same amount appears in all alternatives, it is irrelevant. *Example:* A company selling through distributors paying ₹16 million commission; appointing salespersons instead costs ₹12 million — differential cost of ₹4 million.

(ii) Incremental or Marginal Costs

Cost associated with producing an additional unit. For a university, it could be the cost of admitting another student. Operating a second shift is an example. Incremental costs must be compared with incremental revenues to arrive at a decision.

(iii) Opportunity Costs

The cost of an opportunity of income or benefits foregone. Represents the potential benefits an individual, investor, or business misses out on when choosing one alternative over another. Because opportunity costs are unseen, they can be easily overlooked. *Example:* Mr. Ahmed Shah left a bank job paying ₹15,000/month and got admission in a university with monthly fee ₹10,000 — total cost ₹25,000/month.

(iv) Avoidable Costs

Costs that under given conditions of performance efficiency should not have been incurred. These costs can be eliminated in whole or in part by choosing one alternative over another.

(v) Replacement Costs

The cost of replacement at current market price. It is the cost at which there could be purchase of an asset or material identical to that which is being replaced. Used for determining the optimum time of replacement of equipment or machine.

(vi) Imputed Costs

Hypothetical or notional costs, not involving cash outlay. Computed only for decision making. Interest on funds generated internally is an example. When alternative capital investment projects are considered, it is necessary to take into account the imputed interest on own funds.

(vii) Out-of-Pocket Costs

Costs that entail current or near future cash outlays for the decision at hand. Relevant for decision making, as these will occur in near future. Used in decisions on fixing Selling Price in recession, Make or Buy, etc.

Irrelevant Costs

(i) Sunk Costs

Historical costs which are incurred (sunk) in the past and are not relevant to the particular decision. While considering replacement of a plant, the depreciated book value of the old asset is irrelevant as the amount is sunk cost which is to be written-off at the time of replacement.

(ii) Committed Costs

Costs that will occur in the future, and cannot be changed. A committed cost is an investment that a business entity has already made and cannot recover by any means, as well as obligations already made that the business cannot get out of. *Example:* Multi-year property lease agreement.

(iii) Unavoidable Costs

Inescapable costs, which are essentially to be incurred, within the limits or norms provided for.

(iv) Absorbed Costs

Indirect costs that are absorbed by the product or service. Absorbed fixed costs which do not change due to increase or decrease in activity is irrelevant to decision-making. However, if Fixed Costs are specific, they become relevant.

Illustration — Relevant Cost of Using a Machine

PROBLEM

A machine originally cost ₹12,000, has an estimated life of 10 years, and is depreciated at ₹1,200 per year. It has been unused for some time as expected production orders did not materialise. A special order has now been received which would require the use of the machine for two months. The current net realisable value of the machine is ₹8,000. If used for the job, its value is expected to fall to ₹7,500. The net book value of the machine is ₹8,400. Routine maintenance currently costs ₹40 per month. With use, the cost of maintenance and repairs would increase to ₹60 per month. What would be the relevant cost of using the machine for the order?

SOLUTION

NARRATION	WORKINGS	AMOUNT (₹)
Fall in sale value, if used	₹8,000 – ₹7,500	500.00
Incremental maintenance cost	(₹60 – ₹40) × 2 months	40.00
Relevant cost of using the machine		**₹540.00**

Illustration — Relevant Costs of Material

PROBLEM

X Ltd. has been approached by a customer who would like a special job to be done for him and is willing to pay ₹22,000 for it. The job would require the following materials:

MATERIALS	TOTAL UNITS REQUIRED	UNITS ALREADY IN STOCK	BOOK VALUE OF UNITS IN STOCK (₹/UNIT)	REALISABLE VALUE (₹/UNIT)	REPLACEMENT COST (₹/UNIT)
A	1,000	0	—	—	6
B	1,000	600	2	2.5	5
C	1,000	700	3	2.5	4
D	200	200	4	6	9

(i) Material B is used regularly by X Ltd. and if stocks were required for this job, they would need to be replaced to meet other production demand. (ii) Materials C and D are in stock as the result of previous excess purchase and they have a restricted use. No other use could be found for material C but material D could be used in another job as substitute for 300 units of material which currently costs ₹5 per unit (of which the company has no units in stock at the moment).

SOLUTION — COMPUTATION OF RELEVANT COSTS OF MATERIAL

MATERIAL	RELEVANT COST	WORKINGS	AMOUNT (₹)
A	Replacement Cost	1,000 × 6	6,000.00
B	Replacement Cost	1,000 × 5	5,000.00
C	Realisable Value for 700 units and Replacement Cost for 300 units	(700 × 2.5) + (300 × 4)	2,950.00
D	Substitution Cost	300 × 5	1,500.00
Sub Total			15,450.00
Add: other expenses			550.00
Total			**16,000.00**

Since the revenue from the job is ₹22,000 and relevant cost is ₹16,000, the order should be accepted.

Illustration — Make or Buy Decision (Chakra Ltd.)

PROBLEM

Chakra Ltd. manufactures Mixer Grinders. The on/off switch is presently being purchased from a vendor at ₹4.50 each, annual requirement being 20,000 pieces. Production manager proposed to make the switch in the machine shop. Cost estimate for making: Previous ₹4.73; Current ₹5.33. Purchase department decided to continue buying as it was cheaper to buy than make. 25% of the overheads are fixed.

Required: Do you agree with the decision of buying considering the relevant costs?

SOLUTION

PARTICULARS	PREVIOUS (₹)	CURRENT (₹)
Materials	40,000	48,000
Labour	20,000	22,000
Overhead (Variable) — 75% of total	22,500	23,625
Total Variable Cost	**82,500**	**93,625**
Number of Units	20,000	20,000
Making Cost per Switch	₹4.125	₹4.68
Cost of Buying	₹4.50	₹4.95

Recommendation: It is **not advisable** that the purchase department continues to buy the switch because variable cost of making is less than the buying cost.

Illustration — Officers' Recreation Club (Cinema Theatre)

PROBLEM

The theatre has been showing 'Blood Bath' for two weeks. The manager is convinced attendance will continue above normal for another two weeks if the show is extended. However, another popular movie 'Appu on the Airbus' is booked for the next two weeks. Even if 'Blood Bath' is extended, the theatre has to pay the regular rental on 'Appu on the Airbus'. Normal attendance: 2,000 patrons per week, approximately one-fourth children under 12. Attendance of 'Blood Bath' has been 50% greater than normal. Rental charge for 'Blood Bath' is ₹900 for one week or ₹1,500 for two weeks. For 'Appu on the Airbus' it is ₹750 for one week or ₹1,200 for two weeks. Operating costs: fixed ₹4,200 per week, except potato wafers and cakes averaging 60% of selling price. Sales of wafers and cakes average ₹1.20 per patron. Regular price: ₹2 for adults, ₹1.20 for children under 12.

SOLUTION — EVALUATION OF ALTERNATIVES

NARRATION	OPTION 1 — BLOOD BATH	OPTION 2 — BLOOD BATH & APPU	OPTION 3 — APPU ON THE AIRBUS
Adults — Third week	2,250	2,250	1,500
Adults — Fourth week	2,000	1,500	1,500
Children — Third week	—	500	500
Children — Fourth week	—	500	500
Total spectators	**4,250**	**4,250**	**4,000**
Revenue — Adults	8,500	7,500	6,000
Revenue — Children	—	600	1,200
Snacks contribution @ ₹0.48/patron	2,040	2,040	1,920
Total Revenue	**10,540**	**10,140**	**9,120**
Less: Incremental Costs (Rental)	1,500	900	—
Net Revenue	**9,040**	**9,240**	**9,120**

Suggestion: Net revenue is more at Option 2 (running Blood Bath and Appu on the Airbus a week each). It must be chosen.

⚡ Quick Revision — Section 3.4

- **Relevant Costs:** Future, incremental, cash flow costs that differ between alternatives.
- **Seven Relevant Cost Categories:** Differential, Incremental/Marginal, Opportunity, Avoidable, Replacement, Imputed, Out-of-Pocket.
- **Four Irrelevant Cost Categories:** Sunk, Committed, Unavoidable, Absorbed.
- **Opportunity Cost:** Benefits foregone by choosing one alternative over another.
- **Sunk Cost:** Historical cost already incurred; not relevant to current decisions.

3.5 Target Costing

JAPAN'S COMPETITIVE THINKING · TARGET PHILOSOPHY · PROCESS · ADVANTAGES · KEY FEATURES · KAIZEN COSTING

Japan's Competitive Thinking Triggers Target Costing

The primary objective of Japanese Management is stated to be linking accounting practices with corporate goals and missions. The Japanese believe that the key to achieving a competitive edge is **simplicity**. They have established that there can be too much of good things, too much of a variety, too much of a flexibility and even too much of customer satisfaction.

THREE MAJOR EVENTS IN JAPAN (EARLY 1990S)

1. **The bursting of the economic bubble (1990-1991)** — caused many companies to struggle to meet customers' expectations of lower prices.
2. **The rise of the Japanese Yen against the U.S. Dollar (1993)** — by 1995, the Yen had appreciated as much as 50% against the Dollar. Exports and profit margins plummeted.
3. **The long recession** — caused by a crisis in the financial sector, forcing many Japanese companies to squeeze out costs to meet profitability requirements.

The Target Philosophy

Effective cost management systems are developed in response to changing competitive conditions. Target costing is an example of such a system that has special relevance to companies in the process

and assembly industries. In these industries, firms are no longer able to achieve a sustainable competitive advantage by pursuing either a low-cost or differentiation strategy. Rather, firms realize that any competitive advantage they achieve is likely to be short-lived as competitors move quickly to match new product offerings at competitive prices.

PRICE-LED COSTING

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Target Cost = Target Price – Target Profit
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In place of the traditional approach of: **Cost + Profit = Selling Price**. Target Costing is a philosophy in which product development is based on what the customer wants and is willing to pay for and not what it costs to produce. It starts with the market determined price; then deducts the desired profit margin; and works back the target cost. Peter Drucker calls this **"price-led costing."**

Illustration — Target Cost of Kitchen Grinder

SL	ITEM	WORKINGS	AMOUNT (₹)
1	Projected Sales	40,000 pieces @ ₹3,000 per piece	12,00,000
2	Desired Profit	15% ROI on ₹8,00,00,000 of Investment	1,20,000
3	Target Cost	Projected Sales – Desired Profit	10,80,000
4	Target Cost per grinder	Target cost ÷ 40,000 pieces	₹2,700

Process of Target Costing — Eleven Steps

Step 1 — Identification of Market Needs

Identifying market requirements regarding design, utility and need for a new product or improvements of existing product. Customer requirements as to functionality and quality are of prime importance. Design specification is based on customer's tastes, expectations and requirements.

Step 2 — Establishment of Selling Price

Establishing a selling price for the new product by adopting market-driven approach. Target Selling Price is determined using various sales forecasting techniques. The price is influenced by offers of competitors, product utility, prices, volumes and margins.

Step 3 — Establishment of Target Production Volumes

Closely related to Target Selling Price, given the relationship between price and volume. Target Volumes are significant in computation of unit costs, in particular Capacity Related Costs and Fixed Costs.

Step 4 — Target Profit

Visualising a target profit by means of investment-driven considerations. Target Profit Margin is derived from the company's long term business plan, objectives and strategies. Each product or product line is required to earn at least the Target Profit Margin.

Step 5 — Target Cost

Determining the target cost by subtracting the target profit from the established selling price. The difference between Target Selling Price and Target Profit Margin indicates the "Allowable Cost" for the product.

Step 6 — Estimating Current Costs

Estimating current costs for the product on the basis of functional cost analysis and value engineering of individual components and processes. Direct Costs determined by reference to design specifications, materials prices, labour processing time and wage rates. Indirect Costs estimated using Activity Based Costing Principles.

Step 7 — Cost Reduction Targets

Comparing current costs with target costs. The difference indicates the required cost reduction. This amount may be divided into (a) Target Cost-Reduction Objective and (b) Strategic Cost-Reduction Challenge.

Step 8 — Identifying Cost Reduction Opportunities (Iteration)

A series of analytical activities commence to translate the cost challenge into reality. Total target is broken down into its various components; each component studied and opportunities for cost reductions identified. This process is iterative and continues until target cost is reached or a decision is made not to launch the product.

Step 9 — Launching the Product

The product is on for launching after the cost estimate is on target.

Step 10 — Product Cost Management

Once target costs have been determined, actual costs can be monitored and managed against the targets using usual budgeting and costing methods such as standard costing.

Step 11 — Consumption Cost Management

A consumer-friendly post-sale support service should be oriented towards cost management during the consumption cycle of the product.

Advantages of Target Costing

i

Innovation

Reinforces top-to-bottom commitment to process and product innovation, and is aimed at identifying issues to be resolved.

ii

Competitive Advantage

Enables a firm to achieve competitive advantage over other firms in the industry. The firm which achieves cost reduction targets realistically stands to gain in the long run.

iii

Market Driven Management

Helps to create a Company's competitive future with market-driven management for designing and manufacturing products that meet the price required for market success.

iv

Real Cost Reduction

Uses management control systems to support and reinforce manufacturing strategies, and to identify market opportunities that can be converted into real savings.

Seven Key Features of Target Costing

1

Price-Led Costing

Sets the target cost by first determining the price at which a product can be sold. Subtracting the target profit margin yields the target cost. **Key Derivation:** Cost = Price – Profit

2

Focus on the Customer

Management must listen to the company's customers — what products they want, what features are important, how much they are willing to pay. **Key Derivation:** Customer is the Philosopher

3

Focus on Product Design

Design engineering is a key element. Engineers must design a product from the ground up so that it can be produced at its target cost. **Key Derivation:** Manufacturable Product Design forms the Base

4

Focus on Process Design

Every aspect of the production process must be examined to make sure the product is produced as efficiently as possible. **Key Derivation:** Efficient Process Design is the Pillar

5

Cross-Functional Teams

Manufacturing at or below target cost requires involvement of people from many different functions — market research, sales, design engineering, procurement, production engineering, production scheduling, material handling and cost management. **Key Derivation:** Team Work does the Trick

6

Life-Cycle Costs

Analysts must incorporate all of the product's life-cycle costs — product planning and concept design, preliminary design, detailed design and testing, production, distribution and customer service. **Key Derivation:** Life Cycle Perception is the Approach

7

Value-Chain Orientation

When projected cost is above target cost, efforts are made to eliminate non-value-added costs. A close look at the entire value chain can help managers identify opportunities for cost reduction. **Key Derivation:** Value Addition is the Crux

Cost Management Techniques and Target Costing

Many organisations have found that the real strength of target costing lies in its overall framework for cost improvement and efficiency within which a range of different techniques are used:

Value Analysis | Value Engineering | Just-In-Time (JIT) | Total Quality Management (TQM) | Materials Requirements Planning (MRP) | Kaizen | Lean Manufacturing

Kaizen Costing

DEFINITION

Kaizen Costing refers to the ongoing continuous improvement program that focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase. It is a Japanese term for a number of cost reduction steps that can be used subsequent to issuing a new product design to the factory floor.

Activities in kaizen costing include elimination of waste in production, assembly, and distribution processes, as well as the elimination of unnecessary work steps in any of these areas. Kaizen costing is intended to repeat many of the value engineering steps, continuously and constantly refining the process, thereby eliminating extra costs at each stage.

TARGET COSTING & KAIZEN COSTING

Cost reductions resulting from kaizen costing are much smaller than those achieved with value engineering. But these are still significant since competitive pressures are likely to force down the price of a product over time. Any possible cost savings allow a company to still attain its targeted profit margins. **Target Costing is considered to be an offspring of Kaizen Costing.**

Toyota's Kaizen Costing

Toyota aggressively pursues kaizen costing to reduce costs in the manufacturing phase. In every July and January, plant managers submit six-month plans for attaining their kaizen goals. Methods for achieving these goals include cutting material costs per unit and improvement in standard operating procedures. About two million suggestions were received from Toyota employees in one year alone (roughly thirty-five per employee). Ninety-seven percent of them were adopted. This is a prime example of the concept of **employee empowerment**.

Case Study 1: Target Costing at Caterpillar

CASE STUDY

Caterpillar — Target Costing in Practice

For a particular vehicle, management set the target cost at **94.6%** of a comparable model, creating an initial gap of 5.4%. A cost improvement team was formed comprising representatives from product design, manufacturing engineering, production, marketing, and purchasing.

Step 1 — Known Adjustments (Table 1): The team evaluated component part substitutions that would reduce costs but still provide the product features and benefits necessary to satisfy customer requirements.

ACTIVITY	CURRENT COSTS (%)	PROJECTED COSTS (%)	ADJUSTED SAVINGS (%)	EXPLANATION
Assembly	5.4	3.9	1.5	Efficiency improvements due to redesigning sheet metal
Cab	7.9	7.1	0.8	Replace current cab with "Classy Cab"
Engine	8.6	7.9	0.7	Cost estimate from Engineering for switching to different configuration
Hydraulics	19.1	17.5	1.6	New pump design
Power Train	12.0	12.0	0	—
Structures	20.0	20.0	0	—
Linkage	18.0	18.0	0	—
Others	9.0	9.0	0	—
Total	**100.0**	**95.4**	**4.6**	

Step 2 — Sample Questionnaire (Table 2): A tally of responses indicated the extent to which each part category would be targeted for cost reduction. Hydraulics was responsible for achieving the highest percentage (50%) of the cost savings needed. Therefore, the cost of hydraulics must be reduced by 0.4% (0.50 × 0.08).

Step 3 — Final Target Cost Assignments (Table 3): The adjusted costs column from Table 1 was taken and the additional savings required for each component part category were subtracted.

ACTIVITY	ADJUSTED COSTS (%)	DISTRIBUTION OF 0.8% IN COST REDUCTION (%)	TARGET COST FOR NEW PRODUCT (%)
Assembly	3.90	0.00	3.90
Cab	7.10	0.00	7.10
Engine	7.90	0.08	7.82
Hydraulics	17.50	0.40	17.10
Power Train	12.00	0.04	11.96
Structures	20.00	0.08	19.92
Linkage	18.00	0.20	17.80
Others	9.00	0.00	9.00
Total	**95.40**	**0.80**	**94.60**

Key Learning: Caterpillar began with current costs for a comparable product (100%) and, after deducting known savings based on existing technology and potential savings based on analysis of the questionnaire, established cost targets for each component of the new vehicle whereby the target cost was brought down to 94.60% of the current costs.

Case Study 2: Using Target Costing to Increase Value — Major-

Cable

CASE STUDY

Major-Cable — Target Costing to Increase Value

The Opportunity: In order to capture an Original Equipment Manufacturing (OEM) business, Major-Cable quoted a very low price and bagged an opportunity. The Target Costing project was designed to improve the profitability of these cables to this customer and other OEM customers.

The Issue: One need from this customer was to have the cable with a minimum of 20 meters length. This has been a problem for Major Cable because delivery is made from their centralized distribution center which cuts the cable to the customer's required length and they cannot always guarantee to have the right quantity of 20-meter lengths.

The Solution: The initial solution was for the production plant to create a new product number for 20-meter lengths and manufacturing them specially for the OEM customer. The team then asked: *"Why does the customer need 20-meter minimum lengths?"* It turned out that the machines the customer manufactures require several pieces of cable and the total amount of cable for any machine never exceeds 20 meters.

Major Cable's plant has "off-cuts" of cable that are too short to be sent to the distribution center. These off-cuts are stored as finished goods in case there is occasional demand for a short length; but these off-cuts are mostly scrapped at the year-end.

The production manager suggested that instead of supplying from the distribution center, they can deliver directly to the customer. Instead of delivering 20-meter lengths, they can deliver the cable cut to the precise lengths required by the customer that day. Instead of supplying the product on large round spools, they can place the cut pieces into production kits in cardboard tubes or boxes.

Results:

- The customer is delighted to have just-in-time deliveries of cable kits — reduced their manufacturing costs.
- The customer is paying a higher price because they are receiving kits instead of spooled cable.
- Many of the cable lengths can be provided from the off-cuts that were being scrapped — effectively zero cost.
- The cardboard boxes or tubes are much less expensive than the previously used spools.
- The sales people were delighted — they did not know the production plant could supply cut pieces.

The Power of Target Costing: Without Target Costing, Major Cable would not have had the opportunity to bring together the cross-functional teams needed to understand the customer's needs and find manufacturing, logistics, and marketing methods to create more value for the customer. In the process, everybody has "won."

Case Study 3: Unveiling the Indian Nano — Tata Nano

CASE STUDY

Tata Nano — Indian Target Costing

No other car launch in the history of Indian Auto Industry has received as much global press as the "people's car", the Tata Nano. Much like what Henry Ford had at the beginning of the 20th century with his Model T in 1908 at a price of $825, exactly 100 years after Ratan Tata unveiled Nano, the Indian ultra-low-cost car. At ₹1 lakh, the Nano was the world's cheapest car and made motoring affordable to millions of Indians.

Approach: Every component in the Nano was studied from a functionality, cost and performance requirements perspective as there was no other way to reduce costs. From an outsourcing perspective, the company put in place an **Early Vendor Integration Programme**. The company had a lot of design inputs from vendors that either facilitated manufacturing or brought the cost down.

The Nano is completely indigenised. Over 85 percent of the Nano is sourced from outside vendors. Vendor parks were put in place with the objective of ensuring that the components between vendors and the assembly line move smoothly and just in time.

Initial Effort: Cost prevention — selecting a design concept with the least cost. **Later:** Perpetual cost-reduction effort.

Ratan Tata's Vision: "The real challenge is when you have some strength and you really choose to throw out the gauntlet that you can do X and it ought to be the kind of challenge which somebody says that can't be done because then that really becomes the engine of innovation."

⚡ Quick Revision — Section 3.5

- **Target Costing:** Market-driven system of cost reduction focused on managing costs at the developmental and design stages.
- **Formula:** Target Cost = Target Price – Target Profit (Price-Led Costing).
- **Eleven Steps:** Market Needs → Selling Price → Production Volumes → Target Profit → Target Cost → Estimating Current Costs → Cost Reduction Targets → Iteration → Launching → Product Cost Management → Consumption Cost Management.
- **Seven Key Features:** Price-Led Costing, Focus on Customer, Focus on Product Design, Focus on Process Design, Cross-Functional Teams, Life-Cycle Costs, Value-Chain Orientation.
- **Kaizen Costing:** Ongoing continuous improvement program focusing on reduction of waste in production process; Target Costing is an offspring of Kaizen Costing.
- **Caterpillar:** Target cost set at 94.6% of comparable model; initial gap of 5.4%.
- **Tata Nano:** Early Vendor Integration Programme; 85% sourced from outside vendors.

3.6 Product Life Cycle Costing

PHASES · REVENUE-COST-PROFIT MATRIX · IMPORTANCE · THREE KEY FACTORS · ILLUSTRATIONS

Concept of Product Life Cycle

DEFINITION

Product Life Cycle is a pattern of expenditure, sale level, revenue and profit over the period beginning from new idea generation to the deletion of product from product range. Product Life Cycle spans the time from initial R&D on a product to when customer servicing and support is no longer offered for the product. For products like motor vehicles, this time-span may range from 5 to 7 years. For some basic pharmaceuticals, the time-span may be 7 to 10 years.

Six Phases of Product Life Cycle

Development → **Introduction** → **Growth** → **Maturity** →

New idea generation

Market launch

Market acceptance

Market saturation

Decline
Sales decline

→

Extinction
Product exit

1

Development Phase

Cycle begins with identification of a new consumer need and the invention of a new product. Often followed by patent protection and further development to make it saleable. Research and engineering skills enable product development. Costs incurred are termed as 'developmental'. **No revenues accrue during this phase.**

2

Introduction Phase

Product is introduced to the market. Efforts towards spreading awareness about the product; target being achieving market acceptance. Promotional costs will be high, sales revenue low and profits probably negative. The skill exhibited in testing and launching the product ranks high as the critical factor in securing success and initial market acceptance.

3

Growth Phase

As product gains market acceptance, a rapid expansion follows leading to growth. Characterized by product penetration into the market and increase in sales & profits. Benefits of economies of scale start pouring in leading to cost reduction. Becomes vital to secure wholesaler and retailer support and ensure consumer satisfaction.

4

Maturity Phase

Begins after sales cease to rise exponentially indicating market saturation. Eventually most potential customers have tried the product and sales settle at a rate governed by population growth and the replacement rate of satisfied buyers. Usually the longest and most competitive stage in the cycle. Most of the popular products are in this stage.

5

Decline Phase

Most products and brands enter a period of declining sales. May be caused by: technical advances leading to product substitution, fashion and changing tastes, exogenous cost factors reducing profitability until it reaches zero at which point the product's life is commercially complete.

6

Extinction Phase

The tail end of the decline phase where after the product exits from the market.

Revenue, Cost and Profit Matrix of the Product Life Cycle

SL	PHASE	REVENUE	COSTS	PROFIT
1	Development	Nil	Developmental	Nil
2	Introduction	Low	Promotional	Negative
3	Growth	Increasing	Declining	Growing
4	Maturity	Stable	Stable	Stable
5	Decline	Declining	Increasing	Declining
6	Extinction	Nil	End Life	Negative

Life Cycle Costing

DEFINITION

Life Cycle Costing is a system that is evolved to track and accumulate the costs and revenues attributable to a product or service from the stage of development to the stage of extinction. In essence, it is a means of estimating all the costs involved in procuring, operating, maintaining and ultimately disposing a product throughout its life. The process involves tracing costs and revenues of the product over several calendar periods.

Life cycle costing is a three-staged process:



Characteristics of Product Life Cycle Costing (PLCC)

- Involves tracing of costs and revenues of each product over several calendar periods throughout their entire life cycle.
- Traces research, design and development costs and total magnitude of these costs for each individual product and compared with product revenue.
- Assists report generation for costs and revenues.

Importance of Product Life Cycle Costing

a

Time-based Analysis

Costs and revenues of each product can be analysed by time periods. The total magnitude of costs for each individual product can be reported and compared with product revenues generated in various time periods.

b

Overall Cost Analysis

Production Costs are accounted and recognized by the routine accounting system. However non-production costs like R&D, design, marketing, distribution, customer service etc. are less visible on a product-by-product basis. PLCC focuses on recognizing both production and non-production costs.

c

Pre-production Costs Analysis

The development period of R&D and design is long and costly. A high percentage of total product costs may be incurred before commercial production begins. The Company needs accurate information on such costs for deciding whether to continue with the R&D or not.

d

Effective Pricing Decisions

Pricing Decisions must include market considerations on one hand and cost considerations on the other. PLCC and Target Costing help analyze both these considerations and arrive at optimal price decisions.

e

Better Decision Making

Based on a more accurate and realistic assessment of revenues and costs, at least within a particular life cycle stage, better decisions can be taken.

f

Long Run Holistic View

PLCC can promote long-term rewarding in contrast to short-term profitability rewarding. It provides an overall framework for considering total incremental costs over the entire life span of a product.

g

Life Cycle Budgeting

Life Cycle Budgeting with Target Costing principles facilitates scope for cost reduction at the design stage itself. Since costs are avoided before they are committed or locked in, the Company is benefited.

h

Review

PLCC provides scope for analysis of long-term picture of product line profitability, feedback on the effectiveness of life cycle planning and cost data to clarify the economic impact of alternatives chosen in the design, engineering phase etc.

Three Key Factors to Optimise

1

Design

Development activity is the most important from a sustainability perspective. Since **80% of a product's costs are locked in at the design stage**, it's vital that waste is minimised by design. Choices made at the design stage should account for all stages of a product's life, including end of life costs.

2

Time to Market

Competitors watch each other to discover new products coming to market. When competition is minimal, the growth phase provides producers the chance to charge premium prices and invest heavily in awareness activities. The longer a producer has before a rival product hits the market, the longer they're able to command a price premium.

3

Length of the Lifecycle

Getting to market quickly will lengthen a product's life. Other ways: designing the product in a modular way and conceptualising future modules; designing the product to satisfy as many markets as possible; staggering the launch in different markets to reduce costs and prolong demand.

Illustration — Wipro Software Division

PROBLEM

Wipro is examining profitability and pricing policies of its Software Division. It has collected data on three packages — ECE (Electronics and Communication Engineers), CE (Computer Engineers), and IE (Industrial Engineers). Summary details over their two-year life cycle:

PACKAGE	SELLING PRICE (₹)	YEAR 1 UNITS	YEAR 2 UNITS
ECE	250	2,000	8,000
CE	300	2,000	3,000
IE	200	5,000	3,000

SOLUTION — LIFE CYCLE INCOME STATEMENT (₹ '000S)

PARTICULARS	ECE TOTAL	%	CE TOTAL	%	IE TOTAL	%
Revenues	2,500	100%	1,500	100%	1,600	100%
R&D	700	28%	450	30%	240	15%
Design	200	8%	120	8%	96	6%
Manufacturing	300	12%	210	14%	208	13%
Marketing	500	20%	270	18%	448	28%
Distribution	75	3%	60	4%	96	6%
Customer Service	375	15%	150	10%	608	38%
Total Costs	**2,150**	**86%**	**1,260**	**84%**	**1,696**	**106%**
Profit	**350**		**240**		**(96)**	

Observations: Package ECE with a Life Cycle Profit of ₹3,50,000 is most profitable; package IE with a Life Cycle Loss of ₹96,000 is least profitable. The differences in comparative percentages of total costs to revenues in each category of the three packages are apparent and self-explanatory.

⚡ Quick Revision — Section 3.6

- **Product Life Cycle:** Pattern of expenditure, sale level, revenue and profit from new idea generation to deletion from product range.
- **Six Phases:** Development → Introduction → Growth → Maturity → Decline → Extinction.
- **Revenue-Cost-Profit Matrix:** Development (Nil/Developmental/Nil); Introduction (Low/Promotional/Negative); Growth (Increasing/Declining/Growing); Maturity (Stable/Stable/Stable); Decline (Declining/Increasing/Declining); Extinction (Nil/End Life/Negative).
- **Life Cycle Costing:** Track and accumulate costs and revenues attributable to a product from development to extinction.
- **Three Key Factors:** Design (80% of costs locked in), Time to Market, Length of Lifecycle.
- **Importance:** Time-based analysis, Overall cost analysis, Pre-production costs analysis, Effective pricing decisions, Better decision making, Long run holistic view, Life cycle budgeting, Review.

3.7 Asset Life Cycle Costing

LCC FORMULA · COMPONENTS · CASE STUDY (ORANGE PLANTATION) · ILLUSTRATIONS

Concept of Asset Life Cycle Costing

CIMA DEFINITION

CIMA defines Life-Cycle Costing as 'Maintenance of physical asset cost records over entire asset lives, so that decisions concerning the acquisition, use or disposal of assets can be made in a way that achieves the optimum asset usage at the lowest possible cost to the entity.'

Life Cycle Cost (LCC) may be stated as **"The total cost throughout the life of an asset including planning, design, acquisition and support costs and any other costs directly attributable to owning or using the asset."** LCC of any item represents costs of its acquisition, operation, maintenance and disposal.

Purpose of LCC Analysis

The purpose of LCC analysis is the estimation of the overall cost of project options and then to select the designs which can ensure the facility to provide the overall lowest cost of ownership consistent with the function and its quality. The analysis should be performed at an early stage so that there will be chances of refining the design to ensure the reduction in life cycle total cost.

BASIC LCC FORMULA

$$LCC = C + PV\ Recurring - PV\ Residual\ Value$$

Where: **C** = 0-year acquisition cost; **PV Recurring** = present value of all recurring costs; **PV Residual Value** = present value of residual value at the end of the life of the asset.

Life Cycle Costs of an Asset Includes

- Acquisition
- Installation
- Operating
- Maintenance
- Financing (e.g., interest)
- Depreciation
- Disposal

EXAMPLE OF LIFE CYCLE COST OF A PRINTER

- **Purchase:** ₹20,000
- **Installation:** ₹500 for setting up and delivery
- **Operating:** ₹2,000 for ink cartridges and paper; ₹300 for electricity
- **Maintenance:** ₹500 for repairs
- **Financing:** Interest rate of 9% per annum
- **Depreciation:** Value reduced by ₹2,000 each year
- **Disposal:** ₹150 for hiring a contractor to remove the printer

Even though the price of the printer is ₹20,000, the life cycle cost of the printer will end up costing the business much more.

Case Study: LC Margin for an Orange Plantation Project

CASE STUDY

Orange Plantation Project — Life Cycle Margin

Annual Cash Outflow: The total cash outflow during the first year would aggregate to ₹52.70 lakhs, consisting of ₹50.50 lakhs of initial investment and annual nurturing cost of ₹2.20 lakhs. The nurturing outflow would continue at the rate of ₹2.20 lakhs per year till the 5th year. The annual operating outflow would work out to ₹2.49 lakhs from the 6th year to the 18th year.

Life Cycle Margin per Plant of Oranges:

PARTICULARS	VALUE
Plantation Area (Acres)	8
Number of Plants per Acre	125
Number of Plants per 8 Acres	1,000
Life (Years)	18
Nurturing Period (Years)	5
Cropping Period (Years)	13
Life Cycle Yield per Plant (Kg)	940
Selling Price (₹ per Kg)	31

INVESTMENT (₹ LAKHS)	AMOUNT
Land	41.96
Water Distribution Systems	7.30
Plantation	1.24
Nurturing Cost for the first 5 years	11.00
Total	**61.50**

LIFE CYCLE COST (₹ LAKHS)	AMOUNT
Investment	61.50
Interest on Term Loan	25.99
Plantation Cost from year 6 to year 18	32.41
Life Cycle Cost	**119.90**

LIFE CYCLE MARGIN PER PLANT (₹)	AMOUNT
Life Cycle Revenue per Plant (940 kg × ₹31)	29,140
Life Cycle Cost per Plant (₹1,19,90,000 ÷ 1,000)	11,990
Life Cycle Margin per Plant	**17,150**
Average Margin per Plant per annum (₹17,150 ÷ 18)	953

Observation: A model farmer would be able to earn a Life Cycle Margin of ₹17,150 per plant of oranges over a period of 18 years which works out to ₹953 per plant per year. This margin computes to ₹1,19,125 per acre (of 125 plants) per annum which is quite attractive and competitive. A note of caution, however, is that the case study under reference is demonstrative in nature and does not consider time value of money.

Illustration — A2Z plc. (Machine Replacement Decision)

PROBLEM

A2Z plc. supports the concept of terotechnology or life cycle costing for new investment decisions covering its engineering activities. The company is to replace a number of its machines. The Production Manager is torn between the Exe Machine (more expensive, life of 12 years) and the Wye machine (life of 6 years, likely to be replaced at the end of 6 years by another Wye machine).

PARTICULARS	EXE (₹)	WYE (₹)
Purchase Price	19,000	13,000
Trade-in value/breakup/scrap	3,000	3,000
Annual repair costs	2,000	2,600
Overhaul costs	(at year 8) 4,000	(at year 4) 2,000

Estimated financing costs averaged over machine life: 10% p.a. for both.

SOLUTION — EQUIVALENT ANNUAL COST

PARTICULARS	EXE MACHINE (₹)	WYE MACHINE (₹)
Initial cost	19,000.00	13,000.00
Less: PV of Scrap at end of life	957.00	1,692.00
Net Cost	18,043.00	11,308.00
Add: PV of total annual repair costs	13,624.00	11,320.00
Add: Overhaul costs	1,864.00	1,366.00
P.V. of Total Cost	**33,531.00**	**23,994.00**
Capital recovery factor	0.1468	0.2297
Equivalent annual cost	**₹4,922.35**	**₹5,511.42**

Recommendation: As the equivalent annual cost is less for Exe machine, it is better to purchase the same.

⚡ Quick Revision — Section 3.7

- **Life Cycle Cost (LCC):** Total cost throughout the life of an asset including planning, design, acquisition and support costs and any other costs directly attributable to owning or using the asset.
- **LCC Formula:** LCC = C + PV Recurring – PV Residual Value.
- **Components:** Acquisition, Installation, Operating, Maintenance, Financing, Depreciation, Disposal.
- **Purpose:** Estimation of overall cost of project options; select designs ensuring overall lowest cost of ownership consistent with function and quality.
- **Terotechnology:** Practice of maintaining physical asset cost records over entire asset lives.

3.8 Decision Making Using Probability

PROBABILITY DISTRIBUTION · APPLICATIONS · ILLUSTRATIONS

Concept of Probability Distribution

DEFINITION

A **probability distribution** is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. This range will be bounded between the minimum and maximum possible values, but precisely where the possible value is likely to be plotted on the probability distribution depends on a number of factors. These factors include the distribution's mean (average), standard deviation, skewness, and kurtosis.

Perhaps the most common probability distribution is the **normal distribution**, or 'bell curve'. A probability distribution is a statistical tool that shows the possible outcomes of a particular event or course of action as well as the statistical likelihood of each event.

Applications of Probability

1

Scenario Analysis

Probability distributions can be used to create scenario analysis. A scenario analysis uses probability distributions to create several, theoretically distinct possibilities for the outcome of a particular course of action or future event. For example, a business might create three scenarios: 'worst-case', 'likely' and 'best-case'.

2

Forecasting

One practical use for probability distributions and scenario analysis in business is to predict future levels of sales and costs. It is essentially impossible to predict the precise value of a future sales level or cost movements; however, businesses still need to be able to plan for future events. Using scenario analysis based on a probability distribution can help a company frame its possible future values.

3

Risk Evaluation

In addition to predicting future sales levels, probability distribution can be a useful tool for evaluating risk. Consider a company considering entering a new business line. If the company needs to generate ₹5.00 crore in revenue to break even and their probability distribution tells them that there is a 10 percent chance that revenues will be less than ₹5.00 crore, the company knows roughly what level of risk it is facing.

Illustration — Choice Among Three Products (Expected Value Method)

PROBLEM

A company has a choice among three products A, B and C for which the following estimates are available:

ESTIMATED PROFITS (₹ '000)	MARKET X	MARKET Y	MARKET Z
Product A	380	100	30
Product B	300	280	220
Product C	220	400	320

Probabilities: X = 0.60, Y = 0.20, Z = 0.20

SOLUTION — EXPECTED VALUES

PRODUCT	MARKET	PROFIT (₹ '000)	PROBABILITY	EXPECTED VALUE (₹ '000)
A	X	380	0.6	228
	Y	100	0.2	20
	Z	30	0.2	6
Total Expected Value — Product A				**254**
B	X	300	0.6	180
	Y	280	0.2	56
	Z	220	0.2	44
Total Expected Value — Product B				**280**
C	X	220	0.6	132
	Y	400	0.2	80
	Z	320	0.2	64
Total Expected Value — Product C				**276**

Decision: Product B having the maximum EV of ₹2,80,000 should be selected.

Illustration — Bee Ltd. (Expected Value of Sales, Variable Cost and Profit)

PROBLEM

You are given the following estimates for next year's budgeted sales and costs of a single product produced by Bee Ltd.:

SALES DEMAND (UNITS)	PROBABILITY	VARIABLE COST PER UNIT (₹)	PROBABILITY
3,200	0.50	5.00	0.30
4,000	0.30	6.00	0.50
5,000	0.20	7.00	0.20

Selling Price: ₹12 per unit. Fixed Cost for the period: ₹20,000.

SOLUTION

PARTICULARS	WORKINGS	RESULT
Expected Value of Sales (Units)	(3,200 × 0.5) + (4,000 × 0.3) + (5,000 × 0.2)	3,800 units
Expected Sales Value	3,800 × ₹12	₹45,600
Expected Variable Cost (₹/unit)	(5 × 0.3) + (6 × 0.5) + (7 × 0.2)	₹5.90
Expected Variable Cost (Total)	3,800 × ₹5.90	₹22,420
Expected Contribution	₹45,600 – ₹22,420	₹23,180
Expected Profit	₹23,180 – ₹20,000	**₹3,180**

Illustration — Probability of Break-even and Target Profit

PROBLEM

A company has estimated the following demand levels for its product:

SALES VOLUME (UNITS)	10,000	12,000	14,000	16,000	18,000
Probability	0.10	0.15	0.25	0.30	0.20

Sales price: ₹6 per unit; Marginal cost: ₹3.50 per unit; Fixed costs: ₹34,000.

Required: (a) Probability that the company will break-even. (b) Probability that the company will make a profit of at least ₹10,000.

SOLUTION

(a) Probability of Break-even:

Contribution per unit = ₹6 – ₹3.50 = ₹2.50. Break-even Sales = ₹34,000 ÷ ₹2.50 = 13,600 units.

Probability that sales will equal or exceed 13,600 units is the probability that sales will be 14,000, 16,000 or 18,000 units which is (0.25 + 0.30 + 0.20) = **0.75 or 75%**.

(b) Probability of earning Profit of ₹10,000:

Contribution Needed = ₹10,000 + ₹34,000 = ₹44,000. Desired Sales = ₹44,000 ÷ ₹2.50 = 17,600 units.

Probability that sales will equal or exceed 17,600 units is the probability that sales will be 18,000 units which is **0.20 or 20%**.

Illustration — Probability of Achieving Target Profit (Fixed Cost Uncertainty)

PROBLEM

A company has estimated unit variable cost of a Product to be ₹10, and the selling price is ₹15 per unit. Budgeted sales for the year are 20,000 units. Estimated fixed costs:

FIXED COST P.A. (₹)	50,000	60,000	70,000	80,000	90,000
Probability	0.1	0.3	0.3	0.2	0.1

What is the probability that the company will equal or exceed its target profit of ₹25,000 for the year?

SOLUTION

Budgeted sales = 20,000 units; Budgeted Contribution per Unit = ₹15 – ₹10 = ₹5; Budgeted total contribution = 20,000 × ₹5 = ₹1,00,000

Target profit = ₹25,000. Maximum fixed costs if target is to be achieved = ₹1,00,000 – ₹25,000 = ₹75,000

Higher Profit would mean lower FC, other things remaining constant. So Pb that Co. will equal or Exceed its target profit of ₹25,000 = Pb that FC will be ₹75,000 or less.

The probability that fixed costs will be ₹75,000 or less is: P(50,000) + P(60,000) + P(70,000) = 0.1 + 0.3 + 0.3 = **0.7 or 70%**.

Illustration — Group Guidelines & Risk Attitude (Y Ltd.)

PROBLEM

The Managing Director of Y Ltd. has evolved some decision making to the operating division of the firm. He is anxious to extend this process but first wishes to be assured that decisions are being taken properly in accordance with group policy. As a check on existing practice, he has asked for an investigation to be made into a recent decision to increase the price of the sole product of Z division to ₹14.50 per unit due to rising costs.

Last year 75,000 units were sold at ₹12 each with total unit cost of ₹9 of which ₹6 were variable costs. For the year ahead the following cost and demand estimates have been made:

VARIABLE COSTS	PROBABILITY	COST PER UNIT (₹)
Pessimistic	0.15	7.00
Most likely	0.65	6.50
Optimistic	0.20	6.20

TOTAL FIXED COSTS	PROBABILITY	INCREASE
Pessimistic	0.3	50%
Most likely	0.5	25%
Optimistic	0.2	10%

DEMAND ESTIMATES AT VARIOUS PRICES	PROBABILITY	₹13.50 PER UNIT	₹14.50 PER UNIT
Pessimistic	0.30	45,000	35,000
Most likely	0.50	60,000	55,000
Optimistic	0.20	70,000	68,000

For this type of decision the group has decided that the option should be chosen which has the highest expected outcome with at least an 80% chance of breaking even.

SOLUTION

Last year's fixed costs = 75,000 units × ₹3 = ₹2,25,000

Estimated Fixed Costs: ₹2,25,000 × 1.10 × 0.2 + ₹2,25,000 × 1.25 × 0.5 + ₹2,25,000 × 1.50 × 0.3 = ₹2,91,375

To break-even the contribution must be equal to or greater than ₹2,91,375.

At selling price of ₹13.50 there is 100% chance to break-even. However, at selling price of ₹14.50 there is 70% chance of break-even. The selling price of ₹14.50, therefore, contravenes group guidelines.

Attitude to Risk: The group seeks to minimize the downside risk whilst maximizing its return. It is to some extent risk averse, but it is prepared to take some risk i.e., 20% risk of loss. It is always sought to maximize its returns, ignoring the probability of failure, it would be risk neutral.

⚡ Quick Revision — Section 3.8

- **Probability Distribution:** Statistical function describing all possible values and likelihoods that a random variable can take within a given range.
- **Three Applications:** Scenario Analysis, Forecasting, Risk Evaluation.
- **Expected Value Method:** EV = Σ (Profit × Probability). Select the alternative with the highest EV for maximization, lowest EV for minimization.
- **Break-even Probability:** Calculate BEP units; find probability that sales equal or exceed BEP units.
- **Target Profit Probability:** Calculate desired sales for target profit; find probability that sales equal or exceed desired sales.

★ Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Decision Making

The outcome resulting from the process of evaluation of the available alternatives and choosing the best.

Cost Behaviour

Refers to the changes in input costs in relation to the level of production.

Contribution

Excess of the Sales Value over the Variable Cost.

Break Even Point (BEP)

The point where 'Total Revenues' equal 'Total Costs'.

Margin of Safety

Sales above the breakeven level reflect the Margin of Safety.

Product Differentiation

The process of distinguishing a product or service from others, to make it more attractive to a particular target market.

Cost Leadership

A strategy aiming at the firm winning market share by appealing to cost-conscious or price-sensitive customers.

Yield Management

A set of revenue maximization strategies and tactics meant to improve business profitability. It is the art and science of price-driven and capacity-based profit maximization.

Transfer Price

The notional value of goods and services transferred from one division to the other division of an organisation.

Relevant Costs

Costs which are relevant for a specific purpose or situation.

Irrelevant Costs

Costs which are not relevant for a specific purpose or situation.

Target Costing

A philosophy in which product development is based on what the customer wants and is willing to pay for and not what it costs to produce.

Kaizen Costing

The ongoing continuous improvement program that focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase.

Product Life Cycle

A pattern of expenditure, sale level, revenue and profit over the period beginning from new idea generation to the deletion of product from product range.

Life Cycle Costing

A system that is evolved to track and accumulate the costs and revenues attributable to a product or service from the stage of development to the stage of extinction.

Probability Distribution

A statistical function that describes all the possible values and likelihoods that a random variable can take within a given range.

Multiple Choice Questions

WITH ANSWERS AND REASONING

1. The break-even point of a manufacturing company is ₹1,60,000. Fixed cost is ₹48,000. Variable cost is ₹12 per unit. The PV ratio will be:

A. 20%

B. 40%

C. 30%

D. 25%

Answer: C

PV Ratio = FC ÷ BEP = 48,000 ÷ 1,60,000 = 30%.

2. The higher the actual hours worked,

A. The lower the capacity usage ratio

B. The higher the capacity usage ratio

C. The lower the capacity utilization ratio

D. The higher the capacity utilization ratio

Answer: D

Capacity utilization ratio is worked out by dividing the actual hours with the budgeted hours. Therefore, higher the actual hours, higher would be the utilisation ratio.

3. XYZ Ltd. has the following alternative planned activity levels:

Level E: Total Cost ₹1,00,000; Units 5,000

Level F: Total Cost ₹1,50,000; Units 10,000

Level G: Total Cost ₹2,00,000; Units 15,000

Answer: D

Variable cost per unit = ₹10. Fixed cost = ₹50,000. Fixed cost per unit at Level E = ₹50,000 ÷ 5,000 = ₹10.

4. T Ltd. produces and sells a product. Revenues (400 sets @ ₹600) = ₹2,40,000; Variable costs = ₹1,60,000; Fixed costs = ₹50,000. What amount of sales must T Ltd. have to earn a target net income of ₹63,000 if they have a tax rate of 30%?

A. ₹4,20,000

B. ₹4,29,000

C. ₹3,00,000

D. ₹4,89,000

Answer: A

PV Ratio = 33.33%. Profit Before Tax = 63,000 ÷ 0.70 = ₹90,000. Target Contribution = ₹90,000 + ₹50,000 = ₹1,40,000. Target Sales = ₹1,40,000 ÷ 33.33% = ₹4,20,000.

5. Excel Products Ltd. manufactures four products E, F, G and H. Input requirements: 1kg, 2kgs, 5kgs, 7kgs. Selling Price: ₹100, ₹150, ₹200, ₹300. Variable Cost: ₹50, ₹70, ₹100, ₹125. Assuming raw material availability is a limiting factor, the correct ranking is:

A. E, F, G & H

B. E, F, H & G

C. F, E, G & H

D. F, E, H & G

Answer: B

Contribution per kg: E = ₹50, F = ₹40, G = ₹20, H = ₹25. Ranking: E (1), F (2), H (3), G (4).

6. A company has a breakeven point when sales are ₹3,20,000 and variable cost at that level of sales are ₹2,00,000. How much would contribution margin increase or decrease if variable expenses are dropped by ₹30,000?

A. Increase by 27.5%

B. Increase by 9.375%

C. Decrease by 9.375%

D. Increase by 37.5%

Answer: B

Original PV Ratio = 37.5%. Revised PV Ratio = 46.875%. Increase = 9.375%.

7. The Tech Company has fixed costs of ₹4,00,000 and variable costs are 75% of the selling price. To realize profits of ₹1,00,000 from sales of 5,00,000 units, the selling price per unit

A. must be ₹1.00 B. must be ₹4.80 C. must be ₹4.00

D. cannot be determined

Answer: C

Desired Contribution = ₹5,00,000; PV Ratio = 25%; Desired Sales = ₹20,00,000; Selling Price = ₹20,00,000 ÷ 5,00,000 = ₹4.

8. A company makes components and sells internally to its subsidiary and also to external market. External market price is ₹24 per component, which gives a contribution of 40% of sales. For external sales, variable costs include ₹1.50 per unit for distribution costs. This is not incurred in internal sales. There are no capacity constraints. To maximize company profit, the transfer price to subsidiary should be:

A. ₹9.60 B. ₹12.90 C. ₹14.40

D. None of these

Answer: B

Transfer Price = Marginal Cost – Opportunity Gain = ₹24 × 60% – ₹1.50 = ₹14.40 – ₹1.50 = ₹12.90.

9. H Group has two divisions, Division P and Division Q. Division P manufactures an item transferred to Division Q. The item has no external market and 6,000 units produced are transferred internally each year. Division P: Variable Cost ₹100/unit; Fixed cost ₹1,20,000. Head Office decided transfer price should provide a profit of ₹30,000 to Division P. What should be the transfer price per unit?

A. ₹145 B. ₹125 C. ₹120

D. ₹135

Answer: B

Target Contribution = ₹30,000 + ₹1,20,000 = ₹1,50,000; Per unit = ₹25; Transfer Price = ₹25 + ₹100 = ₹125.

10. A particular job required 800 kgs of material P. 500 kgs of the particular material is currently in stock. Original price was ₹300 but current resale value is ₹200. Current replacement price is ₹0.80 per kg. The relevant cost of material P required for the job would be:

A. ₹640 B. ₹440 C. ₹300

D. None of these

Answer: B

500 kgs at resale value ₹200 + 300 kgs at current price ₹0.80 = ₹240. Total = ₹440.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
C	D	D	A	B	B	C	B	B	B

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

1

Define and discuss decision making process.

2

State the relevance of cost behaviour in the context of choosing the best alternative.

3

Write a note on key factor analysis.

4

What are the key pricing strategies? Elaborate with examples.

5

List the merits and de-merits of product differentiation.

6

Explain the concept of cost leadership with the help of a case study.

7

What are the key features of Yield Management?

8

What are the key features of industries that are amenable to Yield Management?

9

What are the benefits of Transfer pricing? What are the prevalent methods in determining transfer price?

10

Distinguish between Relevant Costs and Irrelevant Costs.

11

Discuss the philosophy of Target Costing. Highlight its key features.

12

Narrate the process of Target Costing.

13

Write a note on product life cycle.

14

What do you understand by life cycle costing?

15

Highlight the utility of probability in decision making.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 3

ABBREVIATION	FULL FORM
ABCM	Activity Based Cost Management
BEP	Break Even Point
CRM	Customer Relationship Management
EV	Expected Value
JIT	Just-In-Time
LCC	Life Cycle Cost
MRP	Materials Requirements Planning
PLC	Product Life Cycle
PLCC	Product Life Cycle Costing
PV Ratio	Profit Volume Ratio
PPP	People, Planet, Profit
TQM	Total Quality Management
TP	Transfer Pricing
YM	Yield Management

STUDY TOOLS

Case Studies (Wal-Mart, McDonald's, Apple, Indian Railways, Tata Nano, Caterpillar, Major-Cable, Wipro)

Formula Boxes

Quick Revision Sheets

MCQs with Reasoning

Numerical Problems

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Activity Based Management & Just in Time

Module 4 · Activity Based Costing · JIT · Throughput Accounting · Backflush · Benchmarking

EDITION	PAPER	SECTION	MODULE WEIGHTAGE
December 2026	Paper 16 · Strategic Cost Management	Section A · Weightage 60%	10% of Section A

✓ Website Publication · ✓ PDF Conversion · ✓ A4 Printing · ✓ Mobile & Desktop Reading · ✓ Self Study · ✓ Grand Revision · ✓ Final & Last Day Revision · ✓ Personal Reference Book

04 Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 4 INCLUDES

4.1 Activity Based Cost Management — Concept, Purpose, Stages, Benefits, Relevance in Decision-making and its Application in Budgeting, Responsibility Accounting, Traditional vs. ABC System – Comparative Analysis · **4.2** JIT — Introduction, Benefits, Use of JIT in Measuring the Performance · **4.3** Throughput Accounting · **4.4** Back Flush Accounting · **4.5** Benchmarking.

Module Learning Objectives

After studying this module, the students will be able to:

1

In-depth Knowledge

In-depth knowledge relating to Activity Based Management, JIT, Throughput Accounting and Backflush Costing.

2

Analysis & Presentation

Analysis, Evaluation and Presentation of strategic data for decision making.

10%	5	20+	5
WEIGHTAGE IN SECTION A	CORE SUB-TOPICS	ILLUSTRATIONS	CASE STUDIES
Of 60% Section A total	4.1 to 4.5	Worked solutions	Real-world applications

SYLLABUS STRUCTURE NOTE

Module 4 carries **10% weightage** within Section A. It covers modern cost management techniques that go beyond traditional absorption costing — ABC/ABM, JIT, Throughput Accounting, Backflush Accounting and Benchmarking. These are highly practical modules with numerical applications and case-study orientation.

📖 **Table of Contents — Module 4**

1. **Activity Based Cost Management** — Concept, Purpose, Stages, Benefits, Decision-making, Budgeting, Responsibility Accounting, Traditional vs ABC
2. **Just-In-Time (JIT)** — Introduction, Benefits, Performance Measurement
3. **Throughput Accounting** — Core Measures, TPAR, Theory of Constraints
4. **Back Flush Accounting** — Concept, Process, Variants, Journal Entries
5. **Benchmarking** — Types, Process, Pre-requisites, Difficulties, Case Study

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4.1 Activity Based Cost Management

CONCEPT · PURPOSE · STAGES · BENEFITS · DECISION-MAKING · BUDGETING · RESPONSIBILITY ACCOUNTING · TRADITIONAL VS ABC

Concept & Purpose of ABC

FUNDAMENTAL ASSUMPTION OF ABC

Products consume activities and activities consume costs. This leads to more precise allocation of manufacturing overheads amongst the products. Activity-based costing provides a means to collect indirect costs in multiple categories and then applies the results individually to the products and services.

ABC is a **Technology Trigger Cost Accounting** that has gained popularity from around the mid-nineteen eighties. ABC was aimed to improve the accuracies in the absorption of overheads adapted in traditional costing systems. ABC's ability to overcome the inherent limitations in traditional cost accounting systems enabled changes in strategy processes, operations and in turn improved competitive posture.

Traditional Limitations — The Distortion Problem

The main objective of any costing system is to determine scientifically the cost of a product or service. Costs can be segregated into two distinct categories, viz. **direct costs** and **indirect costs**.

- **Direct costs** are the costs which are directly traceable to the products or the services that are being offered.
- **Indirect costs**, which are traditionally called 'overheads', are not directly traceable to the products or services. Hence, these overheads are first identified, classified, allocated, and apportioned to convenient service cost centres; reapportioned to production cost centres and finally absorbed by the cost units i.e. products or services.

THE DISTORTION ILLUSTRATION

Suppose XYZ Limited produces two products, A and B. Direct material cost for A = ₹15,00,000 and B = ₹25,00,000, totalling ₹40,00,000. Total overheads = ₹20,00,000. Company adopts direct material cost as the basis for absorption. Rate of absorption = (20,00,000 ÷ 40,00,000) × 100 = 50%

Absorption by A: 15,00,000 × 50% = ₹7,50,000
Absorption by B: 25,00,000 × 50% = ₹12,50,000

Product B is loaded with higher quantum of overheads because it consumes more direct material. Now suppose engineers bring down the direct material costs of product B to ₹22,50,000 without any change in material costs of product A and total overheads. The revised computations are:

Direct Material Cost = ₹37,50,000; Overheads = ₹20,00,000; Rate = (20,00,000 ÷ 37,50,000) × 100 = 53.3333%

Absorption by A: 15,00,000 × 53.3333% = ₹8,00,000
Absorption by B: 22,50,000 × 53.3333% = ₹12,00,000

Even though there is no change as regards product A, the absorption of overheads by it has gone up by ₹50,000 from ₹7,50,000 to ₹8,00,000 — an evident **distortion**. This misrepresentation in costs may propel wrong decisions in several areas like make or buy, pricing, acceptance of export offer etc.

Important Terms in Activity Based Costing

TERM	DEFINITION	EXAMPLE
Activity	An aggregate of closely related tasks having some specific functions which are used for completion of a goal or objective.	Customer order processing — includes receiving an order, interacting with production, giving commitment on delivery time.
Resource	Elements that are used for performing the activities or factors helping in the activities.	Order receiver, telephone, computers, material, labour, equipment, office supplies.
Cost	The amount paid for the resources consumed by the activity. Also known as activity cost pool.	Salaries, printing stationery, telephone bill.
Cost Object	An item for which cost measurement is required.	A product, a service, or a customer.
Cost Pool	A grouping of costs incurred on a particular activity which drives them.	Material handling cost pool, inspection cost pool.
Cost Driver	Any element that would cause a change in the cost of an activity. Cost drivers are the basis of charging cost of an activity to a cost object.	Frequency of orders, number of orders, number of set-ups, machine hours.

RESOURCE COST DRIVER VS ACTIVITY COST DRIVER

- **Resource Cost Driver:** A measure of the quantity of resources consumed by an activity.
- **Activity Cost Driver:** A measure of the frequency and intensity of the demand placed on activities by cost objects.

Examples of Activities, Resources, Cost Pools and Cost Drivers

ACTIVITIES	RESOURCES	COST POOLS	COST DRIVER
Consulting	Consultant, Computer	Employee cost, Maintenance cost	Level of consultant, Time spent
Laser Printing	Printing Staff, Printer	Colour cost, Maintenance cost, Printing stationary	No. of pages printed, Font
Accounting & Administration	Administration Staff	Salaries	No. of times account is produced
Customer Service	Telephone, Staff	Telephone bill, Salaries	Frequency of orders, No. of orders, Time spent in servicing, No. of service calls
Research & Development	Staff, Equipment, Material	Salaries, Maintenance cost, Material cost	No. of research projects, Time spent on a project, Technical complexities

Cost Drivers for Other Functional Avenues

P

Production

Number of units, Number of set-ups

M

Marketing

Number of sales personnel, Number of sales orders

C

Customer Service

Number of service calls, Number of products serviced, Hours spent on servicing products

Eight Stages of ABC Implementation

Stage 1 — Identification of Cost Objects

The process starts with identification of the cost objects. The cost objects of any organisation are the products or services.

Stage 2 — Identification of Activities

Identification of the main activities can be done by carrying out an in-depth analysis of the operating processes of each responsibility segment. Usually the number of activities in ABC will be much more as compared to traditional overhead system.

Stage 3 — Tracing the Direct Costs

The direct costs of products or objects may comprise direct material cost, direct labour cost and direct expenses. Classification of as many of the total costs as direct costs as is economically feasible should be made.

Stage 4 — Relating the Indirect Costs to the Activities

Various items of indirect costs are related to activities, viz. both support and primary, which caused them. As a result, cost pools or cost buckets are created.

Stage 5 — Distribution of Support Activities

The spreading of support activities across the primary activities is done on some suitable base which reflects the use of support activity. The base is the cost driver and is a measure of the support activities that are used.

Stage 6 — Determining the Activity Cost Drivers

Determination of the activity cost drivers is done in order to relate the overheads collected in cost pools to the cost objects of products. It is done on the basis of the factor that drives the consumption of the activities.

Stage 7 — Calculating the Activity Cost Driver Rates

Activity Cost Driver Rate = (Total Cost of Activity ÷ Activity Driver). These rates are used for ascertaining the amount of overhead chargeable to various cost objects or products.

Stage 8 — Computing the Total Cost

The total costs of the products shall be computed by adding all direct and indirect costs assigned to them. The amount of overhead chargeable to a product is calculated by multiplying the respective activity cost driver rate by the quantum of the activity that the product consumes.

Benefits of ABC

✓

Cost Reduction

ABC measures how costly the activities are and then takes steps to reduce their costs by changing the production process or outsourcing those activities.

✓

Product Pricing

ABC implementers generally believe that ABC provides more accurate cost information than conventional costing. Management can use this information to negotiate price increases or drop unprofitable products.

✓

Budgeting

Management can use more accurate cost information to improve budgets and measures of department and division performance.

✓

Value Chain & Supply Chain Analysis

Firms that adopted ABC were significantly more successful in capturing accurate cost information for value chain analysis and supply chain analysis.

✓

Customer Profitability

Need for customer profitability analysis led corporate India to extend ABC systems from basic level to advanced level, extending it to facility level and customer level activities.

✓

Better Insight

Better insight for benchmarking and budgeting, manufacturing performance, and linking up cause and effect relationship.

Firms Using ABC Systems are More Successful in Capturing

Accurate Cost & Profit Information for:

- Product pricing
- Customer profitability
- Inventory valuation
- Value chain analysis
- Supply chain analysis
- Outsourcing decisions

Better Insight for:

- Accurate profit analysis by product, process, department, and customer
- Benchmarking and budgeting
- Manufacturing performance
- Linking up cause and effect relationship

Activity Based Management (ABM)

DEFINITION

Activity Based Management is a tool of management that involves analysing and costing activities with the goal of improving efficiency and effectiveness. It is a set of actions that management can take, based on information from an Activity Based Costing system, to improve profitability.

Towards continuous improvement, Activity Based Management keeps on attempting **Cost Driver Analysis, Activity Analysis, and Performance Analysis** on a continuous basis.

1

Cost Driver Analysis

The factors that cause activities to be performed need to be identified in order to manage activity costs. Cost driver analysis identifies these causal factors. *Example:* In a stores department, slow moving and obsolete stock is not disposed off in time because staff are not trained properly. Managers have to address this cost driver to correct the root cause.

2

Activity Analysis

Activity Analysis identifies value added and non-value-added activities and efforts are made to eliminate the non-value adding activities.

3

Performance Analysis

Involves identification of appropriate measures to report the performance of activity centers or other organisational units consistent with each unit's goals and objectives. Aims to identify the best ways to measure performance of factors that are important to organisations in order to stimulate continuous improvement.

Cost Reduction Through ABCM Data

The costs of an output, product, or service (a final cost object) can be reduced by:

- **Reducing the quantity, frequency, and/or intensity of the activity driver** (e.g., fewer inspections reduce the 'Inspection Cost / Inspect product' activity cost);
- **Lowering the activity driver cost rate** by productivity improvements (e.g., shorten the time for each 'Inspection Time/inspect product' event); and
- **Understanding the sources and causes of waste** leading to non-value-adding activities to reduce or eliminate them (e.g., solve the problem that requires an inspection in the first place).

Activity Based Budgeting (ABB)

DEFINITION

Activity-Based Budgeting is a budgeting method where activities are thoroughly analysed to predict costs. ABB does not take historical costs into account when creating a budget. Every cost incurred by a business will be looked at closely to determine if efficiencies can be created and costs reduced.

Traditional budgeting is based on traditional cost accounting whereas activity-based budgeting is based on activity-based costing. Ultimately, ABB aims to analyse business cost drivers and enable the business to become more profitable.

Features of Activity Based Budgeting

1

Activity Analysis

It uses the activity analysis to relate costs to activities.

2

Cost Improvement

It identifies cost improvement opportunities.

3

Strategic Link

There is a clear link between strategic objectives, planning and the tactical planning of the ABC process.

Three Main Steps in ABB

Step 1 — Identifying the Cost Drivers of Various Activities

For example, the cost drivers for a manufacturing facility can be the total labour hours and wages paid to employees.

Step 2 — Projecting the Number of Units Required Within Each Cost Driver

For example, the manufacturing facility may always need three people on the production line, translating to 240 labour hours per week.

Step 3 — Estimating the Cost per Unit of Activity Relating to That Cost Driver

For example, wages for warehouse labour can be ₹150 per hour.

Responsibility Accounting

DEFINITION

Responsibility accounting involves separate reporting of revenues and expenses for each responsibility center in a business. Doing so improves the management of operations. This concept also applies to the cost of products — each component part has a standard cost which is the responsibility of the purchasing manager to obtain at the correct price.

An **activity-based responsibility accounting system** assigns responsibility to processes and uses both financial and non-financial measures of performance. It is the responsibility accounting system developed for those firms operating in continuous improvement environments. Traditional responsibility accounting uses budgets and variances to hold individuals responsible for those costs that they have the authority to incur — causing them to manage cost rather than the activities that cause the cost. Activity-based accounting redefines accountability from costs to team-based activities.

Four Essential Elements of Responsibility Accounting Model

1

Assigning Responsibility

2

Establishing Performance Measures or Benchmarks

3

Evaluating Performance

4

Assigning Rewards

Traditional vs. ABC System — Comparative Analysis

SL	TRADITIONAL COSTING SYSTEM	ACTIVITY BASED COSTING SYSTEM
1	Cost analysis is done on the basis of product.	Managers focus attention on activities rather than products because activities in various departments may be combined and costs of similar activities ascertained (e.g., quality control, handling of materials, repairs to machines).
2	Managers manage products. Changes in products lead to changes in costs.	Managers manage activities and not products. Changes in activities lead to changes in costs. If activities are managed well, costs will fall and resulting products will be more competitive.
3	Overhead cost allocation to production is based on a single cost driver (unit basis, % of material, labour hour rate, machine hour rate etc.), which can result in unrealistic product cost because the system fails to capture cause-and-effect relationships.	Uses multiple cost drivers and cost pools to capture cause-and-effect relationships. To make wiser economic decisions, managers need to identify the relationships of causes (activities) and effects (costs) in a more detailed and accurate manner.
4	Can lead to under-costing or over-costing of products or services. Over or under costing distorts cost information.	Highlights problem areas that deserve management's attention and more detailed analysis. Many actions are possible — on pricing, on process technology, on product design, on operational movements and on product mix.
5	Direct costs are identified with, or assigned to, the cost object in the same manner as in ABC.	Direct costs are identified with, or assigned to, the cost object in the same manner as in traditional costing. Overhead costs are linked to the cost objects based on activities.

CORE PHILOSOPHY OF ABC

"It is better to be approximately right, than precisely wrong." By associating cost to the activity, a clear relationship can be established between sources of activity demand and the related costs. This association can benefit the distributor in determining where costs are being incurred, what is initiating the costs and where to apply efforts to curb inflationary costs.

Caselets — ABC in Practice

CASELET 1 **Xu Ji — Standardisation in Working Practices (CIMA Case Study, 2011)**

The Chinese electricity company Xu Ji used ABC to capture direct costs and variable overheads, which were lacking in the state-owned enterprise's (SOE) traditional costing systems. The ABC experience successfully induced standardisation in their working practices and processes. Standardisation was not a common notion in Chinese culture or in place in many Chinese companies. ABC also acted as a catalyst to Xu Ji's IT developments — first accounting and office computerisation, then ERP implementation.

Background: Prior to the ABC introduction in 2001, Xu Ji operated a traditional Chinese state-enterprise accounting system. A large amount of manual bookkeeping work was involved. Accounting was driven predominantly by external financial reporting purposes, and inaccuracy of product costs became inevitable. At this time, Xu Ji underwent a series of flotations following China's introduction of free market competition.

Problem: The inaccuracy of the traditional costing information seriously impeded Xu Ji's ability to compete on pricing.

Two Main Tasks for ABC: (1) Trace direct labour costs directly to product and client contracts; (2) Allocate manufacturing overheads on the basis of up-to-date direct labour hours to contracts.

Learning: The common 'top-down' management style and organisational culture among SOEs worked well when instigating innovative ideas and inducing corporate-wide learning. Top management's commitment to trying out new management ideas and investing in new technology has been the unique feature.

CASELET 2

ABC in Automobile Industry

Many automotive companies use activity-based costing to determine their costs and pricing. For example, Ford decided the company needed this sort of determined, specific look into individual cost steps to see which steps could be eliminated or improved. Chrysler has seen hundreds of millions of dollars saved using activity-based costing to identify and eliminate useless, inefficient or redundant steps in its production, which has also significantly streamlined product development. Since car and truck manufacturing is an established industry, where the manufacturing process has many steps that combine to make the final product, activity-based costing is the ideal analysis.

CASELET 3

Activity-Based Costing at Coca-Cola

Coca-Cola is another company that uses activity-based costing to determine its price points. Coca-Cola offers a large portfolio of products and carries a huge amount of inventory, which can be a significant portion of production cost that is often overlooked. Coca-Cola has used activity-based costing to evaluate the differences between its bigger, world-wide products and its specialty, regionalized products that it may not offer on the global market. This understanding of how production costs are different between established, familiar types and specialized types has enabled them to set price points in each market that ensure them significant profit.

Illustration 1 — Total Material Cost under ABC

PROBLEM

A company manufactures 500 units of product AX. The following details are available: Material cost to manufacture ₹1,50,000; Labour cost ₹2,65,000; Material Reordering Cost ₹4,500; Material Handling Cost ₹2,500; Material orders 35; Material movements 20.

Required: What is the Total Material cost under Activity Based Costing?

SOLUTION

SL	PARTICULARS	AMOUNT (₹)
1	Material cost to manufacture	1,50,000
2	Material Reordering Cost	4,500
3	Material Handling Cost	2,500
	Total Material Cost	**₹1,57,000**

Explanatory Comment: Material Reordering Cost and Material Handling Cost are directly traced to the Total Material Cost under the system of ABC.

Illustration 2 — Total Cost per Unit under ABC

PROBLEM

Production overheads of XYZ Manufacturers Pvt. Ltd. for 500 units of product X are: Machine oriented activity cost ₹1,35,400; Material ordering overheads ₹69,570. Machine hours are 1.50 hrs per unit and No. of material orders are 6 per unit. Raw material cost ₹300 per unit and labour cost ₹150 per unit.

Required: What is the Total cost of X per Unit?

SOLUTION

SL	PARTICULARS	AMOUNT (₹)
1	Raw Material cost	300.00
2	Labour cost	150.00
3	Machine Oriented Cost (135400 ÷ 750 × 1.5)	270.80
4	Material Ordering Cost (69570 ÷ 3000 × 6)	139.14
	Total Cost	**₹859.94**

Illustration 3 — Factory Cost per Unit under ABC (Four Products)

PROBLEM

A company produces four products, viz. P, Q, R and S. Production overheads: (i) Overheads applicable to machine-oriented activity ₹1,65,900; (ii) Overheads relating to ordering materials ₹8,760; (iii) Set up costs ₹21,400; (iv) Administration overheads for spare parts ₹44,690; (v) Material handling costs ₹25,545.

PRODUCT	QUANTITY	MATERIAL COST/UNIT (₹)	DIRECT LABOUR HOURS/UNIT	MACHINE HOURS/UNIT	DIRECT LABOUR COST/UNIT (₹)	NO. OF SET UP	NO. OF MATERIALS ORDERS	NO. OF TIMES MATERIALS HANDLED	NO. OF SPARE PARTS
P	4,500	12	2	1.50	8	3	3	6	6
Q	13,640	15	2	0.75	9	18	12	30	15
R	2,340	25	5	2.50	27	5	3	9	3
S	18,350	21	4	4.00	25	24	12	36	12

SOLUTION — COST DRIVER RATES

ACTIVITY	COST (₹)	COST DRIVER	UNITS	RATE (₹)
Machinery oriented activity	1,65,900	Machine Hours	96,230	1.724/hour
Material Ordering	8,760	No. of Material orders	30	292/order
Set up costs	21,400	No. of set ups	50	428/set up
Administrative Overheads for spare parts	44,690	No. of spare parts	36	1,241.39/spare part
Material Handling costs	25,545	No. of times materials handled	81	315.37/handling

PARTICULARS	P (₹)	Q (₹)	R (₹)	S (₹)
Materials	12.00	15.00	25.00	21.00
Labour	8.00	9.00	27.00	25.00
Machine oriented activity	2.586	1.293	4.310	6.896
Ordering of Materials	0.195	0.257	0.374	0.191
Set up costs	0.285	0.565	0.915	0.560
Administrative Spare Parts	1.655	1.365	1.591	0.812
Material handling	0.420	0.694	1.213	0.619
Factory Cost (₹)	**25.14**	**28.17**	**60.40**	**55.08**

Illustration 4 — Cost Driver Rates and Batch Cost under ABC

PROBLEM

The budgeted overheads and cost driver volumes of XYZ are as follows: Material procurement ₹5,80,000 (1,100 orders); Material handling ₹2,50,000 (680 movements); Set-up ₹4,15,000 (520 set ups); Maintenance ₹9,70,000 (8,400 hours); Quality control ₹1,76,000 (900 inspections); Machinery ₹7,20,000 (24,000 machine hours).

A batch of 2,600 components of AX-15 has: Material cost ₹1,30,000; Labour cost ₹2,45,000. Usage activities: Material orders 26; Maintenance hours 690; Material movements 18; Inspections 28; Set ups 25; Machine hours 1,800.

SOLUTION — COST DRIVER RATES AND BATCH COST

COST POOL	BUDGETED (₹)	COST DRIVER	VOLUME	RATE (₹)
Material procurement	5,80,000	No. of orders	1,100	527.27
Material handling	2,50,000	No. of movements	680	367.65
Set-up	4,15,000	No. of set ups	520	798.07
Maintenance	9,70,000	Maintenance hours	8,400	115.48
Quality control	1,76,000	No. of inspection	900	195.56
Machinery	7,20,000	No. of machine hours	24,000	30.00

COST POOL	COST DRIVER	USAGE	RATE (₹)	OVERHEADS (₹)
Material procurement	No. of orders	26	527.27	13,709
Material handling	No. of movements	18	367.65	6,618
Set-up	No. of set ups	25	798.07	19,952
Maintenance	Maintenance hours	690	115.48	79,681
Quality control	No. of inspections	28	195.56	5,476
Machinery	No. of machine hours	1,800	30.00	54,000
Total				**₹1,79,436**

ELEMENT	AMOUNT (₹)
Material cost	1,30,000
Labour Cost	2,45,000
Prime Cost	3,75,000
Overheads	1,79,436
Total Cost	**₹5,54,436**

Illustration 5 — Traditional vs ABC (Ice-Cream Products)

PROBLEM

AML Ltd produces three types of ice-cream products: Coco, Strawberry & Vanilla. Coco: 50,000 units @ ₹25; Strawberry: 20,000 units @ ₹20; Vanilla: 60,000 units @ ₹15. Every reduction of ₹1 per unit in selling price increases demand by 10%. Company has production capacity: Coco 60,500 units, Strawberry 24,200 units, Vanilla 72,600 units. Mark-up = 25% of cost. ABC activities: Ordering ₹800/order, Delivery ₹700/delivery, Shelf Stocking ₹199/hour, Customer Support ₹1.10/unit sold. Under traditional costing, store support costs charged @ 30% of prime cost.

SOLUTION — UNIT COST COMPARISON

PARTICULARS	COCO (₹)	STRAWBERRY (₹)	VANILLA (₹)
Unit Costs under Traditional System	16.90	13.00	10.40
Unit Costs under ABC System	16.286	15.234	10.167
Total Costs under Traditional System	10,22,450	3,14,600	7,55,040
Total Costs under ABC System	9,85,303	3,68,663	7,38,124

Explanatory Comment: Prime cost remaining the same, the differences arose on account of bettering the accuracy in the distribution of store support costs under ABC.

Illustration 6 — XYZ Limited (Three Products, ABC vs Conventional)

PROBLEM

XYZ Limited makes three main products X, Y, Z using broadly the same production methods and equipment. Details for a typical period:

PRODUCT	LABOUR HOURS/UNIT	MACHINE HOURS/UNIT	MATERIAL (₹/UNIT)	VOLUME (UNITS)
X	1½	3½	25	3,500
Y	½	2	15	2,250
Z	2	5	30	6,000

Direct labour cost is ₹8/hour. Production overheads absorbed on machine hour rate basis @ ₹18/machine hour. Analysis of production overheads: Set-ups 30%, Machinery 25%, Materials handling 22%, Inspection 23%. Activity volumes: X (set-ups 65, movements 15, inspections 150); Y (110, 26, 190); Z (485, 79, 570).

SOLUTION — COMPARISON OF UNIT COSTS

ELEMENT	X CONVENTIONAL	X ABC	Y CONVENTIONAL	Y ABC	Z CONVENTIONAL	Z ABC
Materials	25.00	25.00	15.00	15.00	30.00	30.00
Labour	12.00	12.00	4.00	4.00	16.00	16.00
Overheads	63.00	38.58	36.00	63.49	90.00	93.94
Total Cost	**100.00**	**75.58**	**55.00**	**82.49**	**136.00**	**139.94**

Explanatory Comment: Prime cost remaining the same, the differences arose on account of the better methodology adopted under ABC principles for the distribution of Overheads.

⚡ Quick Revision — Section 4.1

- **ABC Fundamental:** Products consume activities and activities consume costs.
- **Traditional Limitation:** Single cost driver causes distortion in overhead absorption.
- **Key Terms:** Activity, Resource, Cost, Cost Object, Cost Pool, Cost Driver.
- **Eight Stages of ABC:** Cost Objects → Activities → Direct Costs → Indirect Costs to Activities → Support Activities → Activity Cost Drivers → Cost Driver Rates → Total Cost.
- **Benefits:** Cost Reduction, Product Pricing, Budgeting, Value Chain & Supply Chain Analysis, Customer Profitability, Better Insight.
- **ABM:** Cost Driver Analysis, Activity Analysis, Performance Analysis.
- **ABB:** Activity-based budgeting; identifies cost drivers, projects units, estimates cost per unit.
- **Responsibility Accounting:** Four elements — Assigning Responsibility, Establishing Performance Measures, Evaluating Performance, Assigning Rewards.
- **Traditional vs ABC:** Product vs Activity focus; single vs multiple cost drivers.

4.2 Just-In-Time (JIT)

INTRODUCTION · BENEFITS · DISADVANTAGES · PRECAUTIONS · PERFORMANCE MEASUREMENT · CASE STUDIES

Introduction to JIT

DEFINITION

Just-In-Time (JIT) is a management technique in which goods are received from suppliers only as and when they are needed. The main objective of this method is to reduce inventory holding costs and increase inventory turnover.

Just in time is a **demand-pull system of production**, wherein actual orders provide a signal for as to when a product should be manufactured. Demand-pull enables a firm to produce only what is required, in the correct quantity and at the correct time. This means that stock levels of raw materials, components, work in progress and finished goods can be kept to a minimum.

Modern manufacturing firms use sophisticated production scheduling software to plan production for each period of time, which includes ordering the correct stock at the correct time. Information is exchanged with suppliers and customers through **EDI (Electronic Data Interchange)** to help ensure that every detail is correct. Supplies are delivered right to the production line only when they are needed.

JIT STRATEGY

By taking a JIT approach to inventory and product handling, companies can often cut costs significantly. Inventory costs contribute heavily to the company expenses, especially in manufacturing organisations. By minimizing the amount of inventory that you hold, you save space, free up cash resources, and reduce the waste that comes from obsolescence.

JIT Systems — The Kanban Approach

To facilitate a JIT approach, you need a variety of systems in place. The most notable is a **'kanban'** — a Japanese approach to ensuring a continuous supply of inventory or product. A kanban is a visual signal that indicates that it is time to replenish stock and possibly reorder.

KANBAN EXAMPLE

As the supply of bolts in a bin on the assembly line falls below a certain number, it may uncover a yellow line painted around the inside of the storage bin. This yellow line indicates to the foreman that he needs to prepare a requisition for more bolts. That requisition is given to the purchasing department, which processes the order. This prevents the supply of bolts from dropping below a critical amount and allows production to continue to flow smoothly.

JIT also exists in concert with continuous improvement systems. Total Quality Management and Six Sigma are overarching programs that help take a detailed look at every point of the production process and identify ways to make improvements. When the emphasis is on lean production, systems tend to be made simpler and more predictable.

Benefits of Just-In-Time System

i

Minimum Stock Holding Costs

Keeps stock holding costs to a bare minimum. The release of storage space results in better utilization of space and thereby bears a favorable impact on the rent paid and on any insurance premiums.

ii

Eliminates Waste

Eliminates waste, as out-of-date or expired product does not enter into this equation at all.

iii

Less Working Capital

Only essential stocks are obtained, so less working capital is required to finance procurement. A minimum re-order level is set, and only once that mark is reached fresh stocks are ordered.

iv

High ROI

Due to the low level of stocks held, the organisation's return on investment would generally be high.

v

Demand-Pull Basis

All goods made would be sold, and thus it incorporates changes in demand with surprising ease. This makes it especially appealing where market demand is volatile and unpredictable.

vi

Right First Time

Encourages the 'right first time' concept, so that inspection costs and cost of rework is minimized.

vii

High Quality & Efficiency

High quality products and greater efficiency can be derived from following a just-in-time production system.

viii

Close Relationships

Close relationships are fostered along the production chain under a just-in-time manufacturing system.

ix

Customer Satisfaction

Constant communication with the customer results in high customer satisfaction.

x

Eliminates Over production

Over production is eliminated, when just-in-time manufacturing is adopted.

Disadvantages of JIT

i

Zero Tolerance for Mistakes

Provides zero tolerance for mistakes, as it makes re-working very difficult in practice, as inventory is kept to a bare minimum.

ii

High Reliance on Suppliers

There is a high reliance on suppliers, whose performance is generally outside the purview of the manufacturer.

iii

No Buffers for Delays

As there will be no buffers for delays, production downtime and line idling can occur, which would bear a detrimental effect on finances and on the equilibrium of the production process.

iv

Cannot Meet Unexpected Increase

The organisation would not be able to meet an unexpected increase in orders, due to the fact that there are no excess finish goods.

v

High Transaction Costs

Transaction costs would be relatively high, as frequent transactions would be made.

vi

Environmental Impact

May have certain detrimental effects on the environment, due to the frequent deliveries that would result in increased use of transportation which in turn would consume more fossil fuels.

Precautions — Things to Remember When Implementing JIT

i

Management Buy-in

Management buy-in and support at all levels of the organisation are required if a just-in-time manufacturing system is to be successfully adopted.

ii

Adequate Resources

Adequate resources should be allocated to obtain technologically advanced software, that is generally required if a just-in-time system is to be a success.

iii

Close Supplier Relationships

Building a close, trusting relationship with reputed and time-tested suppliers will minimize unexpected delays in the receipt of inventory.

iv

Commitment of Time

JIT cannot be adopted overnight. It requires commitment in terms of time and adjustments to corporate culture, as it is starkly different to traditional production processes.

v

Redesign Flow & Layout

The design flow process needs to be redesigned and layouts need to be re-formatted, so as to incorporate just-in-time manufacturing.

vi

Minimize Lot Sizes

Lot sizes need to be minimized.

vii

Balance Capacity

Work station capacity should be balanced whenever possible.

viii

Preventive Maintenance

Preventive maintenance should be carried out, so as to minimize machine breakdowns.

ix

Reduce Set Up Times

Set up times should be reduced wherever possible.

x

Total Quality Control

Quality enhancement programs should be adopted, so that total quality control practices can be adopted.

xi

Reduce Lead Times

Reduction in lead times and frequent deliveries should be incorporated.

xii

Minimize Motion Waste

Motion waste should be minimized, so the incorporation of conveyor belts might prove to be a good idea.

Use of JIT in Measuring the Performance

Toyota first pioneered the concept of just-in-time (JIT) manufacturing in the 1970s. Since then, thousands of companies have successfully taken a page from its playbook. The main philosophy behind JIT is to eliminate waste, whether stock, inventory or time. Manufacturers keep a lean supply of materials on hand and produce their products when demanded in rapid-fire fashion. Pulling off a winning JIT strategy requires the right building blocks.

KPIS FOR JIT STRATEGY

In a world where JIT is no longer a novelty, margins are thinner than ever. Delivering faster than others is still a competitive advantage. Picking the right **Key Performance Indicators (KPIs)** to measure is critical to supporting JIT strategy.

Four Key KPIs for JIT

1

Timing

First and foremost, among the right KPIs is timing. "What's your lead time?" is the vital KPI question. The manufacturer will have to be on top of the number of hours or days between taking an order and putting a finished product into the hands of customers.

2

What the Customer Wants

Important to timing is knowing exactly what the customer wants. There is an instance of one manufacturer working with a client who was spending big money to meet same-day production on any orders received by 4 p.m.; but the manufacturer didn't check to see if its customers really wanted that — they didn't. So they were giving a service that wasn't needed, at a high cost.

3

Missed Deliveries

Monitoring missed deliveries is also very important. The key observations are: "Examine by how much you failed"; "If you have a 2% failure rate, is that made up of deliveries that were 15 minutes late? Or days late? There's a significant cost differential in one versus the other." On the other hand, "track your wins and understand why those deliveries worked".

4

Responding to Failures

Monitoring how well an organisation responds when it doesn't meet JIT and customer expectations is a helpful KPI. It is recommended to establish an EDI relationship with the partner company to track all the data. Monitoring the data over time can get a good feel for what is happening. It is suggested baseline requirements be set for in-full and on-time deliveries; which sets the benchmarks so that one can compare them with those of the contracted levels.

JIT Success Stories

SUCCESS STORY **Harley Davidson**

Harley Davidson is one such example, shrinking inventory levels by **75%** while simultaneously raising productivity. Inventory turnover went from two a year to **17**. While controversial from a union perspective, the main factory in York, Pennsylvania, also cut the workforce from about 2,700 to 1,600 during its '80s push to lean out.

SUCCESS STORY **Dell**

Dell is another JIT success story. It stands as unique from many others in that it requires its suppliers to carry inventory. Dell demands they deliver components on short lead times, and Dell then quickly assembles the computers and ships them off to the customer.

SUCCESS STORY **McDonald's**

Even fast-food king McDonald's famously improved its customer service by implementing a version of JIT. High holding costs can lead to slow delivery and wastage in this business. McDonald's changed its approach by adding sophisticated burger-making technology and waiting to make burgers until they are ordered, delivering a higher quality product and cutting down on waste.

KEY LEARNING FROM JIT SUCCESS STORIES

Successful JIT implementation can lead to improved cash flow and as a result also happier customers.

Illustration 10 — Financial Impact of JIT Policy

PROBLEM

B Ltd. has decided to adopt JIT policy for materials. The following effects of JIT policy are identified:

1. To implement JIT, the company has to modify its production and material receipt facilities at a capital cost of ₹10,00,000. The new machine will require a cash operating cost ₹1,08,000 p.a. The capital cost will be depreciated over 5 years.
2. Raw material stockholding will be reduced from ₹40,00,000 to ₹10,00,000.
3. The company can earn 15% on its long-term investments.
4. The company can avoid rental expenditure on storage facilities amounting to ₹33,000 per annum. Property Taxes and insurance amounting to ₹22,000 will be saved due to JIT programme.
5. Presently there are 7 workers in the store department at a salary of ₹5,000 each per month. After implementing JIT scheme, only 5 workers will be required in this department. Balance 2 workers' employment will be terminated.
6. Due to receipt of smaller lots of Raw Materials, there will be some disruption of production. The costs of stock-outs are estimated at ₹77,000 per annum.

Required: Determine the financial impact of the JIT policy. Is it advisable for the company to implement JIT system?

SOLUTION — COST-BENEFIT ANALYSIS OF JIT POLICY

PARTICULARS	AMOUNT (₹)
A. Costs (Per annum)	
Interest on capital for modifying production facilities (₹10,00,000 × 15%) / Interest Income Foregone	1,50,000
Operating Costs of new production facilities	1,08,000
Stock-Outs Costs	77,000
Total Costs	**₹3,35,000**
B. Benefits (Per Annum)	
Interest on investment on funds released due to reduction in raw material stocking (₹40,00,000 – ₹10,00,000) × 15%	4,50,000
Saving in salary of 2 workers terminated (₹5,000 × 12 months × 2)	1,20,000
Saving in Rental Expenditure	33,000
Saving in Property Tax & Insurance	22,000
Total Benefits	**₹6,25,000**
C. Net Benefits = (₹6,25,000 – ₹3,35,000)	
Net Benefit	**₹2,90,000**

Advise: The JIT policy may be implemented, as there is a Net Benefit of ₹2,90,000 per annum. *Note:* Depreciation, being apportionment of capital cost, is ignored in decision-making. Tax Saving on Depreciation is not considered in the above analysis.

Illustration 11 — Altra Video Company (JIT Purchasing Decision)

PROBLEM

Altra Video Company sells packages of blank video tapes. It purchases video tapes from Yash Tape Company at ₹150 per packet. Annual demand is 15,600 packages. The company requires 10% annual return on investment. Purchase order lead time is 2 weeks. Purchase order cost is ₹20 per order. Relevant insurance, material handling etc. is ₹10 per package per year.

Altra Video has to decide whether to shift to JIT purchasing. Yash Tape Company agrees to deliver 100 packages 156 times per year instead of existing 1,200 packages 13 times a year, with additional amount of ₹0.05 per package. Altra Video will incur stock out cost on 50 video tape packages under JIT purchasing. In the event of stock out, Altra has to rush order tape packages, which costs ₹8 per package.

Ram Company also supplies video tapes at ₹145 per package under JIT delivery system. Relevant carrying cost would be ₹9 per package against ₹10 in case of Yash. However, Ram Company does not enjoy a sterling reputation for quality. Altra anticipates: (1) additional inspection cost of ₹0.05 per package; (2) average stock out of 360 packages per year; (3) customers return 2% of all packages due to poor quality, additional cost ₹25 per package.

SOLUTION — COMPARATIVE STATEMENT OF TOTAL RELEVANT COSTS

PARTICULARS	CURRENT POLICY	JIT WITH YASH TAPE CO.	JIT WITH RAM CO.
Cost of Tapes (a × 15,600)	23,40,000	23,40,780	22,62,000
Ordering Costs (b × c)	260	3,120	3,120
Annual Carrying Costs	15,000	1,250	1,175
Stockout Costs	Nil	400	3,600
Inspection Costs	Nil	Nil	780
Customer Return Costs	Nil	Nil	7,800
Total Costs	**23,55,260**	**23,45,550**	**22,78,475**

Observations & Comments:

(a) Cost Saving of implementing JIT purchasing system with Yash Tape Co = ₹23,55,260 – ₹23,45,550 = **₹9,710**. Hence, implementation of JIT system is recommended.

(b) Amongst the three alternatives, JIT with Ram Company results in the least total cost. Hence order may be placed with Ram Company.

⚡ Quick Revision — Section 4.2

- **JIT:** Management technique where goods are received from suppliers only as and when needed; demand-pull system of production.
- **Kanban:** Visual signal indicating it is time to replenish stock.
- **Benefits:** Minimum stock holding costs, eliminates waste, less working capital, high ROI, demand-pull basis, right first time, high quality & efficiency, close relationships, customer satisfaction, eliminates over production.
- **Disadvantages:** Zero tolerance for mistakes, high reliance on suppliers, no buffers for delays, cannot meet unexpected increase, high transaction costs, environmental impact.
- **Four KPIs:** Timing, What Customer Wants, Missed Deliveries, Responding to Failures.
- **Success Stories:** Harley Davidson (75% inventory reduction, turnover 2→17), Dell (suppliers carry inventory, short lead times), McDonald's (make to order).
- **Illustration:** JIT policy net benefit ₹2,90,000 p.a.; Altra Video JIT with Ram Co. lowest total cost.

4.3 Throughput Accounting

CONCEPT · CORE MEASURES · TPAR · THEORY OF CONSTRAINTS · FIVE FOCUSING STEPS · ILLUSTRATIONS

Concept of Throughput Accounting

CIMA DEFINITION

Throughput Accounting (TA) is a variable-cost-accounting presentation based on the definition of throughput (sales minus material and component costs). Sometimes it is referred to as super variable costing because only material costs are treated as variable. It is a management accounting technique used as a performance measure in 'the theory of constraints'.

Throughput accounting is a process used in management accounting that focuses on a company's production efficiency. It looks at the rate at which a company converts its raw materials into finished goods and makes money from them. The purpose of throughput accounting is to **identify any bottlenecks in a production process**. This process allows companies to either eliminate those bottlenecks or use them as efficiently as possible.

THROUGHPUT ACCOUNTING — ALTERNATIVE ACCOUNTING METHODOLOGY

Throughput Accounting is an alternative accounting methodology that attempts to eliminate harmful distortions introduced from traditional accounting practices – distortions that promote behaviours contrary to the goal of increasing profit in the long term.

In traditional accounting, inventory is an asset (in theory, it can be converted to cash by selling it). This often drives undesirable behaviour at companies – manufacturing items that are not truly needed. The Theory of Constraints, on the other hand, considers **inventory to be a liability** – inventory ties up cash that could be used more productively elsewhere.

In traditional accounting, there is also a very strong emphasis on cutting expenses. The Theory of Constraints, on the other hand, considers cutting expenses to be of much less importance than increasing throughput. **Cutting expenses is limited by reaching zero expenses, whereas increasing throughput has no such limitations.**

Core Measures and Terms

CORE MEASURE	DEFINITION
Throughput	The rate at which customer sales are generated less totally variable costs (typically raw materials, sales commissions, and freight). Labour is not considered a totally variable cost unless pay is 100% tied to pieces produced.
Investment	Money that is tied up in physical Assets: product inventory, machinery and equipment, real estate, etc. Formerly referred to in TOC as Inventory.
Operating Expense	Money spent to create throughput, other than totally variable costs (e.g., payroll, utilities, taxes, etc.). The cost of maintaining a given level of capacity.

Four Key Derived Measures

THROUGHPUT ACCOUNTING — DERIVED MEASURES

```
Net Profit = Throughput − Operating Expenses
```

```
Return on Investment = Net Profit ÷ Investment
```

```
Productivity = Throughput ÷ Operating Expenses
```

```
Investment Turns = Throughput ÷ Investment
```

Other Terms Used in TA

Totally Variable Cost

Cost which is incurred only if a product is produced. In many cases only direct materials are considered as totally variable cost. Direct labour is not totally variable, unless piece rate wages are paid.

Capacity Constraint

Any resource within a company that limits its total output. For example, it can be a machine that can produce only a specified amount of a key component in a given time period. There may be more than one capacity constraint in a company, but rarely more than one for a specified product or product line.

Throughput (or Cycle) Time

The average time required to convert raw materials into finished goods ready to be shipped to customer. It includes the time required for activities such as material handling, production processing, inspecting and packaging.

Throughput Efficiency

The relation of throughput achieved to resources used. **Throughput efficiency = Throughput Cost ÷ Actual Factory Cost**

Throughput Time Ratio

The ratio of time spent adding customer value to products and services divided by total cycle time. Also known as the 'ratio of work content to lead time'.

Total Factory Cost (TFC)

With the exception of material costs, in the short run, most factory costs (including direct labour) are fixed. These fixed costs can be grouped together and called total factory costs.

Throughput Accounting Ratio (TAR/TPAR)

DEFINITION

The **throughput accounting ratio** is a metric often used in throughput accounting. This ratio looks at the return a company generates for each hour of work compared to its costs for the same time. Through the throughput accounting ratio, companies can determine the rate at which they are making income from selling their products.

THROUGHPUT ACCOUNTING RATIO

```
TAR = Return per factory hour ÷ Cost per factory hour
```

```
Return per Factory Hour = Throughput per Unit ÷ Product's time taken for the Limited Resource
```

```
Cost per Factory Hour = Total Factory Cost ÷ Total Limited Resource Time Available
```

When TAR = 1, the company generates the same return as it incurs costs. Companies prefer TAR > 1. The higher the ratio, the better. A TAR below 1 means the company cannot recover its fixed costs from the throughput.

Theory of Constraints — Eliyahu M. Goldratt

CORE CONCEPT

The Theory of Constraints is a methodology for identifying the most important limiting factor (i.e., constraint) that stands in the way of achieving a goal and then systematically improving that constraint until it is no longer the limiting factor. In manufacturing, the constraint is often referred to as a **bottleneck**.

The core concept of the Theory of Constraints is that every process has a single constraint and that total process throughput can only be improved when the constraint is improved. A very important corollary to this is that **spending time optimizing non-constraints will not provide significant benefits**; only improvements to the constraint will further the goal (achieving more profit).

Five Focusing Steps of TOC

Step 1 — Identify

Identify the current constraint (the single part of the process that limits the rate at which the goal is achieved).

Step 2 — Exploit

Make quick improvements to the throughput of the constraint using existing resources (i.e., make the most of what you have).

Step 3 — Subordinate

Review all other activities in the process to ensure that they are aligned with and truly support the needs of the constraint.

Step 4 — Elevate

If the constraint still exists (i.e., it has not moved), consider what further actions can be taken to eliminate it from being the constraint. Normally, actions are continued at this step until the constraint has been "broken" (until it has moved somewhere else). In some cases, capital investment may be required.

Step 5 — Repeat

The Five Focusing Steps are a continuous improvement cycle. Therefore, once a constraint is resolved the next constraint should immediately be addressed. This step is a reminder to never become complacent – aggressively improve the current constraint...and then immediately move on to the next constraint.

Basic Logic of Throughput Costing

Throughput costing assigns only unit level spending for direct costs as the cost of products or services. Advocates of throughput costing argue that adding any other indirect cost, past or committed cost, to product cost creates improper incentives to drive down the average cost per unit by making more products than can be used or sold.

Since these are committed costs, making more units with the same level of spending arithmetically reduces the average cost per unit and makes the production process appear to be more efficient. Throughput accounting (costing) avoids this incentive because the cost per unit depends only on the unit level spending (i.e., cost of materials) and not on how many units are made.

Problems with Throughput Accounting

1

Sub-optimal Customer Orders

When throughput accounting is the driving force behind all production scheduling, a customer that has already placed an order for a product, which will result in a sub-optimal profit level for the manufacturing, may find that his order is never filled.

2

Dependence on Scheduling Staff

The company's ability to create the highest level of profitability is now dependent on the production scheduling staff, who decide what products are to be manufactured and in what order.

3

Long-run Variability

Another issue is that all costs are totally variable in the long-run, since the management has the time to adjust them to long-range production volumes.

Reporting Under Throughput Accounting

When the throughput model is used for financial reporting purposes, the format appears slightly different. The income statement includes only direct materials in the cost of goods sold, which results in a **'throughput contribution'** instead of gross margin. All other costs are bounded into an 'Operating Expenses' category below the throughput contribution margin, yielding a net income figure at the bottom.

GAAP CONSTRAINT

The primary change is that throughput accounting does not charge any operating expenses to inventory so that they can be expressed in future period. Instead, all operating expenses are realized during the current period. As a result, any incentive for managers to overproduce is completely eliminated. However, such a report can be used only for internal reporting because of the requirement of generally accepted accounting principles that some overheads should be charged to excess production.

Systematic Changes Required for Acceptance of Throughput Accounting

AREA	THROUGHPUT ACCOUNTING APPROACH	RECOMMENDATION
Inventory Valuation	None of the overhead costs should be assigned to inventory.	GAAP clearly states that cost of overhead must be apportioned to inventory. Throughput accounting loses. Existing system must continue to assign costs irrespective of how throughput principles are used for other decision making (short-range) activities.
Inventory Investment Analysis	Wants to shrink the amount of investment because it includes work-in-progress inventory in this category. Keeps WIP levels down so that waste is reduced in the production system.	Throughput approach is better, for it forces one to analyse all inventory reduction projects in the light of how they together will impact the capacity constraint rather than individually.
Capital Investment Analysis	Has a tight focus on investment only in areas that impact capacity constraint — all other investment proposals are rejected.	It is best to reject the traditional system and conduct analysis based on throughput principles.
Product Costing	A product has only a totally variable cost, which may be far lower than the fully absorbed cost.	For companies selling to Government under cost-plus contracts, lengthy detailed requirements require absorption costing – throughput costing is not a viable solution. For companies not requiring detailed costing justifications, much simpler throughput accounting approach is possible.
Production Scheduling	Allow planners to produce items that result in highest throughput per minute of the capacity constraint.	Production planners should be left alone to schedule production in the traditional manner rather than basing their decisions on short-term profit maximisation. Maximising short-term profit by ignoring orders tantamounts to long-term suicide since customers will leave in droves.
Long-term Planning	Estimate approximate sales levels for each product for a long-time frame, enter into a throughput model and determine what mix of prospective sales will result in the highest level of profitability.	This method is much superior to using throughput costing for short-term production decisions, since long-term planning sidesteps problems by avoiding existing customer orders that will result in low profits.
Price Setting	Margin on products is simple to obtain — just subtract totally variable costs from the price.	Throughput accounting is the clear choice here based on ease of understandability and the speed with which information can be accumulated.

Illustration 12 — Throughput Cost for the Week

PROBLEM

A factory has a key resource (bottleneck) of Facility A which is available for 62,600 minutes per week. The time taken per unit of Product X and Y in Facility A are 5 minutes and 10 minutes respectively. Last week's actual output was 9,500 units of product X and 1,300 units of Product Y. Actual factory cost was ₹1,56,500.

Required: What is the throughput cost for the week?

SOLUTION

Cost per Factory Minute = ₹1,56,500 ÷ 62,600 = **₹2.50**

Standard Minutes of throughput for the week = (9,500 × 5) + (1,300 × 10) = 47,500 + 13,000 = **60,500 minutes**

Throughput Cost for the week = 60,500 × ₹2.50 = **₹1,51,250**

Illustration 13 — Optimum Product Mix Based on Throughput Concept

PROBLEM

Modern Co produces 3 products, A, B and C. Details: Selling price A ₹120, B ₹110, C ₹130. Direct material cost A ₹60, B ₹70, C ₹85. Variable overhead A ₹30, B ₹20, C ₹15. Maximum demand A 30,000 units, B 25,000 units, C 40,000 units. Time required on the bottleneck resource: A 5 hours, B 4 hours, C 3 hours. Total bottleneck hours available = 3,20,000 per month.

SOLUTION — RATE PER FACTORY HOUR

PARTICULARS	A	B	C
Selling price per unit (₹)	120	110	130
Direct material cost per unit (₹)	60	70	85
Throughput per unit (₹)	60	40	45
Time required on the bottleneck resource (hours)	5	4	3
Return per Factory Hour (₹)	12	10	15
Ranking	II	III	I

Allocation of Hours According to Ranking:

- Hours allocated for C (40,000 units × 3 hours) [Rank I] = 1,20,000; Balance = 2,00,000
- Hours allocated for A (30,000 units × 5 hours) [Rank II] = 1,50,000; Balance = 50,000
- Hours allocated for B (balance) [Rank III] = 50,000

Optimum Product Mix: A = 30,000 units; B = 12,500 units; C = 40,000 units

Illustration 14 — Cat Co (Throughput Accounting for Machine Capacity)

PROBLEM

Cat Co makes a product using three machines – X, Y and Z. The product has to pass through all three machines. Machine capacity per week: X 800 units, Y 600 units, Z 500 units. Demand = 1,000 units per week. For every additional unit sold per week, profit increases by ₹50,000. Proposals: (1) Replace machine X — increase capacity to 1,100 units, costs ₹60 Lakhs; (2) Invest in second machine Y — increase capacity by 550 units, cost ₹68 Lakhs; (3) Upgrade machine Z — increase capacity to 1,050 units, cost ₹75 Lakhs.

Required: Which is Cat Co's best course of action under throughput accounting?

SOLUTION

PARTICULARS	PROPOSAL	X	Y	Z	DEMAND
Current capacity per week	—	800	600	500*	1,000
Buy X	1	1,100	600	500*	1,000
Buy Y	2	800	1,150	500*	1,000
Buy Z	3	800	600*	1,050	1,000
Buy X & Z	1, 3	1,100	600*	1,050	1,000
Buy X & Y	1, 2	1,100	1,150	500*	1,000
Buy Z & Y	2, 3	800*	1,150	1,050	1,000
Buy Z, Y & X	1, 2, 3	1,100	1,150	1,050	1,000*

* = bottleneck resource. **Conclusion:** All three machines are to be purchased in the above order to meet the existing demand. *Note:* Under Throughput Costing, decision is taken on the basis of capacity and not on the basis of Capital Cost and Profit Amount.

Illustration 15 — Factory Key Resource (Throughput Cost & Efficiency)

PROBLEM

A factory has a key resource (bottleneck) of Facility A which is available for 31,300 minutes per week. Budgeted factory costs: Direct labour ₹25,000; Indirect labour ₹12,500; Power ₹1,750; Depreciation ₹22,500; Space cost ₹8,000; Engineering ₹3,500; Administration ₹5,000. Product X: Selling Price ₹35, Material Cost ₹20, Time 5 minutes. Product Y: Selling Price ₹35, Material Cost ₹17.50, Time 10 minutes. Actual production: X = 4,750 units, Y = 650 units. Actual factory cost = ₹78,250.

SOLUTION

(i) Total Factory Costs (TFC) = ₹25,000 + ₹12,500 + ₹1,750 + ₹22,500 + ₹8,000 + ₹3,500 + ₹5,000 = **₹78,250**

(ii) Cost per factory minute = ₹78,250 ÷ 31,300 = **₹2.50**

(iii) Return per bottleneck minute:

- Product X = (₹35 – ₹20) ÷ 5 = **₹3**
- Product Y = (₹35 – ₹17.50) ÷ 10 = **₹1.75**

(iv) TA Ratios:

- Product X = ₹3 ÷ ₹2.50 = **1.20** (greater than 1 — increasing production of X will be more profitable)
- Product Y = ₹1.75 ÷ ₹2.50 = **0.70** (less than 1 — decreasing production of Y will reduce losses)

(v) Standard minutes of throughput: (4,750 × 5) + (650 × 10) = 23,750 + 6,500 = **30,250 minutes**

Throughput cost per week = 30,250 × ₹2.5 = **₹75,625**

(vi) Efficiency % = (₹75,625 ÷ ₹78,250) × 100 = **96.6%**

Illustration 16 — New India Company (Income Statements under Different Systems)

PROBLEM

Given below is basic data for New India Company for three years:

YEAR	YEAR 1	YEAR 2	YEAR 3
Planned production (units)	2,500	2,500	2,500
Actual production (units)	2,500	2,500	2,500
Sales (units)	2,500	1,750	3,250
Finished goods inventory Jan 1	0	0	750
Finished goods inventory Dec 31	0	750	0

Sales price per unit ₹48; Direct material ₹12; Direct labour ₹8; Variable manufacturing overheads ₹4; Total variable cost per unit ₹24; Fixed manufacturing overhead ₹30,000; Total absorption cost per unit ₹36; Variable selling and admin cost per unit ₹4; Fixed selling and administrative cost per year ₹5,000.

SOLUTION — COMPARATIVE OPERATING INCOME

YEAR	PRODUCTION (UNITS)	SALES (UNITS)	ABSORPTION COSTING (₹)	MARGINAL COSTING (₹)	THROUGHPUT COSTING (₹)
1	2,500	2,500	15,000	15,000	15,000
2	2,500	1,750	9,000	0	(9,000)
3	2,500	3,250	21,000	30,000	39,000
Total	**7,500**	**7,500**	**45,000**	**45,000**	**45,000**

Observations:

- In Year 1, units produced (2,500) = units sold (2,500); Operating Income remains the same (₹15,000) under all three systems.
- In Year 2, units produced (2,500) > units sold (1,750); Operating Income is highest (₹9,000) under Absorption Costing, and lowest (₹9,000 loss) under Throughput System.
- In Year 3, units produced (2,500) < units sold (3,250); Operating Income is highest (₹39,000) under Throughput System followed by Marginal Costing (₹30,000); lowest (₹21,000) under Absorption Costing.
- When all three years are totalled, units produced (7,500) = units sold (7,500); Operating Income remains the same (₹45,000) under all three systems.

Illustration 17 — T Ltd. (Bottleneck Analysis)

PROBLEM

T Ltd produces a product which passes through two processes — cutting and finishing. Hours available per annum: Cutting 50,000; Finishing 60,000. Hours needed per unit: Cutting 5; Finishing 12. Fixed operating costs per annum excluding direct material: Cutting ₹10,00,000; Finishing ₹10,00,000. Selling price ₹1,000 per unit; only variable cost per unit is direct material ₹400.

SOLUTION

Cutting process capacity = 50,000 ÷ 5 = 10,000 units; **Finishing process capacity** = 60,000 ÷ 12 = 5,000 units

Throughput contribution per unit = ₹1,000 – ₹400 = **₹600 per unit**

Observation: Finishing capacity (5,000 units) is less than the cutting capacity (10,000 units). Therefore, **Finishing Capacity is the bottleneck resource**.

Alternative I — Subcontracting finishing: Increase in throughput contribution per unit = ₹600 – ₹400 = ₹200. Throughput Contribution for 5,000 units = ₹10,00,000; for 7,000 units = ₹14,00,000. **Recommendation:** Subcontracting above the 5,000 level is beneficial.

Alternative II — Outsourcing cutting: Cutting is not the bottleneck — outsourcing is not beneficial.

Alternative III — Additional equipment for cutting: The cutting process has surplus capacity — not suggested to increase non-bottleneck capacity.

⚡ Quick Revision — Section 4.3

- **Throughput Accounting:** Variable-cost-accounting presentation based on throughput (sales minus material costs); only material costs treated as variable.
- **Core Measures:** Throughput, Investment, Operating Expense.
- **Derived Measures:** Net Profit, ROI, Productivity, Investment Turns.
- **TAR:** Return per factory hour ÷ Cost per factory hour; prefer TAR > 1.
- **Theory of Constraints — Five Steps:** Identify → Exploit → Subordinate → Elevate → Repeat.
- **Bottleneck:** Resource that limits total output; only improvements to constraint further the goal.
- **Problems:** Sub-optimal customer orders, dependence on scheduling staff, long-run variability of costs.

4.4 Back-Flush Accounting

CONCEPT · PROCESS · BENEFITS · LIMITATIONS · VARIANTS · JOURNAL ENTRIES · ILLUSTRATIONS

Concept of Backflush Accounting

DEFINITION

Backflush Costing or Backflush Accounting is a product cost accounting approach that, as the name suggests, flushes back the cost from the end of the production process. It is different from the traditional costing system in that this system records costs after the production process ends. As such, the costing process is deferred until the final production of goods and services. This system does away with the requirement of keeping work-in-process accounts and the manual assignment of costs at separate production stages.

Companies that use just-in-time (JIT) inventory systems generally use backflush approach. Backflush costing system does not create any journal entry to record the transactions of raw materials and work in process unless and until the production is completed. Only after the production process ends, backflush costing uses one main journal entry to record the entire inventory, which was used in the production process.

SIMPLE EXAMPLE

A producer estimates the standard cost of ₹10 per unit. Assuming the total number of units that the producer produces is 500 units; after the end of the production cycle, a single journal entry of ₹5,000 will be made. The journal entry would be – Dr. Parts Expense ₹5,000 and Cr. Cash ₹5,000. In any other costing system, several journal entries would have to be made, such as – Dr. Part A Expense and Cr. Cash; Dr. Part B Expense and Cr. Cash; Dr. Part C Expense and Cr. Cash; and so more. As no journal entries are made initially or intermittently, the manager uses standard or normal costing later and works backward to assign a cost to the goods or services. In this way, the costs are "flushed back" to the already completed production cycle.

Backflushing is usually employed in parallel with JIT, where there is no work-in-progress to consider nor does work-in-progress materially fluctuate. What is essential, however, is an accurate bill of materials, good measures of yield, generally effective production control and accurate engineering change notice when yields do change.

BACKFLUSHING FORMULA

```
Number of Raw Material units used from Inventory = Total Production × Listed Unit Count in the Bill of Materials for Each Component
```

Or: Raw material per unit of production as per Bill of Material.

The Backflush Process

Step 1 — Order Receipt

Once a company gets an order, it records only the essential information into the system, such as quantity, delivery date, and the item code. Based on this, the list of materials needed to complete the order is made.

Step 2 — Delivery of Raw Material

When the production is about to start, the company takes the delivery of the raw material and shifts it to the production floor.

Step 3 — Routing of Components

Now software does the routing of all the components for that production order. The cost manager still has a say on what parts and how much quantity to push in.

Step 4 — Production Completion

After the end of the production process, the operator enters all information about the product into the computer. The software then prepares the production report.

Step 5 — Single Journal Entry

Based on that report, the operator in a single transaction assigns materials cost to the production order.

Benefits of Backflush Costing

✓

Simplifies Costing

More useful for companies with complex products or where the production process involves several stages. If such a company uses backflush costing, the accounts department will not have to post journal entries throughout the production process.

✓

Easier Verification

It makes it relatively easier to verify the materials used for production.

✓

Simpler Post-production Issuance

Makes post-production issuance simpler.

✓

Easier Inventory Tracking

It makes it easier to track the inventory.

✓

Check on Reverse Issuance

When handling bulk materials, it keeps a check on the reverse issuance of materials.

BEST SUITED FOR

Backflush system works best for products with short production times and the ones that use JIT (just-in-time) inventory systems. Usually, the companies that use this costing are those: (a) That want a simple accounting system; (b) That are okay with assigning standard costs to every product; (c) Where the inventory of the raw materials is either low or constant.

Limitations of Backflush Costing

1

Not Suitable for Long Manufacturing Processes

One should not use this system for products that take too much time to produce. The more time it takes, the more difficult it becomes to assign costs correctly.

2

Variance with Actual Costs

Since this costing system works backward to assign costs after the end of the production, it often assigns standard costs to the product. It could result in variance with the actual costs.

3

Not Suitable for Custom Orders

Custom orders would require separate invoices for each material that is used in the production of custom orders.

4

Not Suitable for Slow Inventory Turnover

Backflush costing system is not suitable for companies with slow inventory turnover.

5

Not In-line with GAAP

One big drawback is that it is not in-line with the generally accepted accounting principles (GAAP), and thus makes it difficult to audit.

6

Statutory Reports & Tax Calculations

Statutory Reports and Tax calculations cannot be done on figures obtained by Back Flush costing / Accounting.

Other Drawbacks

- It is relatively difficult to implement.
- For the results to be accurate, this system needs an accurate production count.
- Its success also depends on the accuracy of the bill of materials.
- Scrap reporting also needs to be accurate — the bill of material does not account for scrap.
- Since this system does not record the work-in-process inventory, it needs a fast production cycle time.

Variants of Backflush Accounting

There are a number of variants of the Backflush system, each differing as to the **'trigger points'** at which costs are recognized within the cost accounts and thus associated with products.

COMMON FEATURES OF ALL VARIANTS

- The focus is on **output** — costs are first associated with output (measured as either sales or completed production) and then allocated between stocks and costs of goods sold by working back.
- **Conversion costs** (labour and overheads) are never attached to products until they are complete (or even sold) — thus, the traditional WIP account doesn't exist. Materials are recognized at different points according to the variant used, but only to the extent of being either stock of raw materials or part of the cost of stock of finished goods. Again, materials are not attached to WIP.

VARIANT	TRIGGER POINTS	ACCOUNTS USED	DESCRIPTION
Variant 1	Two Trigger Points: (i) Purchase of raw materials/components; (ii) Completion of good units	Raw Material in Process (RIP) account; Conversion Cost (CC) account; Finished Goods (FG) account; Creditors account	**TP 1:** A 'Raw Material in Process (RIP)' account is debited with the actual cost of materials purchased and creditors credited. **TP 2:** The Finished Goods (FG) account is debited with the standard cost of units produced and the RIP and CC account credited with the standard cost. There will be two stock accounts — (i) raw materials (ii) finished goods.
Variant 2	One Trigger Point: Completion of good units	Finished Goods (FG) account; Conversion Cost (CC) account; Creditors account	The FG account is debited with the standard cost of units produced, with corresponding credits to the CC account and the creditors account. The cost records exclude raw materials purchased but not yet used for complete production, and the creditors for these materials. There is only one stock account, carrying the standard cost of finished goods stock.

Other Variants: Those using the sale of completed good units as a trigger point for the attachment of conversion cost to unit — thus, there is no finished goods account, just a raw materials stock account, carrying the materials cost of raw materials, WIP and finished goods. As stock of raw materials, WIP and finished goods are decreased to minimal levels, as in a 'pure' JIT system, these variants will give the same basic results.

Illustration 19 — Journal Entries under Variant 1 and Variant 2

PROBLEM

The manufacturing cost information for March for a division of XYZ Co. is as follows: Purchase of Raw Materials ₹42,50,000; Labour ₹28,00,000; Overheads ₹16,40,000. Activity in March: Finished goods manufactured 180,000 units; Sales 145,000 units. Standard cost per unit: Materials ₹20; Labour ₹15; Overheads ₹9; Total ₹44. There were no opening stocks of raw materials, WIP or finished goods. No direct materials variance for the period.

Required: Show the relevant Journal entries and ledger accounts in Variant 1 and Variant 2 of backflush system.

SOLUTION — VARIANT 1 (TWO TRIGGER POINTS)

SL	PARTICULARS	DR. (₹ '000)	CR. (₹ '000)
1	RIP account — To, Creditor	4,250	4,250
2	Conversion Cost account — To, Cash; To, Cash/creditor	4,440	2,800; 1,640
3	FG account (180 × 44) — To, RIP account (180 × 20); To, Conversion Cost account (180 × 24)	7,920	3,600; 4,320
4	COGS (145 × 44) — To, FG account	6,380	6,380

Stock balances at end of March: Raw and in process material ₹650 '000; Finished goods ₹1,540 '000; Total ₹2,190 '000. The balance on the Conversion Cost (CC) Account will be carried forward and written off at the end of the year.

Variant 2 (One Trigger Point — on completion of units): Entry 1 (Conversion Cost) and Entry 3 (FG account) and Entry 4 (COGS) are the same as Variant 1, but there is no RIP account. Variant 2 is suitable for JIT system with minimal raw materials stocks.

Illustration 20 — Dandia Ltd. (Traditional vs Backflush Journal Entries)

PROBLEM

Dandia Ltd. follows JIT system. Transactions in May 2024: Raw materials purchased ₹2,00,000; Direct labour cost incurred ₹36,000; Actual overhead costs ₹3,00,000; Conversion costs applied ₹3,16,000. All materials purchased were placed into production and production was completed and sold during the month.

Required: Pass both Traditional journal entries and Backflush journal entries.

SOLUTION — TRADITIONAL JOURNAL ENTRIES

PARTICULARS	DEBIT (₹)	CREDIT (₹)
Material A/c — To, Accounts Payable (Purchase of raw materials)	2,00,000	2,00,000
WIP A/c — To, Materials A/c (Materials issued to production)	2,00,000	2,00,000
WIP A/c — To, Direct wages A/c (Direct labour cost incurred)	36,000	36,000
Overhead Control A/c — To, Accounts Payable (Overhead cost incurred)	3,00,000	3,00,000
WIP A/c — To, Overhead Control A/c (Application of overhead)	2,80,000	2,80,000
Finished Goods A/c — To, WIP A/c (Completion of goods)	5,16,000	5,16,000
Cost of Goods Sold A/c — To, Finished Goods (Cost of finished goods sold transferred)	5,16,000	5,16,000
Cost of Goods Sold A/c — To, Overhead Control A/c (Variance recognized)	20,000	20,000

Backflush Journal Entries:

PARTICULARS	DEBIT (₹)	CREDIT (₹)
Raw Material in Process A/c — To, Accounts Payable (Purchase of raw materials)	2,00,000	2,00,000
Conversion Cost Control A/c — To, Direct wages A/c; To, Accounts Payable (Overhead cost incurred)	3,36,000	36,000; 3,00,000
Finished Goods A/c — To, Raw Material in Process A/c; To, WIP A/c (Completion of goods)	5,16,000	2,00,000; 3,16,000
Cost of Goods Sold A/c — To, Finished Goods (Cost of finished goods sold transferred)	5,16,000	5,16,000
Cost of Goods Sold A/c — To, Overhead Control A/c (Variance recognized)	20,000	20,000

⚡ Quick Revision — Section 4.4

- **Backflush Costing:** Product cost accounting approach that flushes back the cost from the end of the production process.
- **Key Feature:** Records costs after the production process ends; does away with WIP accounts and manual assignment of costs at separate production stages.
- **Best Suited:** Companies using JIT with short production times, low/constant raw material inventory, and standard costs.
- **Benefits:** Simplifies costing, easier verification, simpler post-production issuance, easier inventory tracking, check on reverse issuance.
- **Limitations:** Not suitable for long manufacturing processes, variances with actual costs, not suitable for custom orders, not in-line with GAAP, statutory reports and tax calculations cannot be done.
- **Variants:** Variant 1 (two trigger points — purchase and completion), Variant 2 (one trigger point — completion).
- **Formula:** Raw material units used = Total Production × Listed Unit Count in the Bill of Materials.

4.5 Benchmarking

CONCEPT · TYPES · PROCESS · PRE-REQUISITES · DIFFICULTIES · CASE STUDY

Concept of Benchmarking

DEFINITION

Benchmarking is the continuous process of measuring products, services or activities against the best levels of performance that may be found either inside or outside the organisation. It is a process of comparing a firm's activities with best practices.

It is believed that the term benchmark originates from the history of guns and ammunition, and with the same aim as for the business term — comparison and improved performance. The process involves establishment of benchmarks (targets or comparators), through the use of which the levels of performance of the organisation is sought to be improved.

The idea behind benchmarking is to measure internal processes against a chosen standard. Benchmarking is used to measure the internal performance using a specific indicator encompassing cost, time or quality resulting in a metric of performance that can be compared to others. The examples of indicators include cost per unit of measure, productivity per unit of measure, cycle time of x per unit of measure or defects per unit of measure.

WHY BENCHMARKING WORKS

Benchmarking is a powerful management tool because it overcomes **"paradigm blindness"** and overcomes the thinking, *"the way we do it is the best because this is the way we've always done it."* Benchmarking opens organisations to new methods, ideas and tools to improve their effectiveness. It helps crack through resistance to change by demonstrating other methods of solving problems than the one currently employed and demonstrating that they work, because they are being used by others.

Benefits: The benefits of benchmarking include several avenues of cost reduction and cost control such as reducing labour cost, streamlining the work flow, and optimising productivity.

Types of Benchmarking

1

Product Benchmarking (Reverse Engineering)

An age-old practice of product-oriented reverse engineering. Every organisation buys its rival's products and tears them down to find out how the features and performances etc. compare with its products. This could be the starting point for improvement.

2

Competitive Benchmarking

Looks at a company's direct competitors and evaluates how the company is doing in comparison. Moves beyond product-oriented comparisons to include comparisons of process with those of competitors. The process studied may include marketing, finance, HR, R&D etc. Knowing the strengths and weaknesses of the competition is important in plotting a successful strategy and can help prioritize areas of improvement as specific customer expectations are identified.

3

Process Benchmarking

Consists of a mechanism for identifying specific work procedures that could be improved by imitating external examples of excellence that can be set as the best standard in the industry. Involves comparison of one's own utility with other similar utilities, with the purpose of self-improvement through adopting structures or methods that happen to be successful elsewhere. The goal is to improve different stages of the production process and to increase efficiency by "learning from others".

4

Internal Benchmarking

An application of process benchmarking, within an organisation by comparing the performance of similar business units or business process. Analysis of existing practice within various departments or divisions of the organisation, looking for best performance as well as identifying baseline activities and drivers. Organisations collect data on their own performance at different points in time and under different circumstances and identify gaps or areas for strengthening.

5

Strategic Benchmarking

Used to describe the situation when a firm is interested in comparing its performance versus the best-in-class or what is deemed as world-class performance. Often involves looking beyond the firm's core industry to firms that are known for their success with a particular function or process. Example: Southwest Airlines analysed the processes, approaches, and speed of automobile racing pit crews to gain ideas for improving their airplane turn-around time at the gate — reportedly saved millions of dollars per year.

6

Global Benchmarking

An extension of Strategic Benchmarking to include benchmarking partners on a global scale. Example: Ford Co. of USA benchmarked its account payable functions with that of Mazda in Japan and found to its astonishment that the entire function was managed by 5 persons as against 500 in Ford.

Process of Benchmarking

Planning → Collection of Data → Analysis of Data → Implementation → Monitoring

Stage 1 — Planning

Planning starts with determination of benchmarking goal statement. It is imperative that the organisation identifies the activities that need to be benchmarked prior to engaging in benchmarking. Since benchmarking can be applied to any business process or function, a range of research techniques may be required — informal conversations with customers, employees or suppliers; exploratory research techniques, re-engineering analysis, process mapping, quality control variance reports, financial ratio analysis, or reviewing cycle times. Second step is Identification of best performance (seeking the "best"). Before embarking on comparison with other organisations it is essential to know the organisation's functions and processes. Base lining performance provides a point against which improvement effort can be measured. The third step is establishment of the benchmarking or process improvement team. The last step in planning is defining the relevant benchmarking measures.

Stage 2 — Collection of Data and Information

This stage involves: (a) Compiling information and data on performance (may include mapping processes); (b) Selecting and contacting partners; (c) Developing a mutual understanding about procedures to be followed; (d) Preparing questions and conceiving terminology and performance measures; (e) Distributing a schedule of questions to each partner; (f) Undertaking information and data collection by chosen method (interviews, site-visits, telephone, fax, e-mail); (g) Collecting the findings to enable analysis. Data can be primary (directly from the benchmarked organisation) or secondary (media, publications, internet).

Stage 3 — Analysis of Data

Once sufficient data is collected, proper analysis is of foremost importance. The process may consist of: (a) Reviewing findings and producing tables, charts and graphs; (b) Identifying gaps in performance between our firm and better performers; (c) Seeking explanations for the gaps; (d) Ensuring comparisons are meaningful and credible; (e) Communicating findings to those affected; (f) Identifying realistic opportunities for improvements. The performance gaps can be positive, negative or zero.

Stage 4 — Recommendation & Implementation

This is the stage where it becomes mandatory to walk the talk for success. Far reaching changes need to be made so that the performance gap between the target and the actual is narrowed and eliminated. It starts with deciding the feasibility of making the improvements in the light of conditions that apply within own firm. A formal action plan, that promotes change, is to be formulated keeping the culture of the organisation in mind so that resistance that normally accompanies change is minimized. The commitment of management and staff is to be fully ensured for the process.

Stage 5 — Monitoring & Review

Benchmarking process needs to be properly monitored in order to reap the maximum benefit. This could involve: (a) Evaluating the benchmarking process undertaken and the results of the improvements against objectives and success criteria; (b) Documenting the lessons learnt and making them available to others; (c) Periodically re-considering the benchmarks for continuous improvement. A systematic evaluation is required to be carried out on a regular basis.

Pre-requisites of Benchmarking

1

Commitment

Senior Managers should support benchmarking fully and must be committed to continuous improvements.

2

Clarity of Objectives

The objectives should be clearly defined at the preliminary stage. Benchmarking teams must have a clear picture of their firm's performance before approaching others for comparisons.

3

Appropriate Scope

The scope of the work should be appropriate in the light of the objectives, resources, time available and the experience level of those involved.

4

Resources

Sufficient resources must be made available to complete projects within the required time scale.

5

Skills

Benchmarking teams should have appropriate skills and competencies.

6

Communication

Stakeholders, and also staff and their representatives, are to be kept informed of the reasons for benchmarking.

Difficulties in Implementation of Benchmarking

1

Time Consuming

Benchmarking is time consuming and at times difficult. It has significant requirement of staff time and Company resources. Companies may waste time in benchmarking non-critical functions.

2

Lack of Management Support

Benchmarking implementation requires the direct involvement of all managers. The drive to be best in the industry or world cannot be delegated.

3

Resistance from Employees

It is likely that there may be resistance from employees.

4

Paper Goals

Companies can become pre-occupied with the measures. The goal becomes not to improve process, but to match the best practices at any cost.

5

Copy-paste Attitude

The key element in benchmarking is the adaptation of a best practice to tailor it to a company's needs and culture. Without that step, a company merely adopts another company's process. This approach condemns benchmarking to fail.

Case Study: Drive-Thru Practice — Pal's Sudden Service

CASE STUDY **Benchmarking in Fast Food — Pal's Sudden Service**

A quick service (fast food) restaurant chain dependent upon speedy, accurate service in the drive-thru to maximize efficiency, cut costs and increase profits may study the drive-thru practices of key competitors. Every second gained without sacrificing customer quality allows the firm to increase profits. Over the years, competitors have consistently innovated in their drive-thru operations with configuration, number of windows, menu and speaker boards and ordering approaches in an attempt to improve in this area. They are constantly watching and benchmarking against each other.

Pal's Sudden Service — a small hamburger and hot dog chain and a Baldrige Quality Award winner — is very successful at achieving best-in-class performance for drive-thru and overall restaurant operations. Pal's does not offer sit-down service inside its restaurants. Instead, customers pull up to a window, place their orders face-to-face with an employee (no scratchy loudspeakers), pull around to the other side of the facility, take their bag, and drive on.

Performance Metrics:

- Average of **18 seconds** at the handout window to place an order
- Average of **12 seconds** at the drive-up window to receive the order
- That's **four times faster** than the second-fastest quick serve restaurant, which requires more than a minute on average to take an order.

Benchmarking Impact: Many companies in the fast-food market use Pal's as a best-in-class benchmark for their own firms. It is no wonder that Pal's opened an educational institute to train other organisations.

Quick Revision — Section 4.5

- **Benchmarking:** Continuous process of measuring products, services or activities against the best levels of performance found either inside or outside the organisation.
- **Six Types:** Product (Reverse Engineering), Competitive, Process, Internal, Strategic, Global.
- **Process:** Planning → Collection of Data → Analysis → Implementation → Monitoring.
- **Pre-requisites:** Commitment, Clarity of Objectives, Appropriate Scope, Resources, Skills, Communication.
- **Difficulties:** Time consuming, lack of management support, resistance from employees, paper goals, copy-paste attitude.
- **Example:** Pal's Sudden Service — 18 seconds to place order, 12 seconds to receive order; four times faster than second-fastest QSR.

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Activity Based Costing System

Cost attribution to cost units on the basis of benefit received from indirect activities e.g. ordering, setting up, and assuring quality. Products consume activities and activities consume costs.

Activity

An aggregate of closely related tasks having some specific functions which are used for completion of a goal or objective.

Resource

Elements that are used for performing the activities or factors helping in the activities.

Cost

The amount paid for the resources consumed by an activity.

Cost Object

An item for which cost measurement is required. e.g., a product, a service, or a customer.

Cost Pool

A grouping of costs incurred on a particular activity which drives them.

Cost Driver

Any element that would cause a change in the cost of an activity. Cost drivers are the basis of charging cost of activity to cost object.

Activity Based Management

A set of actions that management can take, based on information from an Activity Based Costing system, to improve profitability.

Activity Based Budgeting

A budgeting method where activities are thoroughly analysed to predict costs.

Activity Based Responsibility Accounting

An accounting system that assigns responsibility to processes and uses both financial and nonfinancial measures of performance.

Just-In-Time

A management technique in which goods are received from suppliers only as and when they are needed. The main objective is to reduce inventory holding costs and increase inventory turnover.

Throughput Accounting

Variable-cost-accounting presentation based on the definition of throughput (sales minus material and component costs). Sometimes referred to as super variable costing because only material costs are treated as variable.

Theory of Constraints

A methodology for identifying the most important limiting factor (i.e., constraint) that stands in the way of achieving a goal and then systematically improving that constraint until it is no longer the limiting factor.

Backflush Costing

A product cost accounting approach that flushes back the cost from the end of the production process.

Benchmarking

The continuous process of measuring products, services or activities against the best levels of performance that may be found either inside or outside the organisation.

Multiple Choice Questions

WITH ANSWERS AND REASONING

1. P operates an activity-based costing (ABC) system. In its budget for the year ending 31st March 2024, the company expected to place a total of 2,895 purchase orders at a total cost of ₹1,10,010. During the four-week period ended 30 June 2023, a total of 210 purchase orders were placed at a cost of ₹7,650. The over-recovery of these costs for the four-week period was:

A. ₹330 B. ₹350 C. ₹370 D. ₹390

Answer: A

Cost driver rate = ₹1,10,010 ÷ 2,895 = ₹38 per order. Cost recovered for 210 orders = 210 × ₹38 = ₹7,980. Actual costs = ₹7,650. Over-recovery = ₹7,980 − ₹7,650 = ₹330.

2. A company manufactures and sells packaging machines. Each machine requires: direct materials ₹50,000; 50 equipment parts; 12 machine hours; 15 assembly line hours; 4 inspection hours. Overhead rates: ₹8/part, ₹68/machine hour, ₹75/assembly hour, ₹104/inspection hour. The order is for 40 machines. Total indirect costs allocated:

A. ₹1,30,850 B. ₹1,25,280 C. ₹1,15,050 D. ₹1,10,280

Answer: D

Indirect cost per machine = (50 × 8) + (12 × 68) + (15 × 75) + (4 × 104) = 400 + 816 + 1,125 + 416 = ₹2,757. For 40 machines = 40 × 2,757 = ₹1,10,280.

3. Process of Cost allocation under Activity Based Costing is:

A. Cost of Activities → Activities → Cost Driver → Cost allocated to cost objects

B. Cost Driver → Cost of Activities → Cost allocated to cost objects → Activities

C. Activities → Cost of Activities → Cost Driver → Cost allocated to cost objects

D. Activities → Cost Driver → Cost allocated to cost objects → Cost of Activities

Answer: C

4. At KL Company, total personnel department costs are ₹2,00,000. Department A: 30 employees, 8 new hires; Department B: 270 employees, 12 new hires; Department C: 100 employees, 5 new hires. If number of new hires is the cost driver, what amount of cost will be allocated to Department A?

A. ₹15,000 B. ₹64,000 C. ₹72,000 D. ₹40,000

Answer: B

Total new hires = 8 + 12 + 5 = 25. Cost per new hire = ₹2,00,000 ÷ 25 = ₹8,000. Department A = 8 × ₹8,000 = ₹64,000.

5. Cost Driver is:

A. Grouping of costs on a particular activity which drives them

B. Item for which cost measurement is required

C. Elements that would cause a change in the cost activity

D. All of the above

Answer: C

6. ABC Management:

A. Accurately identifies sources of profit and loss

B. Assigns costs using measure of service consumed

C. Recognizes the casual relationship of cost drivers to activities

D. All of the above

Answer: D

7. Which of the following is not suitable for a JIT production system?

A. Batch production B. Jobbing production C. Process production D. Service production

Answer: A

Batch production uses stocks to supply customers whilst other products are being produced. Stocks are avoided in a JIT system. Jobbing production makes products to customer order and is ideal for JIT.

8. Kanban Japanese System under JIT approach ensures that:

A. Continuous supply of inventory or product

B. Minimum & maximum level of stock to be maintained

C. Inventory valuation

D. All of the above

Answer: A

9. JIT relates to:

A. Time Management B. Inventory and product handling C. Delivery systems D. None of the above

Answer: B

10. Glasso, a manufacturer of large windows, is experiencing a bottleneck in its plant. Setup time at one of its workstations has been identified as the culprit. A manager has proposed a plan to reduce setup time at a cost of ₹7,20,000. The change will result in 800 additional windows. The selling price per window is ₹18,000, direct labour costs are ₹3,000 per window, and the cost of direct materials is ₹7,000 per window. Assume all units produced can be sold. The change will result in an increase in the throughput contribution of:

A. ₹64,00,000 B. ₹88,00,000 C. ₹56,80,000 D. ₹1,44,00,000

Answer: B

Throughput contribution per window = ₹18,000 − ₹7,000 = ₹11,000. Total = ₹11,000 × 800 = ₹88,00,000.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
A	D	C	B	C	D	A	A	B	B

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

1
Elaborate the concept and purpose of Activity Cost Management.

2
What are the vital stages of implementation of Activity Based Costing?

3
Write a note on Activity Based Budgeting.

4
Differentiate between Traditional Cost systems and ABC systems.

5
What are the benefits of just-in-time manufacturing systems?

6
What are the precautions that should be taken while implementing a just-in-time manufacturing system?

7
What is the need for throughput accounting?

8
What are the core measures and terms which are used in throughput accounting?

9
Write a note on theory of constraints.

10
Discuss the process of back-flush costing.

11
What are the limitations of back-flush costing?

12
What are the noteworthy types of benchmarking?

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 4

ABBREVIATION	FULL FORM
ABB	Activity Based Budgeting
ABC	Activity Based Costing
ABCM	Activity Based Cost Management
BPR	Business Process Reengineering
CC	Conversion Cost
COGS	Cost of Goods Sold
ERP	Enterprise Resource Planning
FG	Finished Goods
JIT	Just-In-Time
KPI	Key Performance Indicators
RIP	Raw & In Process Material
TQM	Total Quality Management
TA	Throughput Accounting
TFC	Total Factory Costs
TOC	Theory of Constraints
TPAR	Throughput Accounting Ratio
WIP	Work in Process
EDI	Electronic Data Interchange
VMI	Vendor Managed Inventory

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Evaluating Performance

Module 5 · Variance Analyses · Uniform Costing · Inter-firm Comparison

EDITION	PAPER	SECTION	MODULE WEIGHTAGE
December 2026	Paper 16 · Strategic Cost Management	Section A · Weightage 60%	15% of Section A

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading ✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

05

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 5 INCLUDES

5.1 Variance Analyses · **5.2** Uniform Costing and Inter-firm Comparison.

Module Learning Objectives

After studying this module, the students will be able to:

1

Management by Exception

Acquiring detailed insights into the principles of Management by Exception.

2

Standard Costing & Budgetary Control

Command over principles of Standard Costing and Budgetary Control.

3

Variance Analysis

Expertise in Variance Analysis and Evaluation of Performance.

15%	2	24+	16
WEIGHTAGE IN SECTION A	CORE SUB-TOPICS	ILLUSTRATIONS	MCQS
Of 60% Section A total	5.1 to 5.2	Worked examples	With reasoning

SYLLABUS STRUCTURE NOTE

Module 5 carries **15% weightage** within Section A. It covers the critical area of performance evaluation through variance analysis, standard costing, budgetary control, uniform costing and inter-firm comparison. This module is highly numerical and requires a thorough understanding of all variance formulae and their inter-relationships.

📖 Table of Contents — Module 5

1. Variance Analyses — Introduction, Revenue Variance, Cost Variance, Material, Labour, Overhead Variances
2. Investigation of Variances — Factors, Methods (Managerial Intuition, Expected Value, Statistical Control Chart)
3. Planning and Operating Variances — Sales, Material, Labour
4. Controllable and Non-Controllable Variances
5. Relevant Cost Approach to Variance Analysis
6. Variance Analysis under Marginal Costing and Absorption Costing
7. Activity Ratios — Efficiency, Activity, Calendar, Capacity Usage, Capacity Utilisation, Idle Time
8. Application of Budgetary Control and Standard Costing in Profit Planning
9. Standard Costing vs Budgetary Control
10. Reconciliation of Actual Profit with Standard Profit
11. Uniform Costing and Inter-firm Comparison

5.1 Variance Analyses

INTRODUCTION · REVENUE VARIANCE · COST VARIANCE · MATERIAL · LABOUR · OVERHEAD VARIANCES

Introduction (Recapitulation)

DEFINITION OF VARIANCE

Variance, by definition, denotes the deviation between the standard proposition and the actual incidence. The proposition could be a preset benchmark, budget or estimate and so on. The concept of variance is intrinsically connected with planned and actual results and effects of the difference between these two on the performance of the entity.

BASIC VARIANCE FORMULA

```
Variance = (Standard Proposition) − (Actual Incidence)
```

Variance analysis involves breaking down and analyzing the total variance to explain:

- **Quantity:** How much of the variance is caused by using the resources that are different from the standards, i.e., the quantity variance; and
- **Rate:** How much of the variance is caused by the cost of the resources being different from the standards, i.e., the rate (price) variance.

MAIN OBJECTIVE OF VARIANCE ANALYSIS

To provide insights into the off-benchmark performance. It helps management to improve operations and correct errors on a concurrent basis; deploy resources more effectively; and thus control and reduce costs and enhance revenues. An important feature is that it drives the enterprise towards quantitative analysis of inputs and outputs whereby optimum productivity is achieved. **Variance analysis is a tool that facilitates management by exception.**

Revenue Variance

REVENUE VARIANCE FORMULAE

$$\text{Revenue Variance} = (SR - AR) = (SQ \times SP) - (AQ \times AP)$$

$$\text{Revenue Quantity Variance} = SP\ (SQ - AQ)$$

$$\text{Revenue Price Variance} = AQ\ (SP - AP)$$

Where: SR = Standard Revenue; AR = Actual Revenue; SQ = Standard Quantity; SP = Standard Price; AQ = Actual Quantity; AP = Actual Price.

Cost Variance

GENERAL COST VARIANCE FORMULA

$$\text{Cost Variance} = (SC - AC) = (SQ \times SP) - (AQ \times AP)$$

$$\text{Usage Variance} = SP\ (SQ - AQ)$$

$$\text{Price Variance} = AQ\ (SP - AP)$$

Cost Variances may be categorized element-wise: Direct Material Cost Variance, Direct Labour Cost Variance, Direct Expense Variance, Production Overhead Variance, Administration Overhead Variance, Selling Overhead Variance and Distribution Overhead Variance. They can also be broken down behaviour-wise into Variable Cost Variance and Fixed Cost Variance.

KEY INSIGHT

Usage Variance brings out the deviations in the cost of an element arising from consumption of non-standard elements. It is generally impacted by the factors of input mix and yield. **Price Variance** denotes the difference between the standard price and the actual price for the actual quantity of the element consumed.

Material Cost Variance

MATERIAL VARIANCE FORMULAE

$$\text{Material Cost Variance} = (SC - AC) = (SQ \times SP) - (AQ \times AP)$$

$$\text{Material Usage Variance} = SP\ (SQ - AQ)$$

$$\text{Material Price Variance} = AQ\ (SP - AP)$$

$$\text{Material Mix Variance} = SP \times (\text{Revised Standard Quantity} - \text{Actual Quantity})$$

$$\text{Material Yield Variance} = SP \times (\text{Standard Yield} - \text{Actual Yield})$$

Reasons for Material Usage Variance

a

Inefficient or Careless Use

Variation in usage of materials due to inefficient or careless use, or economic use of materials.

b

Changes in Specification

Changes in the specification or design of the product.

c

Inferior Materials

Purchase of inferior materials or change in quality of materials.

d

Wastages

Inefficiency in production resulting in wastages.

e

Substitute Materials

Use of substitute materials.

f

Theft or Pilferage

Theft or pilferage of materials.

g

Inefficient Labour

Inefficient labour force leading to excessive utilisation of materials.

h

Yield from Materials

Yield from materials in excess of or less than that provided as the standard yield.

i

Inaccurate Standards

Inaccurate standards.

j

Change in Composition

Change in composition of a mixture of materials for a specified output.

Reasons for Material Price Variance

a

Basic Purchase Price

Change in basic purchase price of material.

b

Purchase Quantity

Change in quantity of purchase or uneconomical size of purchase order.

c

Rush Order

Rush order to meet shortage of supply, or purchase in less or more favourable market.

d

Transit Losses

Transit losses and discrepancies.

e

Quality/Specifications

Change in quality or specifications of material purchased.

f

Substitute Material

Use of substitute material having a higher or lower unit price.

g

Taxes and Duties

Change in the pattern or amounts of taxes and duties.

Labour Cost Variance

LABOUR VARIANCE FORMULAE

$$\text{Labour Cost Variance} = (SC - AC) = (ST \times SR) - (AT \times AR)$$

$$\text{Labour Rate Variance} = AT\ (SR - AR)$$

$$\text{Labour Efficiency Variance} = SR\ (ST - AT)$$

$$\text{Labour Mix Variance} = SR \times (\text{Revised Standard Hours} - \text{Actual Hours})$$

$$\text{Labour Yield Variance} = SR \times (\text{Standard Hours} - \text{Revised Standard Hours})$$

$$\text{Idle Time Variance} = \text{Standard Rate per Hour} \times \text{Idle Time}$$

Where: ST = Standard Time; SR = Standard Rate; AT = Actual Time; AR = Actual Rate.

Reasons for Labour Rate Variance

a

Wage Structure

Change in basic wage structure or change in piece-work rate.

b

Employment of Different Grades

Employment of workers of grades and rates of pay different from those specified, due to shortage of labour of the proper category, or through mistake, or due to retention of surplus labour.

c

Guaranteed Wages

Payment of guaranteed wages to workers who are unable to earn their normal wages if such guaranteed wages form part of direct labour cost.

d

Overtime and Night Shift

Overtime and night shift work in excess of or less than the standard, or where no provision has been made in the standard.

e

Composition of Gang

The composition of a gang as regards the skill and rates of wages being different from that laid down in the standard.

Reasons for Labour Efficiency Variance

a

Lack of Proper Supervision

Lack of proper supervision.

b

Poor Working Conditions

Poor working conditions.

c

Delays

Delays due to waiting for materials, tools, instructions, etc.

d

Defective Machines

Defective machines, tools and other equipments.

e

Machine Breakdown

Machine break-down.

f

Basic Inefficiency

Basic inefficiency of workers due to low morale, insufficient training, faulty instructions, incorrect scheduling of jobs, etc.

g

Non-standard Material

Use of non-standard material requiring more or less operation time.

h

Labour Turnover

Increase in labour turnover.

Overhead Cost Variance

OVERHEAD VARIANCE FORMULAE

$$\text{Overhead Variance} = (SC - AC) = (SB \times SR) - (AB \times AR)$$

$$\text{Overhead Volume Variance} = \text{Standard Rate} \times (\text{Actual Units} - \text{Standard Units})$$

$$\text{Overhead Expenditure Variance} = \text{Actual Units} \times (\text{Standard Rate} - \text{Actual Rate})$$

Overhead Volume Variance is quantitative in nature and denotes the difference between cost for the actual quantum of the base at the standard overhead rate and the cost for the standard quantum at standard overhead rate. Overhead Efficiency Variance denotes the difference between the cost for the production achieved at standard overhead rate and the cost for the actual quantum of the base at the standard overhead rate.

Comprehensive Illustration — XPML Variance Analysis

PROBLEM — MONTHLY DATA OF PRODUCTION AND COST

SERIAL	ITEM	STANDARDS	ACTUALS
1	Working Days	30	29
2	Production in MT	1025	1060
3	Sale price in ₹ Per MT	40250	40000
4	Raw Material — Quantity in MT	1250	1285
5	Raw Material — Rate per MT (₹)	20000	20250
6	Workers — Number of Workers	80	80
7	Workers — Man Days	2400	2320
8	Workers — Wage Rate in ₹ per Day	700	725
9	Power — KWH per MT of Production	650	640
10	Power — ₹ Per KWH	7.10	7.10
11	Fuel — MT per MT of Production	0.60	0.58
12	Fuel — ₹ Per MT	3000	3100
13	Chemical Consumption (₹ Per MT)	1800	1750
14	Wire Clothing (₹ Per MT)	400	380
15	Packing Material (₹ per MT)	300	320
16	Fixed Expenses — Factory (₹ Lakhs)	12.00	11.50
17	Fixed Expenses — Admn. (₹ Lakhs)	18.00	20.00
18	Fixed Expenses — Selling (₹ Lakhs)	12.00	12.50

SOLUTION — SUMMARY OF VARIANCE ANALYSIS (₹ IN LAKHS)

SL	ITEM	STANDARD	ACTUAL	VARIANCE	F/A
1	Revenue	412.56	424.00	11.44	F
2	Raw Material	250.00	260.21	(10.21)	A
3	Direct Wages	16.80	16.82	(0.02)	A
4	Power	46.63	48.16	(1.53)	A
5	Fuel	18.45	19.06	(0.61)	A
6	Chemicals	18.45	18.55	(0.10)	A
7	Wire Clothing	4.10	4.03	0.07	F
8	Packing Material	3.08	3.39	(0.31)	A
9	Contribution	55.05	53.78	(1.27)	A
10	Fixed Expenses	42.00	44.00	(2.00)	A
11	**Margin**	**13.05**	**9.78**	**(3.27)**	**A**

Observation: XPML has planned for a standard revenue of ₹412.56 lakhs for the month with a targeted margin of ₹13.05 lakhs. The company achieved a higher revenue of ₹424.00 lakhs, but fell short of the margin by ₹3.27 lakhs, the primary reason being a disproportionate increase in costs. The computations furnished above would trace the causes element-wise and enable decisions for corrective actions.

⚡ Quick Revision — Section 5.1

- **Variance:** Deviation between standard proposition and actual incidence.
- **Revenue Variance:** Revenue Quantity Variance + Revenue Price Variance.
- **Cost Variance:** Usage Variance + Price Variance.
- **Material Variance:** Material Cost Variance = Material Usage Variance + Material Price Variance; Material Usage Variance = Material Mix Variance + Material Yield Variance.
- **Labour Variance:** Labour Cost Variance = Labour Rate Variance + Labour Efficiency Variance; Labour Efficiency Variance = Labour Mix Variance + Labour Yield Variance + Idle Time Variance.
- **Overhead Variance:** Overhead Volume Variance + Overhead Expenditure Variance.
- **Formulae:** SP (SQ – AQ) for usage; AQ (SP – AP) for price.

5.1.1 Investigation of Variances

FACTORS · COST BENEFIT ANALYSIS · THREE METHODS OF INVESTIGATION

Concept of Investigation

DEFINITION

Investigation of variances implies systematic examination of deviations undertaken for the purpose of initiating corrective actions. As such, Variance analysis is the quantitative investigation of the difference between actual and planned behaviour. Such an analysis is used to maintain control over a business through the investigation of areas in which performance was unexpectedly off the mark.

Since the analysis of variances consumes resources and money, not all variances need to be investigated. Management takes up only the **significant variances** for probing. As a common practice, minor deviations from the propositions and random variances are not considered for investigation.

Factors Influencing Investigation Decision

1

Adverse or Favourable

Adverse variances tend to attract most attention as they indicate problems. However, there is an argument for the investigation of favourable variances so that a business can learn from its successes. Not all adverse variances are bad and not all favourable variances are indicators of efficiency in operation. An adverse variance might result from something good that has happened — e.g., higher production costs may have occurred because sales are significantly higher than budget (favourable budget).

2

Materiality

The size of the variance may indicate the scale of the problem and the potential benefits arising from its correction. Small variations in a single period are bound to occur and are unlikely to be significant. Investigation of such variances is likely to be time-consuming and irritating from the manager concerned.

3

Trends

One adverse variance may be caused by a random event; but a series of adverse variances would definitely need investigation. If an efficiency variance is ₹1,000 adverse in month 1, but the same variance is ₹1,000 adverse every month, the trend indicates that the process is in control and the standard has been wrongly set.

4

Controllability

Controllability must also influence the decision. If there is a general worldwide price increase in the price of an important raw material, there is nothing that can be done internally to control the effect. Uncontrollable variances call for a change in the plan and not an investigation into past.

5

Interdependencies

Sometimes a variance in one area is related to a variance in another. For example, a favourable raw material price variance resulting from the purchase of a lower grade of material may cause an adverse labour efficiency variance. These two variances would need to be considered jointly before making an investigation decision.

6

Inherent Nature

The inherent variability of the cost or revenue. Some costs, by nature, are quite volatile (oil prices, for example) and variances would therefore not be surprising. Other costs, such as labour rates, are far more stable and even a small variance may indicate a problem.

7

Reliability

Reliability and accuracy of the figures warrant due consideration. For example, mistakes in calculating budget figures or in recording actual costs and revenues could lead to a variance being reported inaccurately.

Cost Benefit Analysis of Investigation

Costs of Investigation

- The cost of investigating the variance
- Cost of corrective action (action needed to correct the process and bring it back under control)

Benefits of Investigation

- The cost of allowing the process to continue as it is (in an out-of-control state)

DECISION RULE

Investigation is taken up **only if** the cost of allowing the present state to continue exceeds the costs of investigation and correction.

Three Methods of Deciding Whether to Investigate

1

Managerial Intuition and Judgment

Most firms prescribe limits of variances expressed in terms of: (a) absolute monetary amount (e.g., ₹2,500 per month for stationery); or (b) as a percentage of the standard proposition (e.g., 0.25% of budget); or (c) both, as guidelines. Variances falling within these limits are considered in-control; those beyond are out-of-control and investigated. The intuition method is simple, inexpensive and, though not statistically justified, may be reasonably accurate if fixed with proper care.

2

Expected Value Method

Probabilities of a variance being in out-of-control and in-control states are estimated and a payoff matrix is formed. If ai > a0 → do not investigate; if ai < a0 → investigate; if ai = a0 → indifferent. Break-even probability P2 = [Ci ÷ (Cb – Cc)]. Decision: investigate only if the estimated probability of the out-of-control state is greater than the break-even probability. Limitations: hard to estimate the probability distribution and the value of Cb (cost of allowing out-of-control state).

3

Statistical Control Chart Method

Statistical Quality Control is based on the concept that repetitive processes are subject to a certain amount of chance variability which has a stable pattern. A process is said to be in-control if all measurements fall within this pattern of variability. Items outside the pattern are in out-of-control state needing investigation. Thus, if we build up the parameters within which a standard or budgeted cost item should vary, we can find out whether a variance should or should not be investigated.

Payoff Matrix — Expected Value Method

ACTION	STATE — IN-CONTROL	STATE — OUT-OF-CONTROL
Probability	P1	P2
Investigate, ai	Ci	Ci + Cc
Do not investigate, a0	0	Cb

EXPECTED VALUE METHOD — FORMULAE

$$\text{Expected value, } ai = P1Ci + P2(Ci + Cc)$$

$$\text{Expected value, } a0 = P1 \times 0 + P2Cb$$

$$\text{Break-even probability } P2 = [Ci \div (Cb - Cc)]$$

Where: P1 = Probability of in-control state; P2 = Probability of out-of-control state; Ci = Cost of investigation; Cc = Cost of bringing back the process in-control; Cb = Cost of allowing the out-of-control state to continue.

NUMERICAL EXAMPLE

Let P1 = 0.85, P2 = 0.15, Ci = ₹300, Cc = ₹2,000, Cb = ₹5,000

Break-even probability P2 = [300 ÷ (5000 – 2000)] = 0.1

Since P2 (0.15) is higher than the break-even probability, the decision will be to **investigate**.

Expected value, ai = 0.85 × 300 + 0.15 (300 + 2,000) = ₹600; Expected value, a0 = 0 + 0.15 × 5,000 = ₹750. Since ai < a0, investigate.

⚡ Quick Revision — Section 5.1.1

- **Seven factors:** Adverse/Favourable, Materiality, Trends, Controllability, Interdependencies, Inherent Nature, Reliability.
- **Cost Benefit Analysis:** Investigate only if cost of allowing present state > costs of investigation and correction.
- **Three Methods:** Managerial Intuition and Judgment, Expected Value Method, Statistical Control Chart Method.
- **Break-even probability:** P2 = Ci ÷ (Cb – Cc). Investigate if estimated P2 > break-even P2.

5.1.2 Planning and Operating Variances

SALES · MATERIAL · LABOUR — PLANNING VS OPERATING VARIANCES

Introduction

May it be standard costing, budgetary control or any other system, explaining the causes of variances is a key step in variance analysis. In some cases, the cause is due to poor budgeting and planning (e.g., the planners used an out-of-date price list when setting the standard cost of materials). In some cases, the cause is purely operational (e.g., the price of raw materials went up due to market shortages). Often causes are a mixture of planning and operating factors. Some firms seek to make these distinctions more explicit by separating out planning and operating variances. The basic approach is to have two budgets — the original budget and a revised one that takes into account planning issues so that we can then determine two sets of variances.

PLANNING AND OPERATING VARIANCES

```
Planning Variance = (Original Proposition) – (Revised Proposition)
```

```
Operating Variance = (Revised Proposition) – (Actual Incidence)
```

Planning variances seek to explain the extent to which the original standard needs to be adjusted to reflect changes in operating conditions between the current situation and that envisaged when the standard was originally calculated. Operating variances indicate the extent to which attainable targets (the adjusted standards) have been achieved.

Planning and Operating Variances for Sales

SALES VARIANCE FORMULAE

```
Sales Quantity Planning Variance (Market Size Variance) = Standard Price (Revised Sales Quantity – Original Sales Quantity)
```

```
Sales Quantity Operating Variance (Market Share Variance) = Standard Price (Actual Sales Quantity – Revised Sales Quantity)
```

```
Sales Price Planning Variance = Actual Quantities Sold (Revised Sale Price – Original Sale Price)
```

```
Sales Price Operating Variance = Actual Quantities Sold (Actual Sale Price – Revised Sale Price)
```

Causes for Sales Price Planning Variance

1

Raw Material Price Changes

Change in the raw materials prices, compelling management to revise sale prices significantly.

2

Inefficient Operations

Inefficient operations or unskilled labour, causing the product prices to rise; or conversely efficient production to control the costs.

3

Market Competition

Market competition, with fewer competitors likely to achieve favourable sale price variance and intense competition to result in adverse.

Causes for Sales Price Operating Variance

1

New Competitors

The threat of new competitors compelling management to lower the sales prices.

2

Input Availability

Unavailability of input components or increase in the raw material prices.

3

Operational Inefficiency

Inefficiency in operations leading to wastes and idle labour hours.

4

Seasonal Demand

Seasonal product demand or lack of competition may tempt the management to increase sales prices.

5

Operational Efficiencies

Operational efficiencies and economies of scale can also lead to favourable sales price variances.

Planning and Operating Variances for Material and Labour Costs

In case of materials and labour, planning and operational variances can be calculated by comparing original and revised budgets (planning) and revised budgets with actual results (operational). A material price planning variance is really useful to provide feedback on just how skilled managers are in estimating future prices. The operational variance is more meaningful as it measures the purchasing department's efficiency given the market conditions that prevailed at that time. It ignores factors which cannot be controlled by the purchasing department.

CARE WITH FLEXING THE BUDGET

When applying planning and operating principles to cost variances (material and labour), care must be taken over flexing the budgets. One accepted approach is to **flex both the original and revised budgets to actual production levels**.

Example — Revising the Budget (Rhodes Co)

PROBLEM

Rhodes Co manufactures Stops which are estimated to require 2 kg of material XYZ at ₹100/kg. In week 21, only 250 Stops were produced although budgeted production was 300. 450 kg of XYZ were purchased and used in the week at a total cost of ₹51,000. Later it was found that the standard had failed to allow for a 10% price increase throughout the material supplier's industry. Rhodes Ltd carries no stocks.

SOLUTION

PARTICULARS	WORKINGS	AMOUNT (₹)
(i) Actual Results	450 kgs for ₹51,000	51,000
(ii) Revised flexed budget	250 units @ 2 kg per unit and @ ₹110 per kg	55,000
(iii) Operating Variance	(51,000 – 55,000)	4,000 (A)
(iv) Original flexed budget	300 units @ 2 kg per unit and @ ₹100 per kg	60,000
(v) Planning Variance	(55,000 – 60,000)	5,000 (A)

Example — Revising the Budget (Transport Business)

PROBLEM

A transport business makes a particular journey regularly and has established that the standard fuel cost for each journey is 20 litres of fuel at ₹80 per litre. New legislation has forced a change in the vehicle used for the journey and an unexpected rise in fuel costs. It is decided retrospectively that the standard cost per journey should have been 18 litres at ₹100 per litre.

Required: Calculate the original and revised flexed budgets if the journey is made 120 times in the period.

SOLUTION

Original flexed budget: 120 × 20 × 80 = ₹1,92,000

Revised flexed budget: 120 × 18 × 100 = ₹2,16,000

Illustration 2 — Green Chemicals (Material and Labour Planning & Operational Variances)

PROBLEM

Green Chemicals produces agriculture fertilizers. Standard hours per unit = 1.1; Standard direct labour rate per hour = ₹18.50; Standard usage of material per unit = 1.2 kg; Standard price of material per unit = ₹70. Revised budgets: Revised price of material = ₹71; Revised labour rate = ₹20/hour; Revised hours per unit = 1.05; Revised usage of material per unit = 1.175 kg. Actual results: Actual Production = 15,400 units; Raw Material usage = 16,555 KGs; Actual cost of raw material = ₹11,91,960 or ₹72 per KG; Actual labour costs = 16,632 hours and ₹3,24,324 or ₹19.50 per hour.

SOLUTION — SUMMARY OF VARIANCES

SL	DESCRIPTION	PLANNING (₹)	OPERATIONAL (₹)	TOTAL (₹)
1a	Raw Material Price Variance	16,555 (A)	16,555 (A)	33,110 (A)
1b	Raw Material Usage Variance	26,950 (F)	1,07,800 (F)	1,34,750 (F)
1c	**Sub Total**	**10,395 (F)**	**91,245 (F)**	**1,01,640 (F)**
2a	Direct Labour Rate Variance	24,948 (A)	8,316 (F)	16,632 (A)
2b	Direct Labour Efficiency Variance	14,245 (F)	8,547 (A)	5,698 (F)
2c	**Sub Total**	**10,703 (A)**	**231 (A)**	**10,934 (A)**
3	**Total**	**308 (A)**	**91,014 (F)**	**90,706 (F)**

Observations:

- The planning variance consists of ₹10,395 (Favourable) with respect to Raw Material and ₹10,703 (Adverse) in relation to Direct Labour, both together aggregating to ₹308 (Adverse). Even though the variance looks smaller at the aggregate level, it is substantial at the element level and hence needs further probing.
- The operational variance consists of ₹91,245 (Favourable) with respect to Raw Material and ₹231 (Adverse) in relation to Direct Labour, both together aggregating to ₹91,014 (Favourable).

⚡ Quick Revision — Section 5.1.2

- **Planning Variance:** Original Proposition – Revised Proposition.
- **Operating Variance:** Revised Proposition – Actual Incidence.
- **Sales Quantity Planning Variance (Market Size):** Standard Price (Revised Sales Quantity – Original Sales Quantity).
- **Sales Quantity Operating Variance (Market Share):** Standard Price (Actual Sales Quantity – Revised Sales Quantity).
- **Sales Price Planning Variance:** Actual Quantities Sold (Revised Sale Price – Original Sale Price).
- **Sales Price Operating Variance:** Actual Quantities Sold (Actual Sale Price – Revised Sale Price).
- **Material/Labour:** Flex both the original and revised budgets to actual production levels.

5.1.3 Controllable and Non-Controllable Variances

CLASSIFICATION · IMPORTANCE · REVISED VARIANCE

CONTROLLABLE VARIANCE

- Identified as the primary responsibility of a particular person or department.
- Refers to deviation caused by factors which could be influenced by managerial/executive action.
- **Example:** Excessive use of materials or labour hours than the standards can be attributable to a particular person.
- The size of controllable variance reflects the degree of efficiency of the person/department.

VS

NON-CONTROLLABLE VARIANCE

- Variations due to factors beyond the control of the concerned person or department.
- **Examples:** Rise in prices of materials, increase in wage rates, Government restrictions, etc.
- These factors are not within the control of the management and the responsibility cannot be assigned to any particular person or division.
- Revision of the standard becomes necessary to avoid non-recurrence of such variance in future.

IMPORTANCE FROM MANAGEMENT VIEWPOINT

The division of variance into controllable and uncontrollable is important from the viewpoint of management as it can place more emphasis on controllable variance and thus facilitate the principle of **management by exception**. Standard costing to be more realistic, sometimes the standards set are to be revised on account of changes in uncontrollable factors like wages, materials etc.

Revised Variance

REVISED VARIANCE

```
Revised Variance = (Standard Cost Originally Set) – (Revised Standard Cost)
```

To take into account uncontrollable factors into variance, a 'revised variance' is created and the basic standard is allowed to continue. The size of controllable variance reflects the degree of efficiency of the person/department. It is the controllable variance with which the management is concerned because it needs remedial measures.

KEY INSIGHT

Finding variance is not the ultimate objective of the cost management. Their analysis and finding the causes of variance is the ultimate aim to control cost. Control of cost depends on the corrective action taken by the management. The analysis of variance helps the management to locate deficiency and assign responsibility to a particular person or cost centre.

⚡ Quick Revision — Section 5.1.3

- **Controllable Variance:** Primary responsibility of a particular person/department; influenced by managerial action.
- **Non-Controllable Variance:** Factors beyond the control; e.g., material price rise, wage rate increase, Govt. restrictions.
- **Revised Variance:** Difference between standard cost originally set and revised standard cost.
- **Importance:** Facilitates management by exception; focuses on controllable variances for remedial measures.

5.1.4 Relevant Cost Approach to Variance Analysis

RELEVANCY · INCREMENTAL INVESTIGATION · SCARCE RESOURCES

CONCEPT OF RELEVANCY

'What is the relevancy?' is a contextual question that keeps springing up in many a process of managerial decision making. The term relevancy signifies, 'the quality or state of being closely connected or appropriate' with respect to the contextual situation. The context could be revenues, costs or even variances.

In cost and management accounting, the notion of relevant costing has a lot of significance because these costs are pertinent with respect to a particular decision. A relevant cost for a particular decision is the one that changes the result if an alternative course of action is chosen. Studies have demonstrated that relevant costs will make a difference in decision making. And, therefore, a similar approach is advocated in variance analysis too.

MAIN INTENT OF RELEVANT COSTING

The main intent of relevant costing is to determine the objective cost of a business decision. An objective measure of the cost of a business decision is the degree of profit that shall result from its execution. The fundamental principles of relevant costing are quite simple and managers can perhaps relate the concept of relevancy to variance analysis. **Costs are relevant if they direct the executive towards the decision.** So also, whether particular variances are relevant for decision making depends on decision circumstance and the options available.

Relevant Variance Analysis

Relevant variance analysis may be perceived as an **incremental investigation** which considers only relevant costs (costs that vary between alternatives) and ignores sunk costs (costs which have been incurred, cannot be changed and therefore are inappropriate to the business situation).

The notion of the relevant variance is very helpful to eliminate irrelevant information from a particular decision-making process. Variances arising from committed contractual obligations such as price escalations in material costs; wage increases springing up from agreements with wage boards; power rates spiraling from electricity boards; etc. could become irrelevant for operating controls.

SCARCE RESOURCES & LOST CONTRIBUTION

The conventional approach to variance analysis is to compute variances based on acquisition cost and standard prices for the acquisition of resources. This is misleading when scarce resources exist. Failure to use scarce resources efficiently leads not only to increased acquisition cost but also to a **lost contribution**. Therefore, the meaningful approach is to incorporate the lost contribution in variance analysis. For example, if scarce material is used excessively, it will cause material costs to be high and in addition there will be lost contribution which should be attached to material usage variance. When this approach is used, price or expenditure variances are not affected. Quantity variance is affected by how efficiently the scarce resource is being used.

⚡ Quick Revision — Section 5.1.4

- **Relevant Variance Analysis:** Incremental investigation considering only relevant costs that vary between alternatives.
- **Ignores:** Sunk costs, committed costs, unavoidable costs.
- **Scarce Resources:** Incorporate lost contribution in variance analysis; quantity variance affected by efficiency of scarce resource use.

5.1.5 Variance Analysis under Marginal Costing and Absorption Costing

OVERHEAD VARIANCE CLASSIFICATION · VARIABLE OVERHEAD · FIXED OVERHEAD · CAPACITY · CALENDAR · EFFICIENCY

Absorption of Overheads

ABSORPTION COSTING

- Uses single overhead absorption rate to absorb overheads
- Overheads not segregated into variable and fixed
- Variances occur if the absorption rate is incorrect (over/under-absorption)
- Calculates overhead expenditure variance and overhead volume variance
- Overhead volume variance split into capacity variance and efficiency variance

VS

MARGINAL COSTING

- Distinguishes between fixed costs and variable costs
- Facilitates analysis of variances to their causes and points of incidence
- Takes a leap forward from absorption costing
- Classifies overhead (indirect cost) variance into: Variable Overhead Variance and Fixed Overhead Variance

Variable Overhead Variance

VARIABLE OVERHEAD VARIANCE FORMULAE

```
Variable Overhead Variance = (Actual Output × Standard Variable Overhead Rate) − (Actual Variable Overheads)
```

```
Variable Overhead Expenditure Variance = AU × (SR − AR)
```

```
Variable Overhead Efficiency Variance = Standard Variable Overhead Rate per unit × (Standard Units for Actual Production − Actual Units)
```

Variable Overhead Variance is represented by expenditure variance only because variable overhead cost will vary in proportion to production whereby only a change in expenditure can cause such variance.

Fixed Overhead Variance

FIXED OVERHEAD VARIANCE FORMULAE

```
Fixed Overhead Variance = Fixed Overheads Absorbed − Actual Fixed Overheads = (SU × SR) − (AU × AR)
```

```
Fixed Overhead Expenditure Variance = Budgeted Fixed Overheads − Actual Fixed Overheads = AU × (SR − AR)
```

```
Fixed Overhead Volume Variance = Standard Rate (Actual Units − Budgeted Units) = SR (AU − BU)
```

INTERPRETATION OF FIXED OVERHEAD VOLUME VARIANCE

Volume variance shows the over or under absorption of fixed overheads during a particular period. If the actual output is more than the budgeted output, there is over-recovery of fixed overheads and volume variance is **favourable**. If the actual output is less than the budgeted output, volume variance is **adverse**.

Subdivisions of Volume Variance

1

Capacity Variance

Portion of the volume variance due to working at higher or lower capacity than the budgeted capacity. Related to under and over utilisation of plant and equipment. Arises due to idle time, strikes and lock-out, break-down of machinery, power failure, shortage of materials and labour, absenteeism, overtime, changes in number of shifts. Formula: **Standard Fixed Overhead Rate per Unit × (Budgeted Units – Actual Units)**

2

Calendar Variance

Portion of the volume variance due to the difference between the number of working days in the budget period and the number of actual working days. If the actual working days are more than the standard working days, the variance will be favourable and vice versa. Formula: **Standard Rate Per Hour or Per Day × Excess or Deficit Hours or Days Worked**

3

Efficiency Variance

Portion of the volume variance due to the difference between the budgeted efficiency of production and the actual efficiency achieved. Related to the efficiency of workers and plant. Formula: **Standard Rate per Unit × (Actual Production in Units – Standard Production in Units) = SR × (AU – SU)**

Reporting of Variances

FIVE REQUISITES FOR EFFECTIVE VARIANCE REPORTING

1. The variances arising out of each factor should be correctly segregated. If any part of a variance due to one factor is wrongly attributed to or merged with that of another, the report would be misleading.
2. Variances, particularly the controllable variances, should be reported with promptness as soon as they occur.
3. For effective control, the line of organisation should be properly defined and the authority and responsibility of each individual should be laid down in clear terms.
4. In certain cases, a particular variance may be the joint responsibility of more than one individual or department. Corrective action should be taken jointly.
5. Analysis of uncontrollable variances should be made with the same care as for controllable variances.

Forms of Variance Reports

The forms of reports for the different types of variances should be designed keeping in view the needs of the management and the size of the concern. Variance Analysis Reports prepared for the top management would obviously be more formal and would contain broad details only, while those meant for presentation to the lower levels would contain details showing the causes of each variance and the specific responsibilities of the individuals concerned.

⚡ Quick Revision — Section 5.1.5

- **Absorption Costing:** Single overhead absorption rate; overhead expenditure variance + overhead volume variance.
- **Marginal Costing:** Distinguishes fixed and variable costs; Variable Overhead Variance + Fixed Overhead Variance.
- **Variable Overhead Variance:** Expenditure Variance + Efficiency Variance.
- **Fixed Overhead Variance:** Expenditure Variance + Volume Variance.
- **Volume Variance:** Capacity Variance + Calendar Variance + Efficiency Variance.
- **Reporting:** Correct segregation, promptness, defined authority, joint responsibility, analysis of uncontrollable variances.

5.1.6 Activity Ratios

EFFICIENCY · ACTIVITY · CALENDAR · CAPACITY USAGE · CAPACITY UTILISATION · IDLE TIME

Concept of Activity Ratios

Although absolute monetary terms show the extent of the variances, the information is insufficient if the management wants to study the trend of variances from period to period. Absolute figures in themselves do not give the full picture and it is only by comparison of one item with another that their correct relationship is obtained. **Variance Ratios** serve this need and comparison of these ratios from one period to another can be gainfully made.

A number of ratios are used for reporting to the management with respect to the effective use of capacity, material, labour and other resources:

1

Efficiency Ratio

Reveals the input-output relationship. Input is available in terms of hours worked. Output is converted into standard hours to determine the relationship of input and output.

```
Efficiency Ratio = (Standard Hours ÷ Actual Hours) × 100
```

It reveals the extent of efficiency or inefficiency of production during the related period. Standard hour is the media of expressing output in terms of hours — a hypothetical hour which measures the amount of work which should be performed in one hour according to standard.

2

Activity Ratio

The number of standard hours equivalent to the output produced, expressed as a percentage of the budgeted standard hours.

```
Activity Ratio = (Standard Hours for Actual Work ÷ Budgeted Standard Hours) × 100
```

Three steps: (a) Actual output expressed in terms of standard hours; (b) Budgeted output expressed in standard hours; (c) Percentage relationship of (a) and (b). Highlights the actual level of activity in comparison to budgeted activity level.

3

Calendar Ratio

The relationship between the number of working days in a period and the number of working days in the relative budget period.

```
Calendar Ratio = (Actual Working Days ÷ Budgeted Working Days) × 100
```

Indicates whether all the budgeted working days in a budgeted period have been available in actual practice. If the ratio is more than 100%, actual working days are more than budgeted working days and vice versa.

4

Capacity Usage Ratio

The relationship between the budgeted number of working hours and the maximum possible number of working hours in a budget period.

```
Capacity Usage Ratio = (Budgeted Hours ÷ Maximum Possible Hours in Budget) × 100
```

Indicates the extent to which the budgeted hours have actually been utilised.

5

Capacity Utilisation Ratio

The relationship between actual hours in a budget period and the budgeted working hours in the period.

```
Capacity Utilisation Ratio = (Actual Hours ÷ Budgeted Hours) × 100
```

6

Idle Time Ratio

The ratio of idle time hours to the total hours budgeted.

```
Idle Time Ratio = (Idle Time Hours ÷ Budgeted Hours) × 100
```

Example — Calculation of Control Ratios

PROBLEM

Product X takes 5 hours to make and Product Y requires 10 hours. In a month of 25 effective days of 8 hours a day, 1,000 units of X and 600 units of Y were produced. The company employs 50 workers in the production department, and the budgeted hours for the year are 1,02,000.

Required: Calculate (a) Efficiency ratio, (b) Activity ratio, (c) Capacity ratio.

SOLUTION

Standard Hours = (1,000 × 5) + (600 × 10) = 5,000 + 6,000 = 11,000

Actual Hours = 25 days × 8 hours × 50 workers = 10,000

Budgeted Hours for the Month = 1,02,000 ÷ 12 = 8,500

- **(a) Efficiency Ratio** = (11,000 ÷ 10,000) × 100 = **110%**
- **(b) Activity Ratio** = (11,000 ÷ 8,500) × 100 = **129.41%**
- **(c) Capacity Ratio** = (10,000 ÷ 8,500) × 100 = **117.65%**

Observation: All the ratios are greater than 100%. The performance may be considered as better than the benchmarks.

⚡ Quick Revision — Section 5.1.6

- **Efficiency Ratio:** (Standard Hours ÷ Actual Hours) × 100.
- **Activity Ratio:** (Standard Hours for Actual Work ÷ Budgeted Standard Hours) × 100.
- **Calendar Ratio:** (Actual Working Days ÷ Budgeted Working Days) × 100.
- **Capacity Usage Ratio:** (Budgeted Hours ÷ Maximum Possible Hours) × 100.
- **Capacity Utilisation Ratio:** (Actual Hours ÷ Budgeted Hours) × 100.
- **Idle Time Ratio:** (Idle Time Hours ÷ Budgeted Hours) × 100.
- **Relationship:** Activity Ratio = Efficiency Ratio × Capacity Ratio.

5.1.7 Application of Budgetary Control and Standard Costing in Profit Planning

PROFIT PLANNING · STANDARD COSTING ADVANTAGES · LIMITATIONS · BUDGETARY CONTROL

Profit Planning

PETER DRUCKER ON PROFIT

"Profit is a condition of survival. It is the cost of the future, the cost of staying in a business." Therefore, profit should be planned and managed properly. An organisation should plan profits by taking into consideration its capabilities and resources. Profit planning lays foundation for the future income statement of the organisation.

Profit Planning aims to set a profit objective for a budgeting period. Also, it seeks to establish the main policy decisions regarding how to achieve the objectives. The profit objectives, in principle, reflect the expected return on capital employed. In profit planning, alternatives are evaluated to select the most likely option that will yield the required profit objective. Managers can plan their budgets on this basis.

Outputs Expected from Profit Planning Exercise

1

Setting Profit Objectives

Setting the profit objectives for the budget period.

2

Specifying Policy Decisions

Specifying the policy decisions and course of action to be followed during the budget period.

3

Providing Planning Directives

Providing planning directives for the preparation of detailed operating plans.

Key Factors Considered in Profit Planning

- Changes needed in volume, price, and cost
- Availability of funds for investment
- Capital expenditure proposals
- Changes needed in the level of working capital
- Limits on discretionary expenditure (e.g., R&D)
- Return Required on Capital Employed

Four Steps of Profit Planning Process

Establishing Profit Goals
Alignment with strategic plans

→

Determining Expected Sales Volume
Market & industry trends

→

Estimating Expenses
Past data / industry data

→

Determining Profit
Excess of projected income over expenses

Explanation of Steps

Establishing Profit Goals

Implies that profit goals should be set in alignment with the strategic plans of the organisation. Moreover, the profit goals of an organisation should be realistic in nature based on the capabilities and resources of the organisation.

Determining Expected Sales Volume

Constitutes the most important step of the profit planning process. An organisation needs to forecast its sales volume so that it can achieve its profit goals. The sales volume can be anticipated by taking into account the market and industry trends and performing competitive analysis.

Estimating Expenses

Requires that an organisation needs to estimate its expenses for the planned sales volume. Expenses can be determined from past data. If an organisation is new, then the data of similar organisation in same industry can be taken. The expense forecasts should be adjusted to the economic conditions of the country.

Determining Profit

Helps in estimating the exact value of profit. Estimated profit is calculated as excess projected income over projected expenses.

PROFIT CONTROL

After planning the profit successfully, an organisation needs to control profit. Profit control involves measuring the gap between the estimated level and actual level of profit achieved by an organisation. If there is any deviation, the necessary corrective measures are taken. Profit control, thus, involves continuous and concurrent comparison of the actuals with the estimates and initiating timely corrective actions. **And this is where standard costing as also budgetary control fit in as important tools in profit planning.**

Advantages of Standard Costing

1

Yardstick

Standard Costing system establishes yard-sticks against which the efficiency of actual performances is measured.

2

Incentive & Motivation

The standards provide incentive and motivation to work with greater effort and vigilance for achieving the standard performance. This increases efficiency and productivity all round.

3

Simplification & Standardisation

At the very stage of setting the standards, simplification and standardisation of products, methods, and operations are ensured and waste of time and materials is eliminated.

4

Simplified Costing Procedure

Costing procedure is simplified. There is a reduction in paper work in accounting and a smaller number of forms and records are required.

5

Prompt Cost Availability

Costs are available with promptitude for various purposes like fixation of selling prices, pricing of inter-departmental transfers, ascertaining the value of costing stocks of work-in-progress and finished stock and determining idle capacity.

6

Budgetary Planning

Standard Costing is an exercise in planning — it can be very easily fitted into and used for budgetary planning.

7

Delegation of Authority

Facilitates delegation of authority and fixation of responsibility for each department or individual. This also tones up the general organisation of the concern.

8

Management by Exception

Variance analysis and reporting is based on the principles of management by exception. The top management may not be interested in details of actual performance but only in the variances from the standards.

9

Performance Analysis

Standard costs assist in performance analysis by providing ready means for preparation of information.

10

Prompt Decision-Making

Production and pricing policies may be formulated in advance before production starts. This helps in prompt decision-making.

11

Integration of Accounts

Facilitates the integration of accounts so that reconciliation between cost accounts and financial accounts may be eliminated.

12

Optimizes Resources

Standard Costing optimizes the use of plant capacities, current assets and working capital.

Limitations of Standard Costing

1

Difficult to Establish

Establishment of standard costs is difficult in practice.

2

Rigidity

In course of time, sometimes even in a short period, the standards become rigid.

3

Inaccurate Standards

Inaccurate, unreliable and out of date standards do more harm than benefit.

4

Adverse Psychological Effects

Sometimes, standards create adverse psychological effects. If the standard is set at high level, its non-achievement would result in frustration and build-up of resistance.

5

Random Factors

Due to the play of random factors, variances cannot sometimes be properly explained, and it is difficult to distinguish between controllable and non-controllable expenses.

6

Not Suitable for Small Concerns

Standard costing may not sometimes be suitable for some small concerns. Where production cannot be carefully scheduled, frequent changes in production conditions result in variances.

7

Non-standardized Products

May not be suitable and costly in the case of industries dealing with non-standardized products and for repair jobs which keep on changing in accordance with customer's specifications.

8

Lack of Management Interest

Lack of interest in standard costing on the part of the management makes the system practically ineffective.

Budgets and Budgetary Control

DEFINITION

A **budget** is a profit plan reflecting anticipated financial inflows and outflows. Budgeting helps all sorts of entities to plan and control their operations, and to support their managerial strategies. A budget sets out the benchmark against which performance will be measured.

Main Purposes of Budgeting

a

Translating Long-term Plans

Translating the long-term plans into an annual workable budget.

b

Communicating Plans

Communicating the plans to those who will be held accountable.

c

Coordinating

Coordinating with the various departments of the organisation to ensure that they are working in harmony.

d

Controlling Performance

Controlling the performance by continuous monitoring of the actual results with the budget and initiating timely corrective measures.

Advantages of Budgetary Control System

1

Efficient Business Activities

The use of budgetary control system enables the management of a business concern to conduct its business activities in the efficient manner.

2

Powerful Instrument

It is a powerful instrument used by business houses for the control of their expenditure. It provides a yardstick for measuring and evaluating the performance of individuals and their departments.

3

Reveals Deviations

It reveals the deviations to management, from the budgeted figures after making a comparison with actual figures.

4

Effective Utilisation

Effective utilization of various resources like men, material, machinery and money is made possible, as the production is planned after taking them into account.

5

Review of Trends

It helps in the review of current trends and framing of future policies.

6

Standard Costing Conditions

It creates suitable conditions for the implementation of standard costing system in a business organisation.

7

Cost Consciousness

It inculcates the feeling of cost consciousness among workers.

Limitations of Budgetary Control System

1

Estimates

Budgets may or may not be true, as they are based on estimates. The assumptions about future events may or may not actually happen.

2

Rigidity

Budgets are considered as rigid documents. Too much emphasis on budgets may affect day-to-day operations and ignores the dynamic state of organisational functioning.

3

False Sense of Security

Mere budgeting cannot lead to profitability. Budgets cannot be executed automatically. It may create a false sense of security that everything has been taken care of in the budgets.

4

Lack of Co-ordination

Staff co-operation is usually not available during Budgetary Control exercise.

5

Time and Cost

The introduction and implementation of the system may be expensive.

ASSIMILATION

Both standard costing and budgetary control aim at the objective of maximum efficiency and managerial control.

⚡ Quick Revision — Section 5.1.7

- **Profit Planning:** Setting profit objective for a budgeting period; reflects expected return on capital employed.
- **Four Steps:** Establishing Profit Goals → Determining Expected Sales Volume → Estimating Expenses → Determining Profit.
- **Standard Costing Advantages:** Yardstick, incentive, simplification, reduced paperwork, prompt cost availability, budgetary planning, delegation, management by exception, performance analysis, prompt decisions, integration, optimization.
- **Standard Costing Limitations:** Difficult to establish, rigidity, inaccurate standards, adverse psychological effects, random factors, not suitable for small concerns/non-standardized products, lack of management interest.
- **Budgetary Control Advantages:** Efficient activities, powerful instrument, reveals deviations, effective utilisation, review of trends, standard costing conditions, cost consciousness.
- **Budgetary Control Limitations:** Estimates, rigidity, false sense of security, lack of co-ordination, time and cost.

5.1.8 Standard Costing vs Budgetary Control

SIMILARITIES · KEY DIFFERENCES · ASSIMILATION

Standard Costing

DEFINITION

Standard Costing is a control technique that reports variances by comparing actual costs to pre-set standards thereby facilitating action through management by exception. A standard is a stipulated norm, something set up and established by an authority as a rule for the measure of quantity, weight, extent, value, or quality. Standards are set based on predetermined physical inputs of materials, labour, machine hours, power and other resources which should be consumed while manufacturing a product. In principle, Standard Costing is **Engineered Costing**.

Budgetary Control

DEFINITION

Budgetary Control is the process that facilitates effective implementation of the budgets. The process allows continuous monitoring of actual results versus budget, either to secure by individual action the budget objectives or to provide a basis for budget revision. Budgetary Control refers to how well managers utilize budgets to monitor and control costs and operations in a given budget period.

Similarities Between Standard Costing and Budgetary Control

i

Determination of Cost in Advance

Both aim at the determination of cost in advance.

ii

Predetermined Benchmarks

For both of them predetermined benchmarks are fixed.

iii

Comparison with Benchmarks

In both of them actual costs are compared with the benchmarks.

iv

Periodic Cost Reports

Both require periodic cost reports.

v

Maximum Efficiency

Both aim at the maximum efficiencies and managerial costs.

Key Differences

SL	STANDARD COSTING	BUDGETARY CONTROL
1	Is a cost accounting system, in which performance is measured by comparing the actual and standard costs.	Is a control system in which actual and budgeted results are compared continuously in order to achieve the desired results.
2	Has a restricted scope, limited to costs only.	Budgets are complete in as much as they are framed for all the activities and functions of a concern such as production, purchase, selling and distribution, research and development, capital utilisation, etc.
3	Is a **unit concept**.	Is a **total concept**.
4	Standards are minimum targets to be attained by actual performance at specified efficiency.	Budgets are the ceilings or limits of expenses above which the actual expenditure should not normally rise; if it does, the planned profits will be reduced.
5	A more searching analysis of the variances from standards is necessary.	A less searching analysis of the variations from the budget is necessary.
6	Standards are pointers to further possible improvements.	Budgets are indices, adherence to which keeps a business out of difficulties.
7	Standard costs do not change due to short-term changes in the conditions.	Budgeted costs may change due to short-term changes in the conditions.
8	A system of Standard Costing may be operated independently.	A system of Budgetary Control may be operated even if no Standard Costing system is in use in the concern.

ASSIMILATION

Accurate cost information is fundamental to budgeting. Companies that use accurate cost management techniques and provide budget developers with ready access to cost information improve both the accuracy and the speed of their budget process. Standardizing the cost management system entity-wide is an important step in improving the link between cost management and budgeting. **As such, Standard Costing facilitates better budgetary control.**

⚡ Quick Revision — Section 5.1.8

- **Standard Costing:** Cost accounting system; unit concept; restricted scope; standards are minimum targets.
- **Budgetary Control:** Control system; total concept; complete scope; budgets are ceilings.
- **Similarities:** Determination of cost in advance, predetermined benchmarks, comparison, periodic reports, maximum efficiency.
- **Key Difference:** Standard Costing is Engineered Costing; Budgetary Control is a total concept.

5.1.9 Reconciliation of Actual Profit with Standard Profit and/or Budgeted Profit

PROFIT VARIANCE · PROFIT PRICE VARIANCE · PROFIT VOLUME VARIANCE · ILLUSTRATION

Concept of Profit

DEFINITION

Profit reflects the financial gains wherein benefits derived from a business activity exceed the costs. Profit is considered as the bottom-line for every entrepreneurial activity. **Actual profit** reflects the actual accruals, **budgeted profit** reflects the budgeted accruals and **standard profit** refers to the normative accruals.

Reasons for Actual Profit Being Different from Budgeted/Standard Profit

i

Sales Units

Differences between actual and expected units of sales.

ii

Product Pricing

Differences between actual and expected product pricing.

iii

Material Costs

Changes in the cost of materials.

iv

Labour Costs

Changes in labour costs.

v

Overhead Costs

Changes in the amount of overhead costs incurred.

vi

Scrap & Wastages

Changes in the amount of scrap and wastages.

Profit Variance

PROFIT VARIANCE FORMULAE

```
Profit Variance = (Budgeted Profit) − (Actual Profit)
```

```
Profit Price Variance = (Actual Quantity Sold) × (Standard Rate of Profit − Actual Rate of Profit)
```

```
Profit Volume Variance = (Standard Rate of Profit) × (Budgeted Quantity − Actual Quantity)
```

```
Profit Volume Mix Variance = Revised Standard Profit − Standard Profit
```

```
Profit Volume Quantity Variance = Budgeted Profit − Revised Standard Profit
```

PURPOSE OF RECONCILIATION

Reconciliation refers to the action of making one view or belief compatible with another. The purpose of calculating variances is to identify the diverse reasons for the deviations and to analyse the effect of each item of cost or income on actual profit in comparison to the expected profit. **Reconciliation of actual profit with the standard or budgeted profit throws up the causes of variance and facilitates corrective steps.**

Illustration 4 — Comprehensive Reconciliation

PROBLEM

In a company operating on a standard costing system for a given four-week period, budgeted sales of 10,000 units at ₹50 per unit; actual sales were 9,000 units at ₹51.25 per unit.

COSTS	STANDARD (₹)	ACTUALS (₹)
Materials	2,50,000	2,57,400
Wages	75,000	70,875
Fixed Overheads	20,000	18,810
Variable Overheads	10,000	9,250
Semi-Variable Overheads	2,700	2,430

Standard Hours 50,000; Actual Hours 40,500

- The Standard material content of each unit is estimated at 25 kg at ₹1 per kg; actual figures were 26 kg at ₹1.10 per kg.
- Semi-variable Overhead consists of FIVE-NINTHS fixed expenses and FOUR-NINTHS variable.
- The Standard wages per unit are 5 hours at ₹1.50; per Unit actual wages were 4.5 hours at ₹1.75.
- There were no opening stocks and the whole production for the period was sold.
- The four-week period was normal period.

Required: (a) To compute the variances in Sales, Materials, Labour and Overheads due to all possible causes; and (b) With the help of such a computation draw a statement reconciling the actual profit for the period with the standard profit.

SOLUTION — STATEMENT OF PROFIT

PARTICULARS	BUDGET (₹)	STANDARD FOR ACTUAL QUANTITY (₹)	ACTUAL (₹)
Sales (Units)	10,000	9,000	9,000
Selling Price per Unit (₹)	50.00	50.00	51.25
Value of Sales	5,00,000	4,50,000	4,61,250
Material	2,50,000	2,25,000	2,57,400
Wages	75,000	67,500	70,875
Variable Overheads	10,000	9,000	9,250
Semi-Variable Overheads	2,700	2,430	2,430
Fixed Overheads	20,000	18,000	18,810
Total Cost	**3,57,700**	**3,21,930**	**3,58,765**
Profit	**1,42,300**	**1,28,070**	**1,02,485**

Reconciliation Statement:

ELEMENT	₹	₹
Budgeted Profit		1,42,300
(–) Sales Volume Variance		(50,000)
(+) Diff. in Budgeted Costs & Standard Costs for Actual Quantity		35,770
Standard Profit for Actual Quantity		**1,28,070**
Add: Favourable Variances		
Sales Price Variance	11,250	
Labour Efficiency Variance	6,750	
Variable Overhead Efficiency Variance	1,008	
Fixed Overhead Efficiency Variance	1,935	
Fixed Overhead Budget Variance	1,340	22,283
Less: Adverse Variances		
Material Usage Variance	9,000	
Material Price Variance	23,400	
Labour Rate Variance	10,125	
Variable Overhead Budget Variance	1,258	
Fixed Overhead Capacity Variance	4,085	47,868
Actual Profit		**1,02,485**

Take-home Pack: The illustration clearly demonstrates the multiple reasons that could lead to variances between the budgets, standards and actuals. The reconciliation serves as a tool whereby efforts can be focused on the areas that warrant attention. Variance Analysis is a concept of 'Management by Exception'. In the ultimate, it is the wisdom of decision makers that ensures effective application.

Overview of Variances with Formulae

Material Cost Variance	Labour Cost Variance	Variable Overhead Variance	Fixed Overhead Variance	Sales Variance	Profit Variance
Material Cost Variance: (SQ × SP) – (AQ × AP)	Labour Cost Variance: (SH × SR) – (AH × AR)	Variable Overhead Efficiency Variance: SR × (SU for Actual Production – AU)	Fixed Overhead Variance: (Absorbed Overheads) – (Actual Overheads) = (SR × SU) – (AR × AU)	Sales Variance: (Budgeted Sales – Actual Sales)	Profit Variance: (Budgeted Profit – Actual Profit)
Material Usage Variance: SP × (SQ – AQ)	Labour Efficiency Variance: SR × (SH – AH)	Variable Overhead Expenditure Variance: AU × (SR – AR)	Fixed Overhead Volume Variance: SR × (SU – AU)	Sales Price Variance: AQ × (SP – AP)	Profit Variance due to Selling Price: AQ × (SP – AP)
Material Price Variance: AQ × (SP – AP)	Labour Rate Variance: AH × (SR – AR)	Variable Overhead Variance: (AU × SR) – AVO	Fixed Overhead Expenditure Variance: AU × (SR – AR)	Sales Volume Variance: SP × (BQ – AQ)	Profit Variance due to Sales Volume: SP × (BQ – AQ)
Material Mix Variance: SP × (RSQ – AQ)	Labour Yield Variance: SR × (SH – SRH)		Fixed Overhead Efficiency Variance: SR × (Standard Units for Actual Production – Actual Units)	Sales Sub-Volume Variance: SP × (SQ – RSQ)	Profit Mix Variance: SP × (AQ – RSQ)
Material Yield Variance: SR × (SY – AY)	Labour Mix/Gang Variance: SR × (RSH – AH)		Fixed Overhead Capacity Variance: SR × (AU – BU)	Sales Mix Variance: SP × (RSQ – AQ)	Profit Sub-Volume Variance: SP × (RSQ – SQ)
	Idle Time Variance: Idle Time × SR/Hr.		Fixed Overhead Calendar Variance: (Standard Rate per Hour) × (Excess or Deficit Hours)		

⚡ Quick Revision — Section 5.1.9

- **Profit Variance:** Budgeted Profit – Actual Profit.
- **Profit Price Variance:** AQ × (Standard Rate of Profit – Actual Rate of Profit).
- **Profit Volume Variance:** Standard Rate of Profit × (Budgeted Quantity – Actual Quantity).
- **Reconciliation:** Budgeted Profit → (± Variances) → Actual Profit.
- **Purpose:** Identify reasons for deviations, analyse effect of each item, facilitate corrective steps.

5.2 Uniform Costing and Inter-firm Comparison

UNIFORM COSTING · SCOPE · NEED · REQUISITES · FIELDS COVERED · ADVANTAGES · LIMITATIONS · INTER-FIRM COMPARISON

Uniform Costing — Concept

DEFINITION

Uniform Costing may be defined as the application and use of the same costing principles and procedures by different organisations under the same management or on a common understanding between members of an association. It is the application of the same costing principles, methods or procedures uniformly by various undertakings in the same industry.

It is neither a separate method of cost accounting like specific order costing or operation costing nor a separate technique of costing like marginal costing or standard costing, but is only a particular system of costing which takes the help of both methods and techniques of costing. It is a technique which applies the usual costing techniques like standard costing, marginal costing, and budgetary control uniformly in a number of concerns in the same industry, or even in different but similar industries.

Amalgamation and closer working arrangements between groups of manufacturers in particular industries, and organisation for nationalization have necessitated, to a certain extent, the establishment of some degree of uniform costing by industries.

PURPOSE OF UNIFORM COSTING

The principles and methods adopted for the accumulation, analysis, apportionment and allocation of costs vary so widely from concern to concern that comparison of costs is rendered difficult and unrealistic. Uniform Costing attempts to establish uniform methods so that comparison of performances in various undertakings can be made to the common advantage of all the constituent units. Uniform Costing enables cost and accounting data of the member undertakings to be compiled on a comparable basis so that useful and crucial decisions can be taken.

Scope of Uniform Costing

a

Single Enterprise with Branches

In a single enterprise having a number of branches or units, each of which may be a separate manufacturing unit.

b

Concerns in Same Industry

In a number of concerns in the same industry bound together through a trade association or otherwise.

c

Similar Industries

In industries which are similar in nature such as gas and electricity, various types of transport, and cotton, jute and woollen textiles.

Need for Uniform Costing

The need for uniform costing arises from the fact that different units use different cost procedures and principles for costing. The need also arises because of differences in size of the organisation, wage structure, methods of production, degree of automation, and so on. The basic reasons for the differences may be as follows:

a

Size and Organisational Set Up

The number and size of the departments, sections and services also vary from one concern to another according to their size and organisation. The difficulty in operating Uniform Cost Systems for concerns which vary widely in regard to size and type of business may to some extent be overcome by arranging the various units in a number of size or type ranges, and applying different uniform systems for each such type.

b

Methods of Production

The use of different types of machines, plant and equipments, degree of mechanization, difference in materials mix and sequence and nature of operations and processes are mainly responsible for the difference in costs.

c

Methods and Principles of Cost Accounting

It is in this sphere that the largest degree of difference arises. Undertakings manufacturing identical or similar products and having the same system of cost accounting would generally employ different methods of treatment of expenditure on buying, storage and issue of materials, pricing of stores issues, payment to workers, basis of classification and absorption of overhead, calculation of depreciation, charging rent on freehold or leasehold assets etc.

FUNDAMENTAL REQUIREMENT

In the application of Uniform Costing, the fundamental requirement is to **locate any kind of differences and to eliminate or overcome, as far as practicable, the causes giving rise to such differences.**

Requisites for Installation of a Uniform Costing System

a

Mutual Trust and Co-operation

There should be a spirit of mutual trust, co-operation and a policy of give and take amongst the participating members.

b

Free Exchange of Ideas

There should be a free exchange of ideas and methods.

c

Sharing by Bigger Units

The bigger units should be prepared to share improvements, achievements of efficiency, benefits of research, know-how, etc. with the smaller ones.

d

No Hiding of Information

There should not be any hiding or withholding of information.

e

No Rivalry or Jealousy

There should be no rivalry or sense of jealousy amongst the members.

Fields Covered by Uniform Costing

ESSENTIAL POINTS ON WHICH UNIFORMITY IS NORMALLY REQUIRED

1. Whether costs are required for the individual products i.e., for the cost units or for cost centres.
2. The method of costing to be applied.
3. The technique employed such as Standard Costing, Marginal Costing.
4. Items to be excluded from costs.
5. The basis of departmentalization.
6. The basis of allocation of costs to departments and/or service department costs to production departments.
7. The methods of application administration, selling and distribution overhead to cost of sales.
8. The method of valuation of work-in-progress.
9. Methods of treating cost of spoilage, defective work, scrap and wastage.
10. Methods of accounting of overtime pay bonus and other miscellaneous allowances paid to workers.
11. Whether purchase, material handling and upkeep expenses are added to the cost of stores or are treated as overhead expenses.
12. The system of materials control, pricing of issues and valuation of stock.
13. The system of classification and coding of accounts.
14. The method of recording accounting information.

Advantages of Uniform Costing

i

Comparative Information

Provides comparative information to the members of the organisation/association and helps to reduce or eliminate the evil effects of competition and unnecessary expenses arising from competition.

ii

Tool for Management Control

Uniform Costing is a useful tool for management control. Performance of individual units can be measured against norms set for the industry as a whole.

iii

Detect Weakness

Enables the member concerns to compare their own cost data with that of the others, detect the weakness and to take corrective steps for improvement in efficiency.

iv

Avoids Cut-throat Competition

Avoids cut-throat competition by ensuring that competition among member units proceeds on healthy lines.

v

Easier Pricing Policy

The process of pricing policy becomes easier when Uniform Costing is adopted.

vi

Cost Consciousness

By showing the one best way of doing things, Uniform Costing creates cost consciousness and provides the best system of cost control and cost presentation in the entire industry.

vii

Research Benefits

The benefits of research and development can be passed on to the smaller members of the association which, in turn, leads to economic prosperity of the industry as a whole.

viii

Reliable Data for Statutory Bodies

Enables the industry to submit to statutory bodies reliable and accurate data which might be required to regulate pricing policy or for other purposes.

ix

Prerequisite to Cost Audit

Serves as a prerequisite to Cost Audit and inter-firm comparison.

x

Simplifies Wage Board Work

Simplifies the work of wage boards set up to fix minimum wages and fair wages for an industry.

Limitations of Uniform Costing

i

Presumption of Same Principles

Uniform costing presumes the application of same principles and methods of Costing in each of the member firms. But individual units generally differ in respect of certain key factors and methods.

ii

Cost of Installation for Smaller Units

For smaller units the cost of installation and operation of Uniform Costing System may be more than the benefits derived by them.

iii

Monopolistic Tendencies

Uniform costing may create conditions that are likely to develop monopolistic tendencies within the industry. Prices may be raised artificially and supplies curtailed.

iv

Lack of Complete Agreement

If complete agreement between the members is not forthcoming, the statistics presented cannot be relied upon. This weakens the Uniform Costing System and reduces its usefulness.

Inter-firm Comparison — Concept

DEFINITION

Inter-firm Comparison as the name denotes means the technique of evaluating the performances, efficiencies, deficiencies, costs and profits of similar nature of firms engaged in the same industry or business. It consists of exchange of information, voluntarily of course, concerning production, sales, costs, prices, profits, etc., among the firms who are interested and willing to make the device a success.

The basic purposes of such comparison are to find out the weak points in an organisation and to improve the efficiency by taking appropriate measures to wipe out the weakness gradually over a period of time.

Need for Inter-firm Comparison

THE PROGRESSIVE MANAGEMENT QUESTION

Every progressive management, all over the world, has always asked itself the question — *how is my company performing in comparison to that of others?* The published trading and profit and loss accounts and the balance sheets along with the annual reports provide scanty data for any purposeful study and assessment of the performance of a company. The figures available from these reports just indicate, in a general way, the profitability, stability, solvency and growth of an organisation; but they do not throw light on whether a company has really made the optimum use of all the available resources in men, materials, etc. The answer depends fully on the availability of more detailed data, and the possibility of comparison with the competitive units in the same line of manufacture.

Pre-Requisites for Inter-firm Comparison

1. Adaptation of Uniform Costing

There must be a sound system of uniform costing in the firm where inter-firm comparison scheme is to be implemented. A uniform manual should also be prepared and distributed among the member units to enable the function of the system efficiently.

2. Responsible Organisation

An organisation must be established to run the system efficiently and for better results. Firms of different sizes in an industry should become members of the organisation. In industrially advanced countries independent agencies such as British Centre for Inter-firm Comparison, European Productivity Agency and U.S. Bureau of Labour Statistics are responsible for collection, coordination and presentation of information. In India, some undertakings such as National Productivity Council, the Trade Development Authority, the Bureau of Industrial Costs and Pricing, the Tariff Commission have undertaken, in a limited way, the task of inter-firm comparison. In some cases, trade associations, holding company or parent organisation are doing the work of interfirm comparison.

3. Collection of Relevant Information

The information to be collected must be relevant. The nature of information to be collected from the participating firms depends upon the needs of the management, comparative importance of the information and the efficiency of the central body responsible for the collection of the information. Information is generally collected relating to costs and cost structure, labour or machine efficiency and utilisation, raw material consumption, wastage, inventory, return on capital employed, liquidity, reserves and appropriation of profit, methods of production, creditors and debtors, technical aspects, etc.

4. Methods of Collection

The time and the form in which the information is to be submitted by the member units must be decided in advance. Multiple statistical tools can be used for the purpose of collection of data, its editing, classification, presentation, drawing conclusions and inferences. Ratio analysis for measuring profitability, efficiency and productivity etc. can also be used.

Benefits of Inter-firm Comparison

a

Awareness of Strengths & Weaknesses

Makes the management of the organisation aware of its strengths and weakness in relation to the other organisations in the same industry.

b

Saves Time and Efforts

As only the significant items are reported to the Management, substantial time and efforts are saved.

c

Up-to-date Information

The management is able to keep up-to-date information of the trends and ratios and, therefore, it becomes easier for them to take the necessary steps for improvement.

d

Cost Consciousness

It develops cost consciousness among the members of the industry.

e

Free Information Flow

Information about the organisation is made available freely without the fear of disclosure of confidential data to outside market or public.

f

Specialized Knowledge

Specialized knowledge and experience of professionally run and successful organisations are made available to smaller units who can take the advantage, as otherwise it may not be possible for them to have such an infrastructure.

g

Industry Benefits

The industry, as a whole, benefits from the process due to increased productivity, standardization of products, elimination of unfair comparison and the trade practices.

h

Reliable Data

Reliable and correct data enhance the organisation's power in dealing with various authorities and Government bodies.

i

Industry Sickness Identification

Inter firm comparison assists in a big way in identifying industry sickness and gives a timely warning so that effective remedial steps can be taken to save the organisation.

Limitations of Inter-firm Comparison

a

Top Management Not Convinced

The top management may not be convinced of the utility of inter-firm comparison.

b

Reluctance to Disclose Data

Reluctance to disclose data which a concern considers to be confidential.

c

Sense of Complacence

A sense of complacence on the part of the management who may be satisfied with the present level of profits.

d

Absence of Proper Cost Accounting

Absence of a proper system of Cost Accounting because of which the costing figures supplied may not be relied upon for comparison purposes.

e

Non-availability of Suitable Base

Non-availability of a suitable base for comparison.

Steps to Overcome Difficulties

1

Selling the Scheme

'Selling' the scheme through education and propaganda. Publication of articles in journals and periodicals, and lectures, seminars and personal discussions may prove useful.

2

Complete Secrecy

Installation of a system which ensures complete secrecy.

3

Scientific Cost System

Introduction of a scientific cost system.

Illustration 23 — Computation of Uniform Price

PROBLEM

The share of total production and the cost-based fair price computed separately for each of the four units in industry are as follows:

PARTICULARS	UNIT 1	UNIT 2	UNIT 3	UNIT 4
Share of Production	40%	25%	20%	15%
Material cost (₹/unit)	150	180	170	190
Direct labour (₹/unit)	100	120	140	160
Depreciation (₹/unit)	300	200	160	100
Other overheads (₹/unit)	300	300	280	240
Total Cost (₹/unit)	850	800	750	690
20% Return on Capital employed (₹/unit)	630	430	350	230
Fair price (₹/unit)	1,480	1,230	1,100	920

Required: Suggest a Uniform Price that may be adopted by the industry.

SOLUTION — COMPUTATION OF UNIFORM PRICE

Weighted Average Cost = [850 × 40%] + [800 × 25%] + [750 × 20%] + [690 × 15%] = 340 + 200 + 150 + 103.5 = **₹793.50**

Weighted Average Return (Profit) on Capital Employed = [630 × 40%] + [430 × 25%] + [350 × 20%] + [230 × 15%] = 252 + 107.5 + 70 + 34.5 = **₹464**

Suggested Uniform Price = 793.5 + 464 = **₹1,257.50**

⚡ Quick Revision — Section 5.2

- **Uniform Costing:** Application and use of same costing principles and procedures by different organisations under the same management or on a common understanding between members of an association.
- **Scope:** Single enterprise with branches, concerns in same industry, similar industries.
- **Requisites:** Mutual trust, free exchange of ideas, sharing by bigger units, no hiding of information, no rivalry or jealousy.
- **Advantages:** Comparative information, management control, detect weakness, avoids cut-throat competition, easier pricing policy, cost consciousness, research benefits, reliable data, prerequisite to Cost Audit, simplifies wage board work.
- **Limitations:** Presumption of same principles, cost of installation for smaller units, monopolistic tendencies, lack of complete agreement.
- **Inter-firm Comparison:** Technique of evaluating performances, efficiencies, deficiencies, costs and profits of similar nature of firms engaged in same industry or business.
- **Pre-requisites:** Adaptation of Uniform Costing, Responsible Organisation, Collection of Relevant Information, Methods of Collection.
- **Benefits:** Awareness of strengths & weaknesses, saves time and efforts, up-to-date information, cost consciousness, free information flow, specialized knowledge, industry benefits, reliable data, industry sickness identification.
- **Limitations:** Top management not convinced, reluctance to disclose data, sense of complacence, absence of proper cost accounting, non-availability of suitable base.

★ Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Variance

Denotes the deviation between the standard proposition and the actual incidence. The proposition could be a pre-set benchmark, budget or estimate.

Revenue Variance

The difference between planned, budgeted or standard revenue vis-à-vis the actual revenue generated.

Cost Variance

The difference between a planned, budgeted or standard cost vis-à-vis the actual cost.

Investigation of Variances

Systematic examination of deviations undertaken for the purpose of initiating corrective actions.

Planning Variance

Denotes the deviation between the original proposition and the revised proposition.

Operating Variance

Denotes the deviation between the revised proposition and the actual incidence.

Controllable Variance

Variance identified as the primary responsibility of a particular person or department.

Uncontrollable Variance

When the variations are due to the factors beyond the control of the concerned person or department.

Standard Costing

A control technique that reports variances by comparing actual costs to pre-set standards thereby facilitating action through management by exception.

Budgetary Control

The process that facilitates effective implementation of the budgets.

Profit Variance

The difference between planned, budgeted or standard profit vis-à-vis the actual profit attained.

Uniform Costing

The application and use of the same costing principles and procedures by different organisations under the same management or on a common understanding between members of an association.

Inter-firm Comparison

The techniques of evaluating the performances, efficiencies, deficiencies, costs and profits of similar nature of firms engaged in the same industry or business.

? Multiple Choice Questions

WITH ANSWERS AND REASONING

1. A manufacturing company uses two types of materials, X and Y, for manufacture of a standard product. Standard Mix: X 120 kg @ ₹5; Y 80 kg @ ₹10. Actual Mix: X 112 kg @ ₹5; Y 88 kg @ ₹10. Direct Materials Mix Variance is:

(a) ₹40 (fav.) (b) ₹40 (unfav.) (c) ₹80 (fav.) (d) ₹80 (unfav.)

Answer: B

DM Mix Variance for X = 5(120 – 112) = 40 F; for Y = 10(80 – 88) = 80 UF; Total = ₹40 UF.

2. The information relating to the direct material cost of a company: Standard price per unit ₹7.20; Actual quantity purchased 1,600 units; Standard quantity allowed for actual production 1,450 units; Material price variance on purchase (Favourable) ₹480. What is the actual purchase price per unit?

(a) ₹7.50 (b) ₹6.40 (c) ₹6.5 (d) ₹6.90

Answer: D

MPV = Standard cost of Actual Quantity – Actual Cost; 480 = 7.20 × 1,600 – Actual Cost; Actual Cost = 11,040; Actual Price = ₹6.90.

3. In a factory where standard costing system is followed, the production department consumed 1,100 kgs of a material @ ₹8 per kg for product X resulting in material price variance of ₹2,200 (Fav) and material usage

variance of ₹1,000 (Adv). What is the standard material cost of actual production of product X?

(a) 11,000 (b) 20,000 (c) 14,000 (d) 10,000

Answer: D

Actual Cost = 1,100 × 8 = 8,800; Material Cost Variance = 2,200 F + 1,000 A = 1,200 F; Standard Cost = 8,800 + 1,200 = ₹10,000.

4. AB Ltd. uses standard cost system. Standard rate per hour = ₹8; Actual rate per hour = ₹8.40; Standard hours allowed for actual production = 2,000 hours; Labour Efficiency variance = ₹1,600 (Adverse). What were the actual hours worked?

(a) 1,800 (b) 1,810 (c) 2,200 (d) 2,190

Answer: C

(–) 1,600 = (2,000 – AT) × 8; –1,600 = 16,000 – 8AT; AT = 17,600 ÷ 8 = 2,200 hours.

5. Aderholt uses activity-based costing to allocate overheads. Budgeted units 5,000; Number of employees 75; Budgeted Cost ₹7,500. Actual: Units 5,500; Employees 77; Actual cost ₹8,085. What was the total variance for the pool?

(a) ₹585 Adverse (b) ₹165 Favourable (c) ₹5,550 Favourable (d) ₹385 Adverse

Answer: B

SQ = 75 ÷ 5,000 × 5,500 = 82.5; SP = 7500 ÷ 75 = 100; Standard Cost = 8,250; Variance = 8,250 – 8,085 = ₹165 F.

6. Budgeted O/H ₹10,000 (Fixed ₹6,000, Variable ₹4,000); Budgeted Hours 2,000; Actual O/H ₹10,400 (Fixed ₹6,100, Variable ₹4,300); Actual Hours 2,100. Variable O/H cost variance and Fixed O/H cost variance will be:

(a) 100 (A) and 200 (A) (b) 100 (F) and 200 (F) (c) 100 (A) and 200 (F) (d) 200 (A) and 100 (F)

Answer: C

Overhead Recovery Rate = ₹2/hour. Variable O/H Cost variance = 4,200 – 4,300 = 100 (A); Fixed O/H Cost variance = 6,300 – 6,100 = 200 (F).

7. XYZ Ltd: Actual production 2,75,000 units; Budgeted production 2,50,000 units; Actual fixed production overheads ₹52,60,00,000; Budgeted fixed production overheads ₹50,00,00,000. Fixed overhead volume variance and expenditure variance will be:

(a) ₹5,00,00,000 (A) (b) ₹5,00,00,000 (F) (c) ₹5,00,00,000 (F) (d) ₹5,00,00,000 (A)

Answer: C

Fixed Overhead Volume Variance = SR (AU – BU). Standard Rate = 50,00,00,000 ÷ 2,50,000 = ₹200. Volume Variance = 200 (2,75,000 – 2,50,000) = ₹50,00,000 (F). Fixed Overhead Expenditure Variance = 50,00,00,000 – 52,60,00,000 = ₹2,60,00,000 (A). [Note: The question likely tests understanding of volume variance; the answer ₹5,00,00,000 (F) corresponds to a miscalculation; the correct volume variance is ₹50,00,000 (F).]

8. A company uses standard absorbing costing. Budget: Output and sales 8,700; Selling Price per unit ₹26; Variable Cost per unit ₹10; Total Fixed Overheads ₹34,800. Actual: Output and sales 8,200; Selling Price per unit ₹31; Variable Cost per unit ₹10; Total Fixed Overheads ₹37,000. The sales price variance for October was:

(a) 38,500 (A) (b) 38,500 (F) (c) 41,000 (A) (d) 41,000 (F)

Answer: D

Sales Price Variance = Actual Quantity × (Actual Price − Standard Price) = 8,200 (31 − 26) = ₹41,000 (F).

9. Which of the following may be the cause of Material Price Variance?

(a) Change in quantity of purchase or uneconomical size of purchase order

(b) Failure to take advantage of off-season price or failure to purchase when price is cheaper

(c) Change in basic purchase price of material

(d) All of the above

Answer: D

10. Variance analysis involves breaking down and analysing the total variance to explain:

(a) How much of the variance is caused by using the resources that are different from the standards, i.e., the quantity variance

(b) How much of the variance is caused by using the cost of the resources being different from the standards, i.e., the rate variance

(c) All of the Above

(d) None of the above

Answer: C

11. A standard costing system consists of the following key elements:

(a) Setting standards for each of the operations

(b) Comparing the actual performance with the standard performance

(c) Analyzing and reporting variances arising from the difference between actual and standard performance

(d) All of the Above

Answer: D

12. Which of the following statements is correct?

(a) Standard costing facilitates the integration of accounts so that reconciliation between cost accounts and financial accounts may be eliminated

(b) Standard costs are planned costs determined on a scientific basis and they are based upon certain assumed conditions of efficiency and other factors

(c) Standard costing is defined as the preparation and use of standard cost, their comparison with actual cost and the measurement and analysis of variances to their cause and points of incidence

(d) All of the above

Answer: D

13. Which of the following statements is true?

(a) If the actual cost is more than the standard, we call it adverse variance and if the difference is less than the standard, we call it favourable variance

(b) In case of sales and profit, if the standard is more than actual, it is adverse variance and if the standard is less than the actual, it is favourable variance

(c) Both (a) and (b)

(d) None of the above

Answer: C

14. Standard cost and budgeted cost are:

(a) Interrelated but not interdependent

(b) Interdependent but not interrelated

(c) Interrelated and interdependent

(d) None of the above

Answer: A

15. Efficiency Ratio is:

(a) Available working days ÷ Budgeted working days × 100

(b) Budgeted hours ÷ Maximum hours in budgeted period × 100

(c) Standard hours ÷ Actual hours × 100

(d) None of the above

Answer: C

16. Uniform Costing may not be successfully applied in the following case:

(a) In a single enterprise having a number of branches, each of which manufactures the same set of products with the same facilities

(b) In a number of entities in the same industry bound by a trade association

(c) In a number of units across different geographical locations manufacturing one or more of a given set of products

(d) In different branches of the same company, each branch making a different product using a unique process

Answer: D

Though the entity is the same, different products using different (unique) process cannot follow uniform costing.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
B	D	D	C	B	C	C	D	D	C	D	D	C	A	B	D

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

1

'The main objective of variance analysis is to provide insights into the off-standard performance'. Discuss.

2

State the primary reasons for cost variances.

3

Highlight the significance of investigation of Variances. What are the methods of Investigation?

4

Write a note on Planning and Operating Variances.

5

What do you understand by Controllable and Non-controllable Variances?

6

Write a note on Cost Variance Ratios.

7

Distinguish between Standard Costing and Budgetary Control.

8

What are the merits and demerits of Standard Costing?

9

What are the merits and demerits of Budgetary Control?

10

Write a note on Profit Variance.

11

What are the advantages of Uniform Costing?

12

What is the need for Interfirm Comparison?

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 5

ABBREVIATION	FULL FORM
A	Adverse
AC	Actual Cost
AH	Actual Hours
AQ	Actual Quantity
AP	Actual Price
AR	Actual Rate
AY	Actual Yield
F	Favorable
RBH	Revised Budgeted Hours
RSH	Revised Standard Hours
RSQ	Revised Standard Quantity
SC	Standard Cost
SH	Standard Hours
SQ	Standard Quantity
SP	Standard Price
SR	Standard Rate
SY	Standard Yield
MPV	Material Price Variance
MUV	Material Usage Variance
LEV	Labour Efficiency Variance
LRV	Labour Rate Variance
FOH	Fixed Overhead
VOH	Variable Overhead

Strategic Cost Management — Master Revision Handbook

Module 5 · December 2026 Edition

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5.1 Variance Analyses

5.1.1 Investigation of Variances

5.1.2 Planning and Operating Variances

5.1.3 Controllable and Non-Controllable Variances

5.1.4 Relevant Cost Approach

5.1.5 Marginal vs Absorption Costing

5.1.6 Activity Ratios

5.1.7 Budgetary Control & Standard Costing in Profit Planning

5.1.8 Standard Costing vs Budgetary Control

5.1.9 Reconciliation of Profit

5.2 Uniform Costing & Inter-firm Comparison

Formula Boxes

Variance Tree

Quick Revision Sheets

MCQs with Reasoning

Numerical Problems

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Module Weightage: 15%

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Linear Programming

Module 6 · Formulation · Graphical Method · Simplex Method · Duality

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	15% (with Transportation, Assignment, Game Theory)

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading ✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

06

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 6 INCLUDES

Introduction · Definition of Linear Programming · Requirements of Linear Programming · Advantages of Linear Programming · Limitations of Linear Programming · Application Areas of Linear Programming · Formulation of Linear Programming Problem · Some Definitions · Methods of Solving Linear Programming Problems · Special Cases in Linear Programming · Duality.

Module Learning Objectives

After studying this module, the students will be able to:

1

Mathematical Model in Decision Making

Describe the role of mathematical model in Decision Making. Understand the meaning and requirements of Linear Programming problems or constrained optimization models.

2

Advantages & Limitations

Understand the advantages and limitations of Linear Programming.

3

Application Areas

Understand the application of Linear Programming in different fields. Formulate the Linear Programming Problems.

4

Solving Methods

Solve the problems of Linear Programming using different methods — Graphical and Simplex. Understand the concept of Duality.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Linear Programming techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

15%	10+	12	30
GROUP WEIGHTAGE	KEY CONCEPTS	ILLUSTRATIONS	MCQS
With Transportation, Assignment, Game Theory	Definitions, methods, special cases	Worked examples	With answers

SYLLABUS STRUCTURE NOTE

Module 6 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Linear Programming, along with Transportation, Assignment and Game Theory, forms a group carrying **15% weightage** in the examination. This module introduces the fundamental optimization technique used extensively in business decision making.

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6. Application Areas of Linear Programming
7. Formulation of Linear Programming Problem
8. Some Definitions
9. Methods of Solving Linear Programming Problems
10. Special Cases in Linear Programming
11. Duality

6.1 Introduction to Linear Programming

MATHEMATICAL MODELS · BASIC COMPONENTS · CONSTRAINED OPTIMIZATION

With the advancement of computer technology, use of mathematical models has increased substantially in different areas of business, manufacturing etc. A **model** represents the essential features of an object, system or a problem without unnecessary details. A **mathematical model** is one where the important aspects are represented in mathematical form using variables, parameters and functions.

Instead of constructing and manipulating real life systems, we get a much cheaper, faster and safer solution by using mathematical models. If only limited resources are there then using mathematical model one can find how best those can be used to maximize the profit or minimize the cost. When an organization has a given amount of resources (man, machine, money, space etc.) at its disposal and also the output per unit of resource as well as the return per unit of output are known then the combination of resource to give maximum profit (or to have minimum cost) can be easily determined with the help of mathematical model.

CONSTRAINED OPTIMIZATION PROBLEMS

When a programming analysis problem has the features in which the total effectiveness is expressed as a **linear function** of individual allocations and the restrictions on resources give rise to **linear equalities or inequalities** of the individual allocations, then the same is called a **Linear Programming Problem**. These are also known as **Constrained Optimization Problems** due to the presence of restrictions or constraints under which the problem is solved.

DEFINITION

> Linear Programming is "a technique for specifying how to use limited resources or capacities of a business to obtain a particular objective, such as least cost, highest margin or least time, when those resources have alternate uses."

Three Basic Components of LPP

1

Decision Variables

These are the physical quantities controlled by the decision maker and represented by mathematical symbols. For example, xi is the number of units of product i that can be produced by an organization in a particular month. Decision variables can take any one of a set of possible values.

2

Objective Function

This defines the criteria for evaluating the solution. It is a mathematical function of the decision variables. For example, the objective function may measure the profit or cost as a function of the quantities of various products produced by an organization. It specifies a direction of optimization — to maximize some return (e.g., Profit) or to minimize some cost (e.g., production cost, investment cost etc.)

3

Constraints

These are a set of functional inequalities or equalities that represent physical, economic, technological or other restrictions under which optimization is to be accomplished. For example, constraints might ensure that no more input is used than is available.

⚡ Quick Revision — Section 6.1

- **Linear Programming:** Optimization technique for using limited resources to obtain a particular objective.
- **Three basic components:** Decision Variables, Objective Function, Constraints.
- **Constrained Optimization:** Problems with restrictions/constraints under which optimization is accomplished.
- **Mathematical Model:** Represents essential features using variables, parameters and functions.

6.2 Definition of Linear Programming

KOHLAR · SAMUELSON, DORFMAN & SOLOW · LOOMBA

KOHLAR'S DEFINITION

"A method of planning and operation involved in the construction of a model of a real situation containing the following elements: (a) variables representing the available choices, and (b) mathematical expressions (i) relating the variables to the controlling conditions, and (ii) reflecting the criteria to be used in measuring the benefits derivable from each of the several possible plans, and (iii) establishing the objective. The method may be so devised as to ensure the selection of the best of a large number of alternatives."

SAMUELSON, DORFMAN & SOLOW'S DEFINITION

"The analysis of problems in which a linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of restraints in the form of linear inequalities."

LOOMBA'S DEFINITION

"LP is only one aspect of what has been called a system approach to management wherein all programmes are designed and evaluated in terms of their ultimate effects in the realization of business objectives."

Quick Revision — Section 6.2

- **Kohlar:** Method of planning and operation involving construction of a model with variables and mathematical expressions.
- **Samuelson, Dorfman & Solow:** Analysis of problems where linear function is maximized/minimized subject to linear inequalities.
- **Loomba:** LP is one aspect of system approach to management.

6.3 Requirements of Linear Programming

TEN ESSENTIAL REQUIREMENTS FOR LPP FORMULATION

1

Decision Variables and Their Relationship

The relationship among different Decision variables must be linear.

2

Well Defined Objective Function

Any LPP should have a well-defined objective function to optimize — which may either be to maximize contribution or to minimize the cost by utilizing the available scarce resources.

3

Presence of Constraints or Restrictions

There must be limitations on availability of resources expressed as linear equations or inequalities.

4

Alternative Courses of Action

It must be possible to make a selection between various combinations of the productive factors.

5

Non-Negativity Constraints

All the decision variables must assume non-negative values because these variables represent physical quantities which cannot be negative.

6

Linearity

Both the Objective Function and the constraints must be expressed as linear equation or inequality, as the case may be.

7

Finiteness

The number of decision variables and constraints must be finite.

8

Additivity

The sum of the resources used by different activities must be equal to the total quantity of resources individually and collectively.

9

Divisibility

This implies that the solutions need not be in whole numbers. Rather they are divisible and may take any fractional value.

10

Deterministic

Conditions of certainty is assumed to exist. The coefficients of the decision variables in both Objective Function and Constraints are completely known and do not change during the period of study, i.e., the coefficients are deterministic in nature.

Quick Revision — Section 6.3

- **Ten Requirements:** Linear relationship, Well-defined objective function, Constraints, Alternative courses, Non-negativity, Linearity, Finiteness, Additivity, Divisibility, Deterministic.
- **Key assumptions:** Certainty of coefficients, linearity of relationships, non-negative variables, finite variables and constraints.

6.4 Advantages of Linear Programming

EIGHT DISTINCT ADVANTAGES

1

Clear Picture of True Problem

Any LPP formulated properly gives a clear picture of the true problem. This is as valuable as the solution of the problem. In other words, insight and perspective into problem solution gets proper importance here.

2

Optimal Solution Ensured

Due consideration is given to all possible solutions of the problem. In real life, the management problems are so complex that a lot of difficulty is encountered in arriving at a feasible solution. But whenever Linear Programming technique is used, an optimal solution is ensured irrespective of the degree of difficulty of the problem.

3

Better and More Successful Decisions

Proper formulation of LPP always shows the limitations and restrictions under which one has to operate. Thus if it is necessary to deviate from the best solution then one can easily evaluate the quantum of penalty for the deviation.

4

Reevaluation Based on Changing Conditions

Any plan arrived at through Linear Programming can always be reevaluated based on the changing conditions. Hence LP is a better tool for adjusting to meet the changing conditions.

5

Highlights Bottlenecks

It highlights any bottleneck in the production process.

6

Optimal Use of Productive Factors

It facilitates optimal use of productive factors as well as best use of existing facilities.

7

Flexibility

It provides flexibility in analyzing a variety of multidimensional problems.

8

Information Base for Scarce Resources

An information base is created with its help which ultimately facilitates allocation of scarce resources.

⚡ Quick Revision — Section 6.4

- **Eight Advantages:** Clear picture, optimal solution, better decisions, reevaluation, bottleneck identification, optimal resource use, flexibility, information base.
- **Key Benefit:** LP ensures optimal solution irrespective of complexity of problem.

6.5 Limitations of Linear Programming

SEVEN KEY LIMITATIONS

1

Linearity Restriction

The linear programming models can be applied only in those situations where the constraints and the objective function can be stated in terms of linear expressions. But in real life situations, many objective functions and constraints cannot be expressed linearly.

2

Certainty of Coefficients

In linear programming problems, coefficients of the decision variables in the objective function and the constraint must be completely known and they should not change during the period of study. In practice, it may not be possible to know all the coefficients with certainty.

3

Fractional Valued Answers

Linear programming may give fractional valued answers of the decision variables which in some cases (like number of a specific type of car to be produced) may be redundant.

4

Conflicting Multiple Goals

Linear programming will fail to give a solution if management have conflicting multiple goals.

5

Additivity Requirement

Linear programming problem requires that the total measure of effectiveness and total resource usage resulting from the joint performance of the activities must be equal to the respective sums of these quantities resulting from each activity being performed individually.

6

Complexity of Real-World Problems

Many real-world problems are so complex, in terms of the number of variables and relationships constrained in them, that they tax the capacity of even the largest computer.

7

Other Limitations

• Does not take into consideration the effect of time and uncertainty.
• Parameters appearing in the model are assumed to be constants but in real-life situations they are frequently neither known nor constants.

⚡ Quick Revision — Section 6.5

- **Seven Limitations:** Linearity restriction, certainty of coefficients, fractional answers, conflicting multiple goals, additivity requirement, complexity, time and uncertainty.
- **Key Issue:** Real-life situations often violate the linearity and certainty assumptions.

6.6 Application Areas of Linear Programming

INDUSTRIAL · MANAGEMENT · MISCELLANEOUS · ADMINISTRATIVE · NON-INDUSTRIAL APPLICATIONS

1. Industrial Applications

Linear programming is extensively used to solve a variety of industrial problems. In each of these applications, the general objective is to determine a plan for production and procurement in the time period under consideration. It is necessary to satisfy all demand requirements without violating any of the constraints. Few examples of industrial applications are as follows:

Product Mix Problem | Production Scheduling | Production Smoothing Problem | Blending Problem | Transportation Problem | Product Distribution Problem | Oil Refineries — optimal mix of products | Communication Industry — facilities for transmission, switching, relaying | Rail Road Industry — optimal programming of railway freight and train movements

2. Management Applications

Portfolio Selection | Financial Mix Strategy | Profit Planning | Media Selection | Travelling Salesmen Problem | Determination of equitable salaries | Staffing problem

3. Miscellaneous Applications

a

Farm Planning

- The particular crops to be grown or cattle to keep during a period
- The acreage to be devoted to each
- The particular production methods to be used

b

Airline Routing

Optimization of airline routes and scheduling.

c

Administration, Education and Politics

Have also employed linear programming to solve their problems.

d

Diet Problems

One of the earliest applications of linear programming — originally used by hospitals to determine the most economical diet for patients.

4. Administrative Applications

Linear programming can be used for administrative applications. Administrative applications of Linear Programming are concerned with optimal usage of resources like men, machine and material.

5. Non-Industrial Applications

Agriculture | Environmental Protection | Urban Department | Facilities Location

6. Further Applications of Linear Programming

i

Structural Design

In structural design for maximum product.

ii

Assembly Lines

In balancing assembly lines.

iii

Military Tanker Fleet

In scheduling of a military tanker fleet.

iv

Make or Buy

In determining which parts to make and which to buy to obtain maximum profit margin.

v

Equipment Selection

In selecting equipment and evaluating methods of improvement that maximize profit margin.

vi

Sales Requirements

In planning most profitable match of sales requirements to plant capacity that obtains a fair share of the market.

vii

Purchasing Policies

In design of optimal purchasing policies.

Quick Revision — Section 6.6

- **Industrial:** Product mix, production scheduling, blending, transportation, distribution, oil refineries, communication, railroad.
- **Management:** Portfolio selection, financial mix, profit planning, media selection, travelling salesmen, staffing.
- **Miscellaneous:** Farm planning, airline routing, diet problems.
- **Administrative:** Optimal usage of men, machine and material.
- **Non-Industrial:** Agriculture, environmental protection, urban department, facilities location.

6.7 Formulation of Linear Programming Problem

FIVE BASIC STEPS · ILLUSTRATIONS

Five Basic Steps of Formulation

Step 1 — Find the Key-Decision to be Made

Find the key-decision to be made from the study of the solution. (In this connection, looking for variables helps considerably).

Step 2 — Identify the Variables

Identify the variables and assume symbols x1, x2 ..... for variable quantities noticed in step 1.

Step 3 — Express the Possible Alternatives Mathematically

Express the possible alternatives mathematically in terms of variables. The set of feasible alternatives generally in the given situation is: [(x1, x2); x1 > 0, x2 > 0].

Step 4 — Mention the Objective Quantitatively

Mention the objective quantitatively and express it as a linear function of variables.

Step 5 — Express the Constraints as Linear Equalities/Inequalities

Express the constraints also as linear equalities/inequalities in terms of variables.

Illustration 1 — Shopkeeper's Investment Problem

PROBLEM

A shopkeeper deals in two items — Wall hangings and Artificial plants. He has ₹50,000 to invest and a space to store 100 pieces at the most. Costs of Wall hangings and Artificial plants are respectively ₹450 and ₹200 each. He can sell a Wall hanging at a profit of ₹80 and an Artificial plant at a profit of ₹37. Assuming that he can sell all the items that he buys, formulate a Linear Programming problem in order to maximize his profit.

SOLUTION

Let x1 be the number of Wall hangings and x2 be the number of Artificial plants that the dealer buys and sells.

Objective Function: Maximize $Z = 80x1 + 37x2$

Subject to the constraints:

- $450x1 + 200x2 \leq 50000$ (Investment constraint)
- $x1 + x2 \leq 100$ (Space constraint)
- $x1 \geq 0$ and $x2 \geq 0$ (Non-negativity constraint)

Illustration 2 — Cement Dealer's Transportation Problem

PROBLEM

A dealer of cement has two warehouses M and N with stocks of 30,000 and 20,000 bags of cement respectively. Three customers A, B and C have placed order on the dealer for 15,000, 20,000 and 15,000 bags respectively. Costs of transportation per 1,000 bags of cement from different warehouses to different customers are given below:

TO	A	B	C
From M	40	20	20
From N	20	60	40

The dealer wants to find how to fulfill the orders so that the transportation cost is minimum. Formulate the problem.

SOLUTION

As transportation costs are given per 1,000 bags, we assume 1 unit = 1,000 bags.

Let Warehouse M supply x1 units to A and x2 units to B. As the stock of M is 30,000 bags or 30 units, so C gets (30 – x1 – x2) units from M.

Total requirement of A is 15,000 bags or 15 units. Of this x1 is supplied from M. Thus remaining (15 – x1) units is to be supplied from N.

Similarly B gets (20 – x2) units from N and C gets [15 – (30 – x1 – x2)] = x1 + x2 – 15 units from N.

Objective Function: Minimize Z = 4000x1 – 2000x2 + 150000

Subject to the constraints:

- $x1 + x2 \leq 30$
- $x1 + x2 \geq 15$
- $x1 \leq 15$
- $x2 \leq 20$
- $x1 \geq 0, x2 \geq 0$

Quick Revision — Section 6.7

- **Five Steps of Formulation:** Identify key-decision → Identify variables → Express alternatives mathematically → Mention objective quantitatively → Express constraints as linear equalities/inequalities.
- **Objective Function:** Maximize Z or Minimize Z — linear function of decision variables.
- **Constraints:** Linear inequalities/equalities representing restrictions.
- **Non-negativity:** All decision variables must be ≥ 0.

6.8 Some Definitions

SOLUTION · FEASIBLE · BASIC · OPTIMAL · DEGENERATE

1

Solution

Values of decision variables xj (j = 1, 2 ....., n) which satisfy the constraints of a general L.P.P., is called the Solution to that L.P.P.

2

Feasible Solution

Any solution that satisfies all the constraints (including the non-negative ones) of the general L.P.P., is called a Feasible Solution.

3

Basic Solution

For a set of m simultaneous equations in n unknowns (n > m), a solution obtained by setting (n-m) of the variables equal to zero and solving the remaining m equations in m unknowns is called a Basic Solution. The (n – m) number of variables which are set equal to zero are called Non Basic Variables and the remaining m are called Basic Variables and constitute a basic solution.

4

Basic Feasible Solution

A feasible solution to a general L.P. problem which is also a basic solution is called a Basic Feasible Solution.

5

Optimal Feasible Solution

Any basic feasible solution which optimizes (maximizes or minimizes) the objective function of a general L.P.P. is called an Optimal Feasible Solution to that L.P. problem.

6

Degenerate Solution

A basic solution to the system of equations is called Degenerate if one or more of the basic variables become equal to zero.

⚡ Quick Revision — Section 6.8

- **Solution:** Values of decision variables satisfying constraints.
- **Feasible Solution:** Satisfies all constraints including non-negativity.
- **Basic Solution:** Setting (n-m) variables to zero; solving m equations in m unknowns.
- **Basic Feasible Solution:** Feasible + Basic.
- **Optimal Feasible Solution:** Optimizes the objective function.
- **Degenerate Solution:** One or more basic variables equal zero.

6.9 Methods of Solving Linear Programming Problems

GRAPHICAL METHOD · SIMPLEX METHOD · CORNER POINT · ISO-PROFIT/ISO-COST

Following two methods of finding an optimal solution of a Linear Programming Problem are mostly used:

1

Graphical Method

Generally used for solving Linear Programming Problems having two or three variables.

2

Simplex Method

Applicable to any LPP — no theoretical restriction on the number of Decision Variables or Constraints.

Graphical Method

Graphical Method is generally used for solving Linear Programming Problems having two or three variables. In fact for problems with three variables also the application of this method is rather rare. Due to this limitation of handling only a few variables, this method is not applied for solving industrial problems which normally contain more variables.

Steps in Solving a LPP Graphically

Step 1

Formulate the problem to get a linear Objective Function which is subjected to a number of Constraints. These Constraints are generally in the form of linear inequality. Sometimes they may be in the form of linear equations also.

Step 2

Draw the graphs for each of the Constraints. For inequalities the graphs are represented by regions or space and for equations they are represented by straight lines. For "Greater than" or "Greater than equal to" type inequalities the region will be above the constraint line. For "Less than" or "Less than equal to" type inequalities it will be below the constraint line.

Step 3

Identify the feasible region or solution space. This is actually the area or the space which satisfies all the inequalities simultaneously.

Step 4

Select any one of the following two methods to get the ultimate solution.

CORNER POINT METHOD

- Necessitates identification of each corner point or vertex of the Feasible Region
- Coordinates can be found directly from the graph paper or by solving simultaneous equations of the constraint lines
- Value of the Objective Function computed at each corner point
- For Maximization: take the highest value; For Minimization: take the lowest value

OR

ISO-PROFIT OR ISO-COST METHOD

- Choose any arbitrary numerical value of the Objective Function such that the corresponding straight line falls within the feasible region
- Move the line parallel to itself over the feasible region so that it passes through all the corner points
- For maximization: corner corresponding to Iso-profit line farthest from the origin gives maximum value
- For minimization: corner corresponding to Iso-cost line closest to the origin gives minimum value

Illustration 3 — Maximization Problem (Corner Point Method)

PROBLEM

One kind of cake requires 200 grams of flour and 25 grams of fat and another kind of cake requires 100 grams of flour and 50 grams of fat. Find the maximum number of cakes that can be made from 5 kgs of flour and 1 kg of fat assuming there is no shortage of other ingredients required for making cakes. Formulate LPP based on the information given and solve graphically.

SOLUTION

Let x and y be the number of cakes of the first and second kinds respectively.

Objective Function: Maximize $Z = x + y$

Subject to the constraints:

- $2x + y \leq 50$ (Flour constraint: $200x + 100y \leq 5000$)
- $x + 2y \leq 40$ (Fat constraint: $25x + 50y \leq 1000$)
- $x \geq 0, y \geq 0$

Corner Points of Feasible Region:

COORDINATES	VALUE OF Z = X + Y
(25, 0)	25 + 0 = 25
(20, 10)	20 + 10 = 30
(0, 20)	0 + 20 = 20

Solution: Zmax = 30, with x = 20 and y = 10. Maximum number of cakes = 30 (20 of first type + 10 of second type).

Illustration 4 — Minimization Problem (Corner Point Method)

PROBLEM

Mr. Lal is on a low cholesterol diet. During lunch at the office canteen he always chooses between two particular types of meal — Type A and Type B. The table below lists the amount of protein, carbohydrates and vitamins each meal provides along with the amount of cholesterol (which he is trying to minimize). He needs at least 200 grams of protein, 960 grams of carbohydrates and 40 grams of vitamins for lunch each month. Over this time period, how many days should he have Type A meal and how many days the Type B meal so that he gets adequate amount of protein, carbohydrates and vitamins and at the same time minimizes his cholesterol intake?

	TYPE A MEAL	TYPE B MEAL
Protein (Grams)	8	16
Carbohydrates (Grams)	60	40
Vitamins (Grams)	2	2
Cholesterol (Milligrams)	60	50

SOLUTION

Let x = No. of days Mr. Lal will take Type A meal & y = No. of days Mr. Lal will take Type B meal

Objective Function: Minimize Z = 60x + 50y

Subject to the constraints:

- x + 2y ≥ 25 (Protein constraint)
- 3x + 2y ≥ 48 (Carbohydrates constraint)
- x + y ≥ 20 (Vitamins constraint)
- x ≥ 0, y ≥ 0

Corner Points of Feasible Region:

COORDINATES	VALUE OF Z = 60X + 50Y
A (0, 24)	(60×0) + (50×24) = 1200
B (8, 12)	(60×8) + (50×12) = 1080
C (15, 5)	(60×15) + (50×5) = 1150
D (25, 0)	(60×25) + (50×0) = 1500

Solution: Zmin = 1080 milligrams at B (8, 12). Mr. Lal should take Type A meal for 8 days and Type B for 12 days to intake least cholesterol.

Illustration 5 — Maximization Problem (Iso-profit Method)

PROBLEM

Solve graphically the following LPP — Maximize M = 50x1 + 60x2 subject to the constraints 2x1 + x2 ≤ 300, 3x1 + 4x2 ≤ 509, 4x1 + 7x2 ≤ 812, x1 ≥ 0, x2 ≥ 0

SOLUTION

Corner Points of Feasible Region: O(0,0), F(0,116), J(138.2, 23.6), H, C(150,0)

Point J: Intersection of L1 (2x1 + x2 = 300) and L2 (3x1 + 4x2 = 509)

Solving: x1 = 138.2, x2 = 23.6

Value of Objective Function at J: M = 50 × 138.2 + 60 × 23.6 = 8326

Solution: Mmax = 8326, x1 = 138.2, x2 = 23.6

Note: In this illustration, the solution is unique because there is only one point J on the feasible region through which the Iso-profit line is passing. But there can be situations where the Iso-profit (or Iso-cost) line has the same slope as one of the constraint lines. In that case the Iso-profit line will coincide with one of the outer boundary lines of the Feasible Region and infinite number of optimal solutions will result. This is a special case of LPP called Multiple Optima situation.

Simplex Method

Simplex Method is applicable to any LPP. There is no theoretical restriction on the number of Decision Variables or Constraints present in a particular LPP. This method was developed in mid-1947 by **George Dantzig** while working on planning methods of U.S Army Air Force to have a better utilization of the scarce resources during World War II. The computational procedure is based on the property that **Optimum solution to any LPP, if it exists, always occurs at one of the corner points of the feasible solution space**. It is a systematic and efficient procedure for finding corner point solutions and proceeding to attain optimality. As origin is one of the corner points of the solution space, evaluation always starts from this. This solution is called **Initial Basic Feasible solution**. This is then tested to find out if further improvement in the value of the Objective Function is possible by moving to the adjacent corner point of the feasible solution space. This iterative search is continued till an optimal solution is reached.

Basic Terms Used in Simplex Method

1

Standard Form of LPP

Refers to the form in which all the constraints are written as equalities. The optimum solution of the standard form of LPP is same as the optimum solution of the original formulation.

2

Slack Variable

The variable used to convert a less than or less than equal to type constraint inequality into an equation. Contribution of this variable towards the Objective Function is zero. The value of this variable indicates the quantity of unused resources. For a constraint x1 + x2 + x3 ≤ 20, the form will change to x1 + x2 + x3 + s1 = 20 after introduction of the Slack Variable s1.

3

Surplus Variable

The variable used to convert a more than or more than equal to type constraint inequality into an equation. Contribution of this variable towards the Objective Function is zero. It is interpreted as the quantity over and above the required minimum level. For a constraint 3x1 + 2x2 ≥ 55, the form will change to 3x1 + 2x2 – s2 + A1 = 55 after introduction of the Surplus Variable s2.

4

Artificial Variable

These variables are fictitious and are introduced only for computational purposes. These are used along with the surplus variables (shown as A1 above) for more than or more than equal to type constraints. For equal to type constraints they are introduced alone. Their contribution towards Objective Function is to cause a very high penalty (–M for a maximization problem and +M for a minimization problem).

5

Simplex Table

Used to keep track of the calculations made at each iteration.

6

Product Mix

A column in the simplex table that contains all the variables in the solution.

7

Basis

The set of variables which are not equal to zero in the current basic solution and are listed in the product mix column. The variables which make up the Basis are termed as Basic Variables and the remaining variables are called Non-basic Variables.

8

Zj Row

The row containing the figures for gross profit or loss given up by adding one unit of a variable into the solution.

9

Net Evaluation or Index Row

Contains the net profit or loss that results by introducing one unit of the variable indicated in that column in the solution. Numerical figures in Index Row are also called Shadow Prices or Accounting Prices. A positive (or negative) figure in the Index Row indicates an algebraic increment (or reduction) in the Objective Function if one unit of the variable at the head of that column is introduced in the Basis. Index Row is also called (Cj – Zj) Row.

10

Pivot or Key Column

The one with largest positive number in the Index Row for a maximization problem or the largest negative number for a minimization problem. It indicates which variable will enter in the next step of solution.

11

Pivot or Key Row

Corresponds to the variable that will depart from the Basis to accommodate the new variable to be entered as per the Key Column determined earlier. The departing variable will correspond to the minimum positive ratio found by dividing the Quantity Column values by the Key Column values for each row.

12

Pivot or Key Element

The element of Simplex Table at the junction of the Key Row and Key Column.

Steps to be Followed for Solution of LPP (Maximization Case) by Simplex Method

Step 1

Formulate the problem mathematically.

Step 2

Express the mathematically formulated problem in the Standard Form by introducing Slack variables to convert the constraint inequalities into equalities.

Step 3

Set up the Initial Basic Feasible Solution by assigning zero value to all the decision variables and represent it in the initial Simplex Table. Complete the Table by adding the two final rows for Cj and (Cj – Zj) or the Index Row. Here Cj = Contribution per unit and these are nothing but the coefficients of the variables in the Objective Function in the Standard Form. Also Zj = S(CBj.aij) where CBj = Coefficients of the current basic variables in the Objective function and aij = Values of the elements in the matrix.

Step 4

Examine if all the elements in the Index Row are negative. If that is so then the current solution is optimum. In case there exist some positive value then the current solution is not optimal and scope of further improvement is there. For this one Basic Variable has to be removed from the Basis and to be replaced by a Non Basic variable.

Step 5

Determine which variable to enter into the Solution mix next. For this, identify the column in which the numerical figure in the Index Row is highest positive. This gives the Pivot Column and the variable corresponding to this column will enter into the next table.

Step 6

Determine which variable to depart from the Solution mix next. For this divide each number in the Quantity Column by the corresponding number in the Pivot Column to get the respective ratios. Identify the row having minimum of these ratios. Consider only the positive ratios here. This Row is the Pivot Row. The variable corresponding to the Pivot Row will depart from the Basis to accommodate for the entering variable obtained in step (5) at its place.

Step 7

Identify the Key element which is at the junction of the Pivot Row and Pivot Column.

Step 8

Compute new values for the Pivot Row by simply dividing each number in the Pivot Row by the Key element.

Step 9

Compute the new values for each of the other rows by using the following formula — New row number = (Old row No.) – [(Corresponding No. in the Pivot Row) × (Corresponding Fixed Ratio)] where, Corresponding Fixed Ratio = Old No. in the Pivot Column / Key No.

Step 10

Make new entries in the CB column and XB column in the new table of the current solution.

Step 11

Compute Zj and (Cj – Zj) as explained in step (3) above. If all the elements in the Index Row or (Cj – Zj) row are zero or negative then an optimal solution has been achieved.

Step 12

If any value in the Index Row is positive then repeat the steps (5) through (11) given above.

Illustration 6 — Simplex Method (Alpha and Beta Products)

PROBLEM

A firm manufactures and sells two products Alpha and Beta. Each unit of Alpha requires 1 hour of machining and 2 hours of skilled labour, whereas each unit of Beta uses 2 hours of machining and 1 hour of labour. For the coming month the machine capacity is limited to 720 machine hours and the skilled labour is limited to 780 hours. Not more than 320 units of Alpha can be sold in the market during a month.

(i) Develop a suitable model that will enable determination of the optimal product mix.

(ii) Determine the optimal product-mix and the maximum contribution if Unit contribution from Alpha is ₹6 and from Beta is ₹4.

(iii) What will be the incremental contribution per unit of the machine hour, per unit of labour, per unit of Alpha saleable?

SOLUTION

Let x1 = no. of units of Alpha produced and **x2** = no. of units of Beta produced.

Objective Function: Maximize Z = 6x1 + 4x2

Subject to the constraints:

- x1 + 2x2 ≤ 720 (Machining)
- 2x1 + x2 ≤ 780 (Skilled Labour)
- x1 ≤ 320 (Alpha saleable)
- x1, x2 ≥ 0

Introducing Slack Variables S1, S2, S3:

- x1 + 2x2 + S1 = 720
- 2x1 + x2 + S2 = 780
- x1 + S3 = 320

Optimal Solution: x1 = 280, x2 = 220, Maximum Z = 2560

Shadow Prices: Incremental contribution per unit of Machining hour = ₹2/3 (loss); Incremental contribution per unit of Labour hour = ₹8/3 (loss); Incremental contribution per unit of Alpha saleable = 0.

Illustration 7 — Simplex Table Analysis

PROBLEM

During solution of a problem manually, at one stage the following Simplex Table is obtained:

CB	PRODUCT MIX	QUANTITY	X1	X2	X3	S1	S2	S3	A1
2	**x1**	4	1	2	1/2	0	0	1/4	0
0	**s2**	12	0	0	–1	0	1	–1/2	0
0	**s1**	12	0	6	0	1	0	1	–1
	Cj		2	4	1	0	0	0	–M
	Zj	8	2	4	1	0	0	½	0
	Cj – Zj		0	0	0	0	0	–½	–M

Answer the following questions with proper explanation: (i) How many varieties of very Premium quality tea are considered in the problem? (ii) Is the solution given in the Table above Optimal? (iii) What is the Objective Function? (iv) Is there any alternate solution to the problem? (v) Is the solution feasible? (vi) What is the optimum product mix and the maximum profit? (vii) If any alternate solution is possible then find it.

SOLUTION

(i) From the table it is clear that there are three decision variables x1, x2 and x3. So 3 varieties of very Premium quality tea are considered.

(ii) The given table has either zero or negative entries in the Index Row or (Cj – Zj) Row. Also this is a problem of maximization. So the criteria of optimality is satisfied. Hence the solution is optimal.

(iii) Cj row represents the contribution per unit of different variables. Thus from the table, respective contributions of the Decision variables x1, x2 and x3 are 2, 4 and 1. So the Objective Function is Z = 2x1 + 4x2 + x3.

(iv) In the Optimal Table only decision variable x1 is present in the Basis. Thus the other two decision variables x2 and x3 are Non-basic variables and corresponding to both the entries in the Index Row or (Cj – Zj) Row are zero. This indicates the presence of multiple optimum solutions. Hence there exists alternate solution to the problem.

(v) As no Artificial Variable is present as Basic Variable in the final table, infeasibility of the solution is not applicable. Hence the solution is feasible.

(vi) Optimum product mix is: Quantity of Type 1 quality premium tea to be produced = x1 = 4 quantity units and no units for the other two varieties of tea. Maximum profit = 8 money units.

(vii) From the answer given in (v) above, we can say that two alternate solutions are possible — either that will contain x2 or x3. So a new solution is found by arbitrarily choosing the column of x2 as the Key Column. The alternative solution gives: Quantity of Type 2 quality premium tea to be produced = x2 = 2 quantity units and no units for the other two varieties. Maximum value of Objective Function remains 8 money units.

Illustration 8 — Cat Co (Throughput Accounting for Machine Capacity)

PROBLEM

A company possesses two manufacturing plants each of which can produce three products X, Y and Z from a common raw material. The proportions in which the products are produced are different in each plant and so are the plant's operating costs per hour. Data on production per hour and costs are given below, together with current orders in hand for each product.

PRODUCT	OPERATING COST PER HOUR (₹)	X	Y	Z
Plant A	9	2	4	3
Plant B	10	4	3	2
Orders on hand		50	24	60

Use the simplex method to find the number of production hours needed to fulfill the orders on hand at minimum cost. Interpret the main features of the final solution.

SOLUTION

Let α be no. of hours of plant A in use and β be no. of hours of plant B in use.

Objective Function: Minimize $Z = 9\alpha + 10\beta$

Subject to the constraints:

- $2\alpha + 4\beta \geq 50$ (Product X)
- $4\alpha + 3\beta \geq 24$ (Product Y)
- $3\alpha + 2\beta \geq 60$ (Product Z)
- $\alpha, \beta \geq 0$

Optimal Solution: Minimum Operating Cost = ₹205; α = 20 hours (Plant A), β = 5/2 hours (Plant B).

⚡ Quick Revision — Section 6.9

- **Graphical Method:** For 2-3 variables; Corner Point Method or Iso-profit/Iso-cost Method.
- **Simplex Method:** Applicable to any LPP; developed by George Dantzig (1947); iterative search for corner point solutions.
- **Slack Variable:** Converts ≤ constraint to equation; zero contribution; indicates unused resources.
- **Surplus Variable:** Converts ≥ constraint to equation; zero contribution; indicates excess over minimum.
- **Artificial Variable:** Fictitious; introduced for computational purposes; high penalty (–M for maximization, +M for minimization).
- **Index Row (Cj – Zj):** Net profit/loss by introducing one unit; all negative/zero indicates optimality for maximization.
- **Pivot Column:** Largest positive (maximization) or largest negative (minimization) in Index Row.
- **Pivot Row:** Minimum positive ratio (Quantity ÷ Key Column).
- **Key Element:** Junction of Pivot Row and Pivot Column.

6.10 Special Cases in Linear Programming

INFEASIBLE SOLUTION · UNBOUNDED SOLUTION · MULTIPLE OPTIMUM SOLUTION

1

Infeasible Solution

Situation of infeasibility occurs when there is no solution to an LPP which satisfies all the constraints simultaneously. Graphically it means that no feasible region exists and the same can occur if the problem is formulated with conflicting constraints.
Simplex Method Indication: When at least one of the Artificial variables remains present as the Basic variable in the final solution table then it is considered as a situation of Infeasible solution.

2

Unbounded Solution

It means the feasible region is not bounded in any respect. For LPP with constraints such that the feasible region extends to infinity on the upper right side of the graph, it is said that there is unbounded solution to the problem.
Simplex Method Indication: The situation of unbounded solution occurs if all the ratios in the Minimum Ratio column (which is the last column of any Simplex table) are either negative or infinite.

3

Multiple Optimum Solution

Graphically this case is encountered when the slope of Iso-profit or Iso-cost lines of the Objective Function matches that of any one of the Constraint lines. In other words the Iso-profit or Iso-cost lines will be parallel to any one of the constraint lines. In fact there will be infinite number of optimum solutions.
Simplex Method Indication: This situation can be recognized when one of the non-basic variables in the (Cj – Zj) row will have a zero value in the final solution table. To get the other solution of the problem one has to bring in the non-basic variable into the Basis.

Quick Revision — Section 6.10

- **Infeasible Solution:** No solution satisfies all constraints; Artificial variable remains in final table.
- **Unbounded Solution:** Feasible region extends to infinity; all minimum ratios negative or infinite.
- **Multiple Optimum Solution:** Iso-profit/Iso-cost line parallel to constraint line; zero value in (Cj – Zj) row for non-basic variable.

6.11 Duality

PRIMAL · DUAL · FORMULATION · INTER-RELATIONSHIP · CHARACTERISTICS

Concept of Duality

DEFINITION

Every LPP is associated with another mirror image problem based on the same data. While the original problem is called **Primal Problem**, the other is called its **Dual problem**. In fact either problem can be considered as Primal and the other as the Dual. Actually both the problems can be derived from each other. The format of the Simplex Method is such that when Primal is solved, the associated Dual is also solved at the same time.

Important Points Related to Formulation of Dual Problem

1

Objective Direction

If the Primal Problem is having the objective of Maximization then the Dual will have the objective of Minimization and vice versa.

2

Unit Contributions as RHS

The unit contributions (c) of the decision variables in the Objective Function of Primal appear as constants in the right-hand side of constraints of Dual.

3

Constants as Unit Contributions

Constants (bi) at the right-hand side of the constraints of Primal appear as the unit contributions of the decision variables in the Objective Function of Dual.

4

Number of Variables and Constraints

The number of decision variables in the Primal are equal to the number of constraints in the Dual. The number of constraints in the Primal are equal to the number of decision variables in the Dual. While considering the number of constraints in the Primal, one must not consider the Non-negativity constraint.

5

Constraint Type Reversal

For the Primal Problem with less than or less than equal to type constraints, the Dual will have more than or more than equal to type constraints and vice versa.

6

Coefficient Transposition

The coefficients (aij) for the Dual decision variables in the constraints are the coefficients of the Primal decision variables in the constraints with rows and columns interchanged.

7

Constraint Type Conversion

Any Primal Problem of Maximization should have only LESS THAN or LESS THAN EQUAL TO type Constraints. In case that is not so, then those should be converted to Less than equal to type. Similarly any Primal Problem of Minimization should have only MORE THAN or MORE THAN EQUAL TO type Constraints. If it is not so then the constraints should be converted to More than equal to type.

Illustration 9 — Minimization Type Primal Problem

PROBLEM

Obtain the Dual of the following Primal Problem: Minimize Z = 6000x1 + 4000x2 subject to the constraints 4x1 + x2 ≥ 12, 9x1 + x2 ≥ 20, 7x1 + 3x2 ≥ 18, 10x1 + 40x2 ≥ 40 and x1 & x2 ≥ 0.

SOLUTION

Dual Problem:

Maximize Z* = 12y1 + 20y2 + 18y3 + 40y4

Subject to the constraints:

- 4y1 + 9y2 + 7y3 + 10y4 ≤ 6000
- y1 + y2 + 3y3 + 40y4 ≤ 4000
- y1, y2, y3 and y4 ≥ 0

Key Points: Primal is Minimization having Objective Function Z, while Dual is Maximization having Objective Function Z*. Number of Decision Variables in Primal is 2 → Number of Constraints in Dual is 2. Number of Constraints in Primal is 4 → Number of Decision Variables in Dual is 4. Primal has ≥ type constraints → Dual has ≤ type constraints.

Illustration 10 — Maximization Type Primal Problem with Mixed Type Constraints

PROBLEM

Write the Dual of the following: Maximize Z = 5x1 + 10x2 subject to 2x1 – 3x2 ≤ 7, x1 + 2x2 = 4 and x1, x2 ≥ 0

SOLUTION

Given Primal is a problem of Maximization. So its Dual can be written only if all the Constraints are less than or equal to type. But one of the Constraints is equal to type. Thus the same needs to be converted to less than type which is done as shown below:

$x1 + 2x2 = 4$ can be replaced by two inequalities $x1 + 2x2 \leq 4$ and $x1 + 2x2 \geq 4$. Again the inequality $x1 + 2x2 \geq 4$ can be re-written as $-(x1 + 2x2) \leq -4$ or, $-x1 - 2x2 \leq -4$.

Dual Problem:

Minimize $Z^* = 7y1 + 4y2 - 4y3$

Subject to:

- $2y1 + y2 - y3 \geq 5$
- $-3y1 + 2y2 - 2y3 \geq 10$
- $y1, y2, y3 \geq 0$

Considering $y2 - y3 = y4$ the Dual problem can be written as: Minimize $Z^* = 7y1 + 4y4$ subject to $2y1 + y4 \geq 5$, $-3y1 + 2y4 \geq 10$, $y1 \geq 0$ and $y4$ is unrestricted in sign.

Inter-relationship of the Optimum Solutions of Primal and Dual

KEY RELATIONSHIPS

1. The value in the Index Row or (Cj – Zj) Row corresponding to the Slack or Surplus variables (as the case may be) of the final table of the solution of Dual problem corresponds to the values of the Basic Variables, with sign changed, in the final table of Primal solution.
2. Values for the Slack/Surplus variables of the optimal Primal solution are given by the values, with sign changed, in the Index Row under the non-basic variables of the optimum Dual solution.
3. The optimum value of the Objective Function is same for both Primal and Dual solutions.

Characteristics of the Dual Problem

1

Dual of a Dual is the Primal

Dual of a Dual is the Primal.

2

Solution Existence

If either of the Primal or Dual has a solution, then the other should definitely give a solution. Also optimum values of both the solutions are equal.

3

Infeasibility and Unboundedness

If any of the two types of problems has Infeasible solution, then the Objective Function of the other should have Unbounded optimal solution and vice versa.

4

Feasible Solution Values

The value of the Objective Function for any feasible solution of the Primal is less than the value of the Objective Function for any feasible solution of the Dual. But the optimum values of the objective functions of both are same.

5

Finite Optimum Solution

If the Primal has a feasible solution but the Dual does not have, then the Primal will not have a finite optimum solution and vice versa.

Quick Revision — Section 6.11

- **Duality:** Every LPP has a mirror image problem — Primal and Dual.
- **Formulation Rules:** Max ↔ Min; c → RHS of Dual; b → Objective coefficients of Dual; m variables ↔ m constraints; ≤ ↔ ≥.
- **Coefficient Matrix:** Transpose of Primal coefficient matrix.
- **Characteristics:** Dual of Dual is Primal; equal optimum values; infeasibility ↔ unboundedness; feasible solution values bounded by optimum.

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Linear Programming

A technique for specifying how to use limited resources or capacities of a business to obtain a particular objective, such as least cost, highest margin or least time, when those resources have alternate uses.

Decision Variables

Physical quantities controlled by the decision maker and represented by mathematical symbols.

Objective Function

Defines the criteria for evaluating the solution; a mathematical function of the decision variables.

Constraints

A set of functional inequalities or equalities that represent physical, economic, technological or other restrictions.

Feasible Solution

Any solution that satisfies all the constraints including non-negativity.

Basic Solution

A solution obtained by setting (n-m) variables equal to zero and solving the remaining m equations in m unknowns.

Basic Feasible Solution

A feasible solution which is also a basic solution.

Optimal Feasible Solution

Any basic feasible solution which optimizes the objective function.

Degenerate Solution

A basic solution where one or more of the basic variables become equal to zero.

Slack Variable

Variable used to convert a less than or less than equal to type constraint into an equation; contribution towards Objective Function is zero.

Surplus Variable

Variable used to convert a more than or more than equal to type constraint into an equation; contribution towards Objective Function is zero.

Artificial Variable

Fictitious variables introduced for computational purposes; cause a very high penalty (–M for maximization, +M for minimization).

Pivot Column

The column with largest positive number in Index Row for maximization or largest negative number for minimization.

Pivot Row

Row corresponding to the minimum positive ratio of Quantity to Key Column values.

Key Element

Element at the junction of Pivot Row and Pivot Column.

Duality

Every LPP has a mirror image problem — Primal and Dual.

Multiple Choice Questions

WITH ANSWERS AND REASONING

1. A constraint in an L.P. Model restricts:

a. Value of the Objective Function | b. Values of the Decision Variables | c. Use of the available resources | d. All the above

Answer: C

2. In graphical method of solution of LPP if the Iso-cost line coincide with a side of the Feasible Region then we get:

a. Unique optimum solution | b. Unbounded optimum solution | c. No feasible solution | d. Infinite number of optimum solutions

Answer: D

3. A feasible solution of LPP:

a. Must satisfy all the constraints simultaneously | b. Need not satisfy all the constraints, only some of them | c. Must be a corner point of the feasible region | d. All the above

Answer: A

4. The Objective Function of a LPP is Z = 3x1 + 2x2. If x1 = 10 and x2 = 5 then the value of Z is:

a. 35 | b. 40 | c. 45 | d. 50

Answer: B

Z = 3(10) + 2(5) = 30 + 10 = 40.

5. Multiple solution exist in a Linear Programming problem when:

a. One of the constraints is redundant | b. Objective Function is parallel to one of the constraints | c. Two constraints are parallel | d. All of the above

Answer: B

6. The linear function of the variables which is to be optimized is called:

a. Constraints | b. Objective Function | c. Decision variables | d. None of the above

Answer: B

7. If the value of the Objective Function can be increased or decreased indefinitely then the solution is called:

a. Unbounded | b. Bounded | c. Infeasible | d. None of the above

Answer: A

8. The first step in formulating a LPP is:

a. Identify the upper and lower boundaries of the decision variables | b. State the constraints as linear combinations of the decision variables | c. Understand the problem | d. Identify the Decision Variables

Answer: D

9. The best use of Linear Programming is to find the optimal use of:

a. Manpower | b. Material | c. Money | d. All of the above

Answer: D

10. Which of the following is assumption of Linear Programming Model?

a. Divisibility | b. Proportionality | c. Additivity | d. All of the above

Answer: D

11. Non-negativity condition of Linear Programming implies:

a. A positive coefficient of variables in Objective Function

b. A positive coefficient of variables in any constraint

c. Non-negative value of resource

d. None of the above

Answer: C

12. If the constraints of a Linear Programming problem are $x1 + x2 \leq 1$, $3x1 + x2 \geq 3$ and $x1, x2 \geq 0$ then:

a. There are two feasible regions

b. There are infinite feasible regions

c. No feasible region

d. None of the above

Answer: C

13. For any LPP the intermediate solutions must be checked by substituting them back into the:

a. Objective Function

b. Constraints

c. Either of (a) and (b)

d. This is not required.

Answer: D

14. The feasible solution of any LPP should belong to:

a. Both first and second quadrant

b. Only first quadrant

c. Only second quadrant

d. Both first and third quadrant

Answer: B

15. The true statement related to the graphs of $3x1 + 2x2 \leq 5$ and $6x1 + 4x2 > 10$ is:

a. Both the graphs are disjoint.

b. Both contain the point (1,1)

c. Both (a) and (b) above are true

d. Both (a) and (b) are not true simultaneously

Answer: A

16. In which quadrant the bounded region of the inequalities $x1 + x2 \leq 1$ and $x1 - x2 \leq 1$ is situated?

a. First and third

b. Second and third

c. First and second

d. All the four quadrants

Answer: D

17. Objective function of LPP is:

a. A relation between the variables

b. A function to be optimized

c. A constraint

d. None of the above

Answer: B

18. The optimal value of the Objective Function is attained at the points:

a. Given by intersection of inequations with axes only

b. Given by intersection of inequations with x axis only

c. Given by intersection of inequations with y axis only

d. Given by corner points of the feasible region.

Answer: D

19. If the constraints in a Linear Programming problem are changed then:

a. The problem is to be re-evaluated.

b. Solution is not defined

c. The Objective Function has to be modified.

d. The change in constraints is to be ignored.

Answer: A

20. The constraints $y - x \leq 1$, $3y - x \leq 9$ and $x, y \geq 0$ are defined on:

a. Bounded feasible space

b. Unbounded feasible space

c. Redundant space

d. None of the above.

Answer: B

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
C	D	A	B	B	B	A	D	D	D	C	C	D	B	A	D	B	D	A	B

State True or False

SL	STATEMENT	ANSWER
1	The feasible region of the constraints $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$ and $y \geq 0$ does not contain the point (60,0) on its boundary.	**True**
2	In the optimal table of a LPP when there exists a non-basic variable with zero value of relative profit [i.e (Cj – Zj)], then an alternate solution to the problem is possible.	**True**
3	The LPP, Maximize $Z = 60x + 50y$ subject to $x + 2y \leq 40$, $3x + 2y \leq 60$, $x, y \geq 0$ has unbounded feasible solution.	**False**
4	Artificial variables are introduced to maintain non-negativity of Surplus variables in the Initial Basic Feasible Solution of LPP.	**True**
5	Every Linear Programming Problem has a unique solution.	**False**
6	If (3,3), (20,3), (20,10), (18,12) and (12,12) are the corner points of the feasible region of the Linear Programming Problem – Maximize Z = 2x1 + 3x2 then maximum value of the Objective Function is 70.	**False**
7	For the LPP – Minimize $Z = 5x + 3y$, Iso-cost lines are a family of straight lines having slope (– 5/3).	**True**
8	Simplex Method does not provide any indication about infeasibility of solution of a LPP.	**False**
9	Linear Programming can provide solution of the situations even if management have conflicting goals.	**False**
10	Linear Programming is used for solving problems related to portfolio selection.	**True**
11	A redundant constraint is one that does not affect the feasible solution region.	**True**
12	In Simplex Method, the optimality condition is said to be arrived when each and every entry in the Index Row of the table is negative.	**False**
13	An Artificial Variable has an infinitely large positive contribution towards the Objective Function for a problem of maximization.	**False**
14	For the LPP – Maximize $Z = 3x + 6y$ subject to $x + y \leq 20$, $2x + y \leq 30$, $x \leq 35$, $x \geq 0$ and $y \geq 0$ the redundant constraint is $x \leq 35$.	**True**
15	Linear Programming Problems of maximization have multiple optimum solution if the Iso-profit lines are parallel to any one or more of the Constraint lines.	**True**
16	Pivot column is decided on the basis of the maximum positive entry in the Index Row of a Simplex Table for a maximization problem of LP.	**True**
17	Airline Routing problems cannot be solved by Simplex Method.	**False**
18	Situation of degeneracy occurs for a LPP when one or more of the basic variables become zero.	**True**
19	3 of the points (1,1), (–1,1), (1,–1), (–1,–1), (–2,1), (2,–1), (–2,–1) & (2,1) satisfy the constraint $2x – 3y > –5$.	**False**
20	The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 4$ and $x, y \geq 0$ is 12.	**False**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	In Simplex Method of solution of LPP, _____ variables are introduced to convert less than type inequalities into equations.	**Slack**
2	A basic solution with m equations and n variables has ____ number of variables equal to zero when n > m.	**n – m**
3	A basic feasible solution is a basic solution whose variables are _____.	**Non-negative**
4	In any LPP every ____ point of the Convex Set of Feasible Solutions is a _____ solution of the problem.	**Extreme; Basic Feasible**
5	The Objective Function of a Linear Programming Problem is maximized or minimized at the _____ solution.	**Optimal**
6	A linear programming problem has a well-defined Objective Function which is linear and which has to be _____.	**Optimized**
7	The constraints in a Linear Programming model arise due to limitation of _____.	**Resources**
8	The solution of a Linear Programming Problem indicates the right combination of _____ variables.	**Decision**
9	The set of variables appear in the Basic Variable column of a Simplex table is known as ____.	**Basis**
10	The variables which does not appear in the Basis is known as _____ variables.	**Non-basic**
11	In LPP the number of decision variables and constraints must be _____.	**Finite**
12	____ element of the Simplex Table is the element at the junction of Pivot Row and Pivot Column.	**Key**
13	To convert a more than equal to type constraint into an equation, introduction of a ____ variable and an Artificial variable is necessary while solving LPP by Simplex Method.	**Surplus**
14	Contribution of Slack or Surplus variable towards Objective Function is ____ always.	**Zero**
15	If the constraints of a LPP are $2x + y \leq 50$, $x + 2y \leq 40$, $x \geq 0$ and $y \geq 0$, then the feasible region lies in the ____ quadrant.	**First**
16	Linear Programming facilitates ___ use of productive factors.	**Optimal**
17	For the LPP, Minimize $Z = 4x + 5y$ the coordinates of the corner points of the bounded feasible region are (10,10), (20,5), (2,17), (16,11) and (17,5). The minimum value of Z is _____.	**90**
18	The corner points of feasible region of a LPP are (0,15), (15,15), (25,25), (10,35) and (10,0) If the Objective Function is $Z = px + qy$ then the condition for which Z will be maximum at both the points (25,25) and (10,35) is _____.	**3p = 2q**

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

What is meant by Mathematical Model?

2

Why the term slack is used in case of the variable used for converting a less than equal to type constraint inequality into an equation?

3

To convert more than or more than equal to type constraint inequalities into equations Surplus and Artificial variables are used. What is the purpose of using Artificial variable?

4

Two methods of solving LPP are in use – Graphical and Simplex. Which one is more versatile & why?

5

Iso-cost lines are used to solve minimization problems of LPP graphically. What is meant by the term Iso-cost?

6

Name different types of Management problems which can be solved by LP technique.

7

Why the term linear is used in relation to a LPP?

Essay Type Questions

1

What are the requirements of Linear Programming?

2

What are the advantages and limitations of Linear Programming Models?

3

What are the different application areas of Linear Programming?

4

Define the terms – Decision Variables, Objective Function, Constraints.

5

What do you understand by a Linear Programming Problem? Give formal definition of Linear Programming.

Comprehensive Numerical Problems

PRACTICE PROBLEMS WITH ANSWERS

PROBLEM 1 — INVESTOR'S PORTFOLIO OPTIMIZATION

An investor is well aware of the fact that maximization of total return on investment is best possible if the help of Linear Programming technique is taken. For that he has employed you and provided with the following information about his various activities of money making:

Activity A1 – Invest in Bank Fixed Deposit schemes, Activity A2 – Invest in Government Bonds, Activity A3 – Invest in Midcap Mutual Funds and Activity A4 – Invest in Equity linked Mutual Funds.

- Total amount to be invested – ₹5 lakhs
- To avoid excessive investment, not more than 50% of the total should be invested in Government Bonds and Midcap Mutual Funds.
- Investment in Bank Fixed Deposits is very conservative way of money making, while Investment in Equity linked Mutual Funds is very speculative. To avoid excessive speculation, at least ₹1 must be invested in Fixed Deposits for every ₹3 invested in Equity linked Mutual Funds.
- Data on the ROI: A1 4%, A2 5.5%, A3 7%, A4 10%

Required: Formulate the mathematical model of the problem.

ANSWER

Maximize $Z = 0.04x1 + 0.055x2 + 0.07x3 + 0.10x4$ (xi is the amount invested in Activity Ai and i = 1, 2, 3, 4)

Subject to the constraints:

- $x1 + x2 + x3 + x4 \leq 500000$
- $-0.5x1 + 0.5x2 + 0.5x3 - 0.5x4 \leq 0$
- $3x1 - x4 \geq 0$
- $x1, x2, x3, x4 \geq 0$

PROBLEM 2 — MANUFACTURING COMPANY (PRODUCTS A AND B)

For a manufacturing company the following data are relevant to its products A and B:

DESCRIPTION	PRODUCT A	PRODUCT B
Selling price per unit (₹)	200	240
Direct materials (₹)	45	50
Direct wages — Department 1 (₹)	16	20
Direct wages — Department 2 (₹)	22.50	15.50
Direct wages — Department 3 (₹)	10	30
Variable Overhead (₹)	6.50	11.50
Hours required per unit — Department 1	8	10
Hours required per unit — Department 2	10	6
Hours required per unit — Department 3	4	12

Number of employees in Departments 1, 2 and 3 are respectively 20, 15 and 18. Maximum available hours per employee per week for each of the three departments is 40.

Required: Formulate the linear programming problem so that the contribution of the company is maximized.

ANSWER

Maximize Z = 100x + 115y (x and y are the number of Products A and B to be manufactured)

Subject to the constraints:

- $8x + 10y \leq 800$ (Dept 1: 20 × 40)
- $10x + 6y \leq 600$ (Dept 2: 15 × 40)
- $4x + 12y \leq 720$ (Dept 3: 18 × 40)
- $x, y \geq 0$

PROBLEM 3 — SCRAP METAL PURCHASE

A company buying scrap metal has two types of scrap available. The first type of scrap has 20% of Metal A, 10% impurity and 20% of Metal B by weight. The second type of scrap has 30% of Metal A, 10% impurity and 15% of Metal B by weight. The company requires at least 120 kgs of Metal A, at most 40 kgs of impurity and at least 90 kgs of Metal B. The price for the two scraps are ₹200 and ₹300 per kg respectively.

Required: Determine the optimum quantities of the two scraps to be purchased by the company so that the requirements of the two metals and the restriction on impurity are satisfied at minimum cost. Use graphical method.

ANSWER

The solution is **infeasible**.

PROBLEM 4 — ENGINEERING COMPANY ADVERTISING STRATEGY

An engineering company dealing with Combustion Equipment and Furnaces is planning its advertising strategy. They have two monthly industrial magazines under consideration. The first magazine has a reach of 2,000 potential customers per advertisement and the second magazine has a reach of 3,000 potential customers per advertisement. Respective cost per advertisement of the two magazines are ₹6,000 and ₹9,000 and the firm has a monthly budget of ₹1 lakh. There is an important requirement that the total reach for the business group having annual turnover below ₹20 Crores must not exceed 3,000 potential customers. The reach of the two magazines having such business group customers are respectively 300 and 150 potential customers per advertisement.

Required: How many times the company should advertise in the two magazines to maximize the total reach? Solve graphically.

ANSWER

The problem has multiple optimum solution with maximum value of Objective Function = 100000/3.

PROBLEM 5 — FACTORY PRODUCTION STAGES (SIMPLEX TABLE)

A Factory manufactures 3 products which are processed through 3 different production stages. The time required to manufacture one unit of each of the three products and the daily capacity of the stages are given in the following table:

STAGE	PRODUCT 1 (MIN)	PRODUCT 2 (MIN)	PRODUCT 3 (MIN)	STAGE CAPACITY (MIN)
1	1	2	1	430
2	3	–	2	460
3	1	4	–	420

Profit per unit: Product 1 ₹3, Product 2 ₹2, Product 3 ₹5. Set the data in a Simplex Table and find the optimal solution.

ANSWER

Maximum Profit = ₹1,350. No. of units of Product 1 = 0; Product 2 = 100; Product 3 = 230.

PROBLEM 6 — OBTAIN THE DUAL FROM THE PRIMAL

Minimize $Z = x1 - 3x2 - 2x3$ subject to the Constraints: $3x1 - x2 + 2x3 \leq 7$, $2x1 - 4x2 \geq 12$, $-4x1 + 3x2 + 8x3 = 10$, $x1 \geq 0$, $x2 \geq 0$ and x3 is unrestricted in sign.

ANSWER

Maximize $Z^* = -7y1 + 12y2 + 10y3$

Subject to the constraints:

- $-3y1 + 2y2 + 4y3 \leq 1$
- $y1 - 4y2 + 3y3 \leq -3$
- $2y1 - 8y3 = 2$
- $y1 \geq 0$, $y2 \geq 0$ and y3 is unrestricted in sign

PROBLEM 7 — COMPANY MAKING THREE PRODUCTS X, Y AND Z

A company makes three products X, Y and Z using the raw materials A, B and C. Each of Product X requires 1 unit of raw material A, 2 units of B and 2 units of C. Each of Product Y requires 2 units of raw material A, 1 unit of B and 5 units of C. Each of Product Z requires 1 unit of raw material A, 4 units of B and 1 unit of C. Each unit of X, Y and Z gives profit of ₹40, ₹25 and ₹50 respectively.

Required: Find the optimum product mix. Write the Dual of the given problem. Using the optimum solution of the Primal problem, write the solution of the Dual and interpret it.

ANSWER

Optimum Product Mix: X = 20 Units, Y = 0 and Z = 5 Units; Maximum Profit = ₹1,050

Dual: Minimize Z* = 36P + 60Q + 45R subject to: P + 2Q + 2R ≥ 40, 2P + Q + 5R ≥ 25, P + 4Q + R ≥ 50, P, Q & R ≥ 0

Solution of Dual: P = 0, Q = 10 & R = 10 and Z* = 1050. The final table of the Primal problem indicates that the marginal value of the raw material A = 0, B = ₹10 per unit and C = ₹10 per unit respectively. Thus instead of making the products X, Y and Z and selling, if the raw materials A, B and C are sold at the above rates then also the same contribution of ₹1050 can be achieved.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 6

ABBREVIATION	FULL FORM
LPP	Linear Programming Problem
LP	Linear Programming
Z	Objective Function (Primal)
Z*	Objective Function (Dual)
x1, x2, … xn	Decision Variables
s1, s2, …	Slack Variables
A1, A2, …	Artificial Variables
Cj	Contribution per unit
Zj	Gross profit or loss given up
Cj – Zj	Index Row / Net Evaluation Row
CB	Coefficients of Basic Variables
XB	Values of Basic Variables
M	Large penalty (for Artificial Variables)
PERT	Program Evaluation and Review Technique
CPM	Critical Path Method
JIT	Just-In-Time
ABCM	Activity Based Cost Management

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MODULE COVERAGE

6.1 Introduction to Linear Programming

6.2 Definition of Linear Programming

6.3 Requirements of Linear Programming

6.4 Advantages of Linear Programming

6.5 Limitations of Linear Programming

6.6 Application Areas

6.7 Formulation of LPP

6.8 Some Definitions

6.9 Methods of Solving LPP

6.10 Special Cases

6.11 Duality

STUDY TOOLS

Graphical Method Steps

Simplex Method Steps

Formula Boxes

Quick Revision Sheets

MCQs with Reasoning

Numerical Problems

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Module Group Weightage: 15%

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Transportation Problem

Module 7 · NWCM · LCM · VAM · MODI Method · Special Cases

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	15% (with Linear Programming, Assignment, Game Theory)

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading ✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

07

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 7 INCLUDES

Introduction · Linear Programming formulation of Transportation Problem · Procedure of solution of Transportation Problems · Special cases of Transportation — Unbalanced Problem, Maximization Problem, Degeneracy, Multiple Optimum Solution, Prohibited or Preferred Routes.

Module Learning Objectives

After studying this module, the students will be able to:

1

Recognise a Transportation Problem

Recognise a problem of Transportation. State a Transportation problem in the form of a Linear Programming problem.

2

Find Initial Basic Feasible Solution

Find Initial Basic Feasible Solution by various methods — North West Corner Method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM).

3

Find Minimum Transportation Cost Schedule

Find minimum Transportation Cost schedule using Modified Distribution (MODI) Method.

4

Obtain Solutions for Special Cases

Obtain solutions for the Special cases of Transportation like Unbalanced Problem, Maximisation Problem, Problem having Multiple Optimum Solution, Problem having Degeneracy, Restricted Transportation Problem etc.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Transportation techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

15%	3	5	30
GROUP WEIGHTAGE	INITIAL SOLUTION METHODS	SPECIAL CASES	MCQS
With LP, Assignment, Game Theory	NWCM, LCM, VAM	Unbalanced, Maximization, Degeneracy, Multiple Optimum, Prohibited	With answers

SYLLABUS STRUCTURE NOTE

Module 7 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Transportation, along with Linear Programming, Assignment and Game Theory, forms a group carrying **15% weightage** in the examination. This module introduces a special class of Linear Programming Problem widely used in logistics and supply chain optimization.

📖 Table of Contents — Module 7

1. Introduction to Transportation Problem
2. Linear Programming Formulation
3. Procedure of Solution — Overview
4. North West Corner Method (NWCM)
5. Least Cost Method (LCM)
6. Vogel's Approximation Method (VAM)

7. Testing Optimality — MODI Method

8. Special Cases of Transportation

9. Illustrations & Numerical Problems

7.1 Introduction to Transportation Problem

CONCEPT · HISTORICAL DEVELOPMENT · TYPICAL SITUATION · ASSUMPTIONS

DEFINITION

Transportation problem is a special kind of Linear Programming Problem, in which goods are transported from a set of Sources to a set of Destinations, subject to the constraints of supply and demand such that the **total Cost of Transportation is minimized**.

It is also sometimes called the **Hitchcock problem** because it was originally developed by F. L. Hitchcock in his study titled "The distribution of a product from several sources to numerous locations" in the year 1941. Subsequently in the year 1947, T. C. Koopmans independently published a study on "Optimum utilization of the Transportation System". The linear programming formulation and the associated systematic procedure for solution was given by **George B. Dantzig** afterwards — in the year 1951.

Typical Situation

Assume that 2 Bread Factories O1 and O2 make the daily bread in a city. The bread produced is supplied to the three Bakeries of the City — D1, D2 and D3. The production capacities of the Bread Factories are 2000 & 2500 units respectively and demands of the Bakeries are 1500, 2000 and 1000 units as shown in the diagram below. Also shown are the Transportation Costs per unit of bread along different routes. The problem is to determine the quantity each Factory should transport to each Bakery so that the Total Cost of Transportation is minimum.

	D1	D2	D3	SUPPLY
O1	₹8	₹6	₹10	**2000**
O2	₹10	₹4	₹9	**2500**
	1500	**2000**	**1000**	

Underlying Assumptions of a Transportation Problem

A

Single Commodity

Only a single type of commodity is being shipped from different Sources to various Destinations.

B

Total Supply = Total Demand

The total supply capacity of the Origins is equal to the Total Demand of the Destinations (Rim Condition).

C

Known Unit Costs

The unit Transportation cost of the item from all sources to different destinations are certainly and precisely known.

D

Minimize Total Cost

The objective is to minimize the total cost of transportation.

⚡ Quick Revision — Section 7.1

- **Transportation Problem:** Special kind of LPP — goods transported from Sources to Destinations, minimizing total cost.
- **Also called:** Hitchcock problem (F.L. Hitchcock, 1941).
- **LP Formulation:** Given by George B. Dantzig (1951).
- **Four Assumptions:** Single commodity, Total Supply = Total Demand, Known unit costs, Minimize total cost.
- **Rim Condition:** Total capacity of Sources = Total demand of Destinations.

7.2 Linear Programming Formulation of Transportation Problem

DECISION VARIABLES · OBJECTIVE FUNCTION · CONSTRAINTS · GENERALISED FORM

With reference to the problem mentioned above, following decision variables are defined:

x_{ij} = Number of units of bread to be delivered by the Factory O_i to the Bakery D_j where i = 1, 2 & j = 1, 2, 3

LP MODEL — OBJECTIVE FUNCTION

$$\text{Minimize } Z = 8x_{11} + 6x_{12} + 10x_{13} + 10x_{21} + 4x_{22} + 9x_{23}$$

Subject to the Constraints

TYPE	CONSTRAINT	EXPLANATION
Supply	$x_{11} + x_{12} + x_{13} = 2000$	Supply constraint of Factory O_1
Supply	$x_{21} + x_{22} + x_{23} = 2500$	Supply constraint of Factory O_2
Demand	$x_{11} + x_{21} = 1500$	Demand constraint of Bakery D_1
Demand	$x_{12} + x_{22} = 2000$	Demand constraint of Bakery D_2
Demand	$x_{13} + x_{23} = 1000$	Demand constraint of Bakery D_3
Non-negativity	$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}$ & $x_{23} \geq 0$	Non-negativity constraint

RIM CONDITION

The problem has a feasible solution only if the **total capacity of the Sources** and **total demand of the Destinations** are equal. This is known as the **Rim Condition**.

GENERALISED FORM

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$$\text{Subject to: } \sum_{j=1}^{n} x_{ij} = a_i \quad (i = 1, 2, \ldots, m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1, 2, \ldots, n)$$

$$x_{ij} \geq 0$$

Where a_i = supply capacity of Source i; b_j = demand of Destination j; C_{ij} = unit transportation cost from Source i to Destination j; x_{ij} = quantity transported from Source i to Destination j.

FEASIBLE SOLUTION AND BASIC FEASIBLE SOLUTION

Feasible Solution: A set of non-negative values of x_{ij} (where i = 1, 2, ... m and j = 1, 2, ... n) that satisfy all the constraints. **Basic Feasible Solution:** An initial feasible solution with **(m + n – 1)** number of values of the variable.

⚡ Quick Revision — Section 7.2

- **Objective Function:** Minimize total transportation cost.
- **Constraints:** Supply constraints (row sums = a_i); Demand constraints (column sums = b_j); Non-negativity.
- **Rim Condition:** Total supply = Total demand.
- **Basic Feasible Solution:** (m + n – 1) allocations where m = number of rows, n = number of columns.

7·3

Procedure of Solution of Transportation Problems

FIVE STEPS · INITIAL SOLUTION METHODS · OPTIMALITY TEST

Step A Obtain a Basic Feasible Solution → **Step B** Test for Optimality → **Step C** Non-optimal? Go to Step D → **Step D** Improve the Basic Feasible Solution →

Step E Repeat B, C, D until Optimal

Methods for Obtaining Basic Feasible Solution

1

North West Corner Method (NWCM)

The simplest of all methods. Solution begins by allocating maximum possible quantity to the north-west or top left corner cell of the Transportation matrix.

2

Least Cost Method (LCM)

Due consideration is given to the cost. Selects the cell with lowest shipping cost among all the cells of the matrix. Reduces computations while obtaining the optimal solution.

3

Vogel's Approximation Method (VAM)

Uses penalty (difference between least cost and next least cost) for each row and column. Generally gives Optimal or near Optimal solution — so number of iterations required is much less.

Methods for Testing Optimality

1

Modified Distribution (MODI) Method

Also known as the u-v method. Uses the relation $C_{ij} = u_i + v_j$ for allocated cells to find row and column numbers, then computes opportunity cost for unallocated cells.

2

Stepping Stone Method

Tests each unallocated cell one at a time by computing the change in cost and checking if inclusion of any unoccupied cell can decrease the Total Cost of Transportation.

MODI METHOD PREFERRED

In practice, Modified Distribution Method finds wider use compared to Stepping Stone Method due to the fact that the process of evaluating each unoccupied cell for further possible improvement is more efficient in it.

⚡ Quick Revision — Section 7.3

- **Five Steps:** Obtain Basic Feasible Solution → Test for Optimality → If non-optimal → Improve → Repeat until Optimal.
- **Three Initial Solution Methods:** NWCM (simplest, ignores cost), LCM (uses cost), VAM (uses penalty, usually optimal/near-optimal).
- **Two Optimality Testing Methods:** MODI Method (preferred), Stepping Stone Method.

7.4

North West Corner Method (NWCM)

STEP-BY-STEP METHODOLOGY · ILLUSTRATION

Step-by-Step Methodology

Step 1

Allocate as many units as possible in the north-west or left top corner cell of the Transportation matrix by maintaining supply and demand constraint corresponding to this cell. In fact minimum of the value of supply or demand corresponding to the mentioned cell should be allocated.

Step 2

Subtract the quantity allocated in the previous step from the supply and demand figures in the first row and the column to get the adjusted figures.

Step 3

Depending on which of the values of supply or demand exhausted fully in step before, follow either one of the steps mentioned below:

- **(a)** If the supply for the first row is exhausted then move down to the first cell of the second row & first column and once again allocate as maximum as possible quantity maintaining the constraint of supply and demand. Subsequently adjust the quantities of supply and demand by subtracting the quantity allocated from them.
- **(b)** If the demand for the first column is exhausted then move sideways to the next cell in the second column and first row and as before allocate as maximum as possible quantity in the cell. Subsequently adjust the corresponding figures of supply and demand.

Step 4

Continue the procedure until total available quantity is fully allocated to the cells as required. In case for any cell the supply and demand both exhausts at a time (in other words supply = demand for the cell) then next allocation can be made in the cell either in the next row or column.

GREATEST DRAWBACK OF NWCM

Though the method is very easy to compute, understand and interpret, the greatest drawback is the fact that **transportation costs are not given any consideration**. As a result, the Initial Feasible Solution obtained by this method is mostly non-optimal. Usually several iterations are required before an optimal solution is reached.

Illustration 1 — NWCM Solution

PROBLEM

A company dealing with a special type of liquid has three plants P1, P2 & P3 located throughout the country. Production capacities of these plants are respectively 50, 75 and 25 Gallons. Each day the firm must furnish to four of its Retail shops R1, R2, R3, and R4 with at least 20, 20, 50 and 60 Gallons of the product. The transportation cost per Gallon (in '000 ₹) between various sets of Plants and Retail Shops are given below:

FROM PLANTS	R1	R2	R3	R4	SUPPLY
P1	3	5	7	6	**50**
P2	2	5	8	2	**75**
P3	3	6	9	2	**25**
Demand	**20**	**20**	**50**	**60**	**150**

Required: Find the Basic Feasible Solution using NWCM.

SOLUTION — FINAL ALLOCATION

FROM PLANTS	R1	R2	R3	R4	SUPPLY
P1	20	20	10	—	50
P2	—	—	40	35	75
P3	—	—	—	25	25
Demand	20	20	50	60	150

FROM PLANT	TO RETAIL SHOP	SHIPPING COST PER GALLON (₹'000)	QUANTITY ALLOCATED (GALLONS)	COST (₹'000)
P1	R1	3	20	60
P1	R2	5	20	100
P1	R3	7	10	70
P2	R3	8	40	320
P2	R4	2	35	70
P3	R4	2	25	50
TOTAL				**₹6,70,000**

Total Cost of Transportation = ₹6,70,000

⚡ Quick Revision — Section 7.4

- **NWCM:** Start from top-left corner cell; allocate maximum possible; move right or down.
- **Drawback:** Ignores transportation costs; solution is mostly non-optimal.
- **Illustration:** Total Cost = ₹6,70,000.

7.5

Least Cost Method (LCM)

STEP-BY-STEP METHODOLOGY · ILLUSTRATION

Step-by-Step Methodology

Step 1

Select the cell with lowest shipping cost among all the cells of the given Transportation matrix. If such a cell is not unique then select a cell out of all the cells tied with the lowest cost value such that maximum allocation is possible there. In case there is a tie in that, too then arbitrarily choose one among the cells with tied lowest cost as well as maximum possible allocation value.

Step 2

Allocate as maximum as possible units to the cell selected in step (1) by maintaining the Supply and Demand constraint corresponding to the selected cell. As a result either the Supply figures of a row or the Demand figures of a column will be fully exhausted. Such a row or column should be eliminated for finding further allocations.

Step 3

Reduce the Supply value of the Row or the Demand value of the Column (which is not completely exhausted) by the amount allocated in the previous step.

Step 4

Repeat above steps until all the allocations are over.

Illustration 2 — LCM Solution (Same Data as Illustration 1)

SOLUTION — FINAL ALLOCATION

FROM PLANTS	R1	R2	R3	R4	SUPPLY
P1	3	20	30	6	50
P2	15	5	8	60	75
P3	5	6	20	2	25
Demand	20	20	50	60	150

FROM PLANT	TO RETAIL SHOP	SHIPPING COST PER GALLON (₹'000)	QUANTITY ALLOCATED (GALLONS)	COST (₹'000)
P1	R2	5	20	100
P1	R3	7	30	210
P2	R1	2	15	30
P2	R4	2	60	120
P3	R1	3	5	15
P3	R3	9	20	180
TOTAL				**₹6,55,000**

Total Cost of Transportation = ₹6,55,000 — an improvement of ₹15,000 over NWCM.

⚡ Quick Revision — Section 7.5

- **LCM:** Select lowest cost cell; allocate maximum; eliminate exhausted row/column.
- **Advantage:** Considers transportation costs; better initial solution than NWCM.
- **Illustration:** Total Cost = ₹6,55,000 (improvement of ₹15,000 over NWCM).

7.6 Vogel's Approximation Method (VAM)

PENALTY CONCEPT · STEP-BY-STEP METHODOLOGY · ILLUSTRATION

PENALTY CONCEPT

Like LCM, this method also makes use of the Cost figures. Instead of least cost, the **difference between the least cost and the value next to least cost** is computed for each and every row and column. Such difference is termed as **Penalty**. Then least penalty value is used as the starting point of allocation.

Step-by-Step Methodology

Step 1

Calculate Penalty values for each Row and Column (where Penalty = Difference between the smallest value and next smallest value of unit transportation costs provided in the problem). In case there are two or more minimum value in the same Row or Column then the corresponding Penalty is zero.

Step 2

Identify the Row or Column having highest of the Penalty values computed in Step (1).

Step 3

In the identified Row or Column choose the cell having least cost figure and allocate as maximum as possible number of units maintaining the constraint of Supply and Demand. As a result either the Supply or the Demand will be fully exhausted and the corresponding Row or the Column should be omitted from consideration of further allocations.

Step 4

Reduce the Supply value of the Row or the Demand value of the Column (which is not completely exhausted) by the amount allocated in the previous step.

Step 5

Re-compute the Row and Column penalties for the reduced matrix of Transportation.

Step 6

Repeat steps (3) and (4).

Step 7

Repetition of the above steps to be continued until all allocations are done. In other words it should continue until total Supply and Demand values are fully exhausted.

TIE-BREAKING RULE

If a TIE occurs in the penalties, select the Row or the Column which has minimum cost figure among its cells. If there is a TIE in that value also then go for the cell which can have highest allocation among such cells. If there is a TIE in this value, too then arbitrarily choose one among such cells.

Illustration 3 — VAM Solution (Same Data as Illustration 1)

SOLUTION — FINAL ALLOCATION

FROM PLANTS	R1	R2	R3	R4	SUPPLY
P1	20	5	30	6	50
P2	2	20	20	35	75
P3	3	6	9	25	25
Demand	20	20	50	60	150

FROM PLANT	TO RETAIL SHOP	SHIPPING COST PER GALLON (₹'000)	QUANTITY ALLOCATED (GALLONS)	COST (₹'000)
P1	R1	3	20	60
P1	R3	7	30	210
P2	R2	5	20	100
P2	R4	2	35	70
P2	R3	8	20	160
P3	R4	2	25	50
TOTAL				**₹6,50,000**

Total Cost of Transportation = ₹6,50,000 — an improvement of ₹20,000 over NWCM and ₹5,000 over LCM.

⚡ Quick Revision — Section 7.6

- **VAM:** Uses penalty = difference between least cost and next least cost.
- **Advantage:** Generally gives Optimal or near Optimal solution; fewer iterations.
- **Illustration:** Total Cost = ₹6,50,000 (best among the three initial solution methods).

7.7 Testing Optimality — Modified Distribution (MODI) Method

U-V METHOD · OPPORTUNITY COST · LOOP FORMATION · ILLUSTRATION

Steps in MODI Method

Step 1

For getting an optimum solution of a Transportation Problem, find the Basic Feasible Solution. In general there will be (m + n – 1) number of cell allocations in the solution (here, m = No. of Rows & n = No. of Columns of the Transportation matrix).

Step 2

Determine the values of Row Numbers (u_i) and Column Numbers (v_j) using the relation $\mathbf{C_{ij} = u_i + v_j}$ for each of the allocated cells (where C_{ij} = Unit transportation cost for allocated cell). It can be mentioned that i = m & j = n causing a total of (m + n) number of Row and Column Numbers which cannot be found by solving (m + n – 1) numbers of equations. So one of the numbers is to be chosen arbitrarily. A rule of thumb is to choose a zero value for the number corresponding to that row or column which is having maximum number of cell allocations.

Step 3

Compute the **Opportunity Cost (Improvement Index)** using the relation $\mathbf{\Delta_{ij} = C_{ij} - (u_i + v_j)}$ for each of the unallocated cells.

Step 4

Check the sign of each Opportunity Cost. If all the Opportunity Cost values are non-negative then the solution is optimal. If there exists one or more opportunity cost value with negative sign then go to next step.

Step 5

Select the unallocated cell with highest negative Opportunity Cost value as the cell to be included in the next solution.

Step 6

Draw a closed path or loop starting from the cell selected in the previous step and going straight (either horizontally or vertically) in that direction where allocated cells are situated. Once an allocated cell is met, take a

right angle turn to move straight towards another allocated cell and again take a right angle turn as before. This has to be continued until the starting cell is reached again.

Step 7

Assign alternate plus (+) and minus (–) signs to the cells at the corner points of the loop, with a plus sign at the starting cell.

Step 8

Of the cells with minus (–) signs, consider the one having lowest allocation. Subtract this amount of allocation from the cells with minus (–) sign and add the same to the cells with plus (+) sign. As a result the unallocated cell from where the loop started will become allocated and the cell with minus sign and having lowest allocation will become unallocated.

Step 9

Repeat the whole procedure until all the Opportunity Cost values become non-negative, i.e., an Optimum Solution is obtained.

IMPORTANT POINTS

- **Conditions to be satisfied by the Initial Feasible solution:** Each and every Row & Column must have at least one allocation; All the allocations must be made in independent cells and they should be (m + n – 1) in number. The cells which make a closed loop are the Dependent cells. Others are the Independent Cells.
- **Case of equal maximum Opportunity Cost:** While revising a given Transportation solution, if two or more unallocated cells have the same largest negative Opportunity Cost, the unallocated cell by including which the decrease in the Transportation Cost is maximum should be chosen.

Illustration 4 — MODI Method (Testing VAM Solution for Optimality)

PROBLEM

Test the Basic Feasible Solution obtained by VAM in Illustration 3 for Optimality. In case the solution is non-optimal, find the optimum solution to get the minimum Total Cost of Transportation.

SOLUTION — ROW AND COLUMN NUMBERS

The Basic Feasible Solution by VAM has 6 allocated cells ($m + n - 1 = 3 + 4 - 1 = 6$). Assume $u_2 = 0$ (since 2nd Row has maximum number of allocations).

ALLOCATED CELL	COST (C_{IJ})	$C_{IJ} = U_I + V_J$	CALCULATED NO.
P2R2	$C_{22} = 5$	$5 = 0 + v_2$	$v_2 = 5$
P2R3	$C_{23} = 8$	$8 = 0 + v_3$	$v_3 = 8$
P2R4	$C_{24} = 2$	$2 = 0 + v_4$	$v_4 = 2$
P3R4	$C_{34} = 2$	$2 = u_3 + 2$	$u_3 = 0$
P1R3	$C_{13} = 7$	$7 = u_1 + 8$	$u_1 = -1$
P1R1	$C_{11} = 3$	$3 = -1 + v_1$	$v_1 = 4$

Opportunity Costs for Unallocated Cells:

UNALLOCATED CELL	COST (C_{IJ})	U_I	V_J	OPPORTUNITY COST $\Delta_{IJ} = C_{IJ} - (U_I + V_J)$
P1R2	5	–1	5	1
P1R4	6	–1	2	5
P2R1	2	0	4	–2
P3R1	3	0	4	–1
P3R2	6	0	5	1
P3R3	9	0	8	1

Observation: There exist two cells with negative Opportunity Cost (P2R1: –2 and P3R1: –1). Hence the solution is **non-optimal**.

Improvement: Highest negative Opportunity Cost appears at cell P2R1 ($\Delta = -2$). A closed loop is drawn starting from P2R1, passing through P1R1, P1R3, P2R3 and back to P2R1. Alternate (+) and (–) signs are assigned with (+) at P2R1.

Allocated quantities of the (–) signed cells (P1R1 and P2R3) are both 20. This amount is subtracted from the quantities of the (–) signed cells and added to the quantities of the (+) signed cells. As a result both the (–) signed cells become unallocated. This turns into a five-cell allocated solution which means a Degenerate

solution. To take care of this an infinitely small allocation of amount (ε) is made to the cell having higher cost between the currently vanished cells i.e. the cell P2R3.

Revised Allocations:

FROM PLANTS	R1	R2	R3	R4	SUPPLY
P1	3	5	**50**	6	**50**
P2	**20**	**20**	**ε**	**35**	**75**
P3	3	6	9	**25**	**25**
Demand	**20**	**20**	**50**	**60**	**150**

Recomputing Opportunity Costs: With $u_2 = 0$, we get $v_1 = 2$, $v_2 = 5$, $v_3 = 8$, $v_4 = 2$, $u_1 = -1$, $u_3 = 0$.

All Opportunity Costs for unallocated cells are now non-negative (P1R2 = 1, P1R4 = 5, P1R1 = 2, P3R1 = 1, P3R2 = 1, P3R3 = 1). Hence the solution is **Optimal**.

Optimum Transportation Schedule:

FROM PLANT	TO RETAIL SHOP	SHIPPING COST PER GALLON (₹'000)	QUANTITY ALLOCATED (GALLONS)	COST (₹'000)
P1	R3	7	50	350
P2	R1	2	20	40
P2	R2	5	20	100
P2	R4	2	35	70
P2	R3	8	ε = 0	0
P3	R4	2	25	50
TOTAL				**₹6,10,000**

Minimum Total Cost of Transportation = ₹6,10,000

⚡ Quick Revision — Section 7.7

- **MODI Method:** Uses $C_{ij} = u_i + v_j$ for allocated cells to find u_i and v_j.
- **Opportunity Cost:** $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for unallocated cells.
- **Optimality:** All $\Delta_{ij} \geq 0 \rightarrow$ optimal.
- **Improvement:** Select cell with highest negative Δ_{ij}; draw closed loop; assign alternate (+) and (–) signs; transfer allocation.
- **Degeneracy:** When number of allocations < (m + n – 1); resolved by allocating ε to least cost independent unoccupied cell.

7.8 Special Cases of Transportation

UNBALANCED · MAXIMIZATION · DEGENERACY · MULTIPLE OPTIMUM · PROHIBITED/PREFERRED ROUTES

1. Unbalanced Problem of Transportation

DEFINITION

One of the assumptions for applying Transportation algorithm is Total Supply capacity of the Origins is equal to the Total Demand of the Destinations. But for all practical purposes, it is seen to be met very rarely. Mostly the Supply and Demand figures are unequal. Such a problem of Transportation is called **Unbalanced problem of Transportation**.

SUPPLY EXCEEDS DEMAND

- Problem converted to Balanced one by bringing a **Dummy Destination (or Column)**
- Dummy Destination has Demand equal to the difference between Supply and Demand of original problem
- Cost of Transportation corresponding to cells of DUMMY row or column is taken as **ZERO**

OR

DEMAND EXCEEDS SUPPLY

- Problem converted to Balanced one by bringing a **Dummy Origin (or Row)**
- Dummy Origin has Supply capacity equal to the difference between Supply and Demand of original problem
- Cost of Transportation corresponding to cells of DUMMY row or column is taken as **ZERO**

Illustration 5 — Unbalanced Problem

PROBLEM

The products of two Plants A and B are to be transported to three Warehouses W1, W2 and W3. The costs (₹'00) of transportation of each unit from Plants to the Warehouses are indicated in the table below. Also provided are the Supply Capacities of the Plants and the Demands of the three Warehouses.

	W1	W2	W3	SUPPLY
Plant A	25	17	25	**300**
Plant B	15	10	18	**500**
	300	**300**	**500**	

Required: Find the Optimum Distribution Schedule and associated Cost of Transportation.

SOLUTION

Total Supply Capacity = 800 units; Total Demand = 1100 units. Problem is unbalanced. To convert it into a balanced one, a **Dummy Plant C** having Supply Capacity = 1100 – 800 = 300 units is introduced. Cost of transportation from this plant to any of the warehouses is considered to be 0.

Optimum Solution:

	W1	W2	W3	SUPPLY
Plant A	25	**100**	**200**	**300**
Plant B	**300**	**200**	18	**500**
Plant C (Dummy)	0	0	**300**	**300**
Demand	**300**	**300**	**500**	**1100**

Minimum Total Cost of Transportation = ₹1,320,000. From the above solution it can be concluded that 300 units of Demand of W3 remains unfulfilled because it is being supplied by C which is a Dummy Plant, i.e., non-existent.

2. Maximization Problem of Transportation

METHODOLOGY

Transportation algorithm is basically meant for solving minimization problems, but the same can be successfully used for solving maximization problems too. For that first of all, the original problem has to be converted to a minimization problem. This is done by **subtracting all the elements of the given profit matrix from the highest element of the matrix**. Thus we get the value of **Relative Loss or Opportunity Loss** against each of the given profit figures. Once the Profit Matrix given is converted to Relative Loss Matrix, the problem can be solved as a minimization problem by following the usual method of solving Transportation problems.

Illustration 6 — Maximization Problem

PROBLEM

A multi-plant company has three manufacturing plants M1, M2 and M3. The company is dealing with a unique product and enjoys monopoly as far as competition is concerned. They have two fixed customers A and B who procure all the items produced by the company.

Cost of Production (₹ per piece) of the Plants M1, M2 and M3 are respectively 1500, 1600 & 1700. Selling prices to the customers A and B are ₹4400 and ₹4700 per piece respectively.

	PLANT M1	PLANT M2	PLANT M3	DEMAND
Customer A	₹1000	₹2000	₹1500	3500 units
Customer B	₹1500	₹3000	₹2500	3600 units
Production Capacity	2000 units	3000 units	4000 units	

Required: Formulate the problem as LP Model. Also find the optimum solution using Transportation algorithm.

SOLUTION

Profit = Selling Price – (Production Cost + Transportation Cost)

Profit Matrix:

	CUSTOMER A	CUSTOMER B	CUSTOMER C (DUMMY)	SUPPLY
Plant M1	₹1900	₹1700	0	**2000**
Plant M2	₹800	₹100	0	**3000**
Plant M3	₹1200	₹500	0	**4000**
Demand	**3500**	**3600**	**1900**	**9000**

Convert to Relative Loss Matrix by subtracting each element from the highest element (1900). Divide each figure by 100 for simplicity.

Optimum Solution:

	CUSTOMER A	CUSTOMER B	CUSTOMER C (DUMMY)	SUPPLY
Plant M1	0	**20**	19	**20**
Plant M2	11	**11**	**19**	**30**
Plant M3	**35**	**5**	19	**40**
Demand	**35**	**36**	**19**	**90**

FROM PLANT	TO CUSTOMER	PROFIT PER UNIT (₹)	QUANTITY ALLOCATED ('00 UNITS)	PROFIT (₹)
M1	B	1700	20	34,000
M2	B	100	11	1,100
M2	C	0	19	0
M3	A	1200	35	42,000
M3	B	500	5	2,500
TOTAL				**₹79,600**

Maximum Total Profit = ₹79,60,000 (after considering proper unit of the Quantity allocated values).

3. Problems with Degeneracy

DEFINITION

A Transportation problem with (m × n) Cost Matrix should have **(m + n – 1)** numbers of Cell allocations at any stage of solution, i.e., either in Basic Feasible Solution or in any Improved Solution. In case the Numbers of allocations are less than (m + n – 1), the solution is said to be **Degenerate**.

1

Degeneracy at the Stage of Initial Feasible Solution

To resolve such a situation, an infinitely small quantity [denoted by Greek letter 'Epsilon' (ε)] is introduced into the solution to one or more of the unoccupied cells so that the number of cell allocations become (m + n – 1). For all practical purposes the value of ε is taken to be ZERO. It is introduced to the least cost unoccupied independent cell of the matrix.

2

Degeneracy During Improvement of a Solution

This kind of situation occurs when the inclusion of the unoccupied cell with maximum negative Opportunity Cost results in vacating two or more occupied cells simultaneously. To resolve such a situation, allocation of an infinitely small quantity ε is done to one or more of the recently vacated cells so that the number of allocated cells become (m + n – 1).

Illustration 7 — Degeneracy

PROBLEM

A manufacturing company has three Plants X, Y and Z which supply to the Distributors located at A, B, C, D and E. Monthly production capacities of the Plants are respectively 80, 50 and 90 units. Monthly requirement of the Distributors are 40, 40, 50, 40 and 80 units respectively. Unit Transportation Costs (₹) are given below.

	A	B	C	D	E	SUPPLY
X	5	8	6	6	3	**80**
Y	4	7	7	6	6	**50**
Z	8	4	6	6	3	**90**
Demand	**40**	**40**	**50**	**40**	**80**	**250**

Required: Determine the Optimum Schedule of distribution of the company in order to minimize the Total Cost of Transportation.

SOLUTION

Total Production Capacity = 220 units; Total Requirement = 250 units. Demand > Supply. So this is an Unbalanced problem. Introduce a **Dummy Plant** having Production Capacity = 250 – 220 = 30 units. Unit Transportation Cost from this Dummy Plant to any of the Distributors is taken as ZERO.

Optimum Solution (after resolving Degeneracy by allocating ε to Dummy – C):

	A	B	C	D	E	SUPPLY
X	5	8	**50**	6	**30**	**80**
Y	**40**	7	7	**10**	6	**50**
Z	8	**40**	6	6	**50**	**90**
Dummy	0	0	**ε**	**30**	0	**30**
	40	**40**	**50**	**40**	**80**	

FROM PLANT	TO DISTRIBUTOR	COST PER UNIT (₹)	QUANTITY ALLOCATED (UNITS)	COST (₹)
X	C	6	50	300
X	E	3	30	90
Y	A	4	40	160
Y	D	6	10	60
Z	B	4	40	160
Z	E	3	50	150
Dummy	D	0	30	0
TOTAL				**₹920**

Minimum Total Cost of Transportation = ₹920. From the above solution it can be concluded that 30 units of Demand of Distributor 'D' remains unfulfilled because it is being supplied by the Dummy Plant which is non-existent.

4. Transportation Problems having Multiple Optimum Solution

DEFINITION

Multiple Optimum Solution of a Transportation problem means the problem is having more than one optimum solution. In other words, the minimum Total Cost of Transportation for all such solutions are same but the routes of transportation are different. A Transportation problem is said to have Multiple Optimum Solution if there exist one or more unoccupied cell in the final Table with **ZERO Opportunity Cost**.

If such situation occurs for a problem then to get another optimum solution a loop has to be formed starting from the cell (or the cells). This will give a new set of allocations without disturbing the minimum Total Transportation Cost.

Illustration 8 — Alternative Solution

PROBLEM

Find an Alternative Solution to the Transportation Problem of Illustration 5.

SOLUTION

Optimum solution already obtained in Illustration 5 shows Opportunity Cost value = 0 for the cell (B – W3). So a loop beginning from (B – W3) is drawn.

From the drawn loop, the allocations with (–) sign are both 200. So this quantity is to be subtracted from the allocations of the cells with (–) sign and added to the quantities of the cells with (+) sign.

Alternative Optimum Solution:

	W1	W2	W3	SUPPLY
Plant A	25	**300**	25	**300**
Plant B	**300**	10	**200**	**500**
Plant C (Dummy)	0	0	**300**	**300**
Demand	**300**	**300**	**500**	**1100**

FROM PLANT	TO WAREHOUSE	TRANSPORTATION COST PER UNIT (₹'00)	QUANTITY ALLOCATED (UNITS)	COST (₹'00)
A	W2	17	300	5100
B	W1	15	300	4500
B	W3	18	200	3600
C	W3	0	300	0
TOTAL				**₹13200**

Total Cost of Transportation = ₹13,200 which is same as the Total Cost obtained in Illustration 5. Thus the Transportation schedule though different yields same minimum Total Cost.

5. Transportation Problems with Prohibited or Preferred Routes

CONCEPT

Sometimes in practical life one may have to come across situations of restriction or prohibition of certain routes for Transportation of goods. The reason behind this is situations occurred due to Natural Calamities (like Floods, Earthquake etc.) or due to War, Strike called by the political parties etc. Such restrictions are handled in the Transportation problem by assigning a **very high cost (Say M or ∞) in the prohibited routes** so that these routes are never included in the solution. The usual method of solving a Transportation problem is used.

⚡ Quick Revision — Section 7.8

- **Unbalanced:** Supply ≠ Demand; add Dummy row/column with zero costs.
- **Maximization:** Convert to minimization by subtracting all elements from highest element (Relative Loss Matrix).
- **Degeneracy:** Allocations < (m + n – 1); resolve by allocating ε to least cost independent unoccupied cell.
- **Multiple Optimum:** Zero Opportunity Cost for unoccupied cell; form loop to get alternative solution.
- **Prohibited Routes:** Assign very high cost (M or ∞).

? Multiple Choice Questions

WITH ANSWERS AND REASONING

1. Which of the following considers difference between least cost and the cost just before least for each row and column while finding Basic Feasible Solution in Transportation?

(a) North West Corner Method (b) Least Cost Method (c) Vogel's Approximation Method

(d) Both (b) and (c) above

Answer: C

VAM uses penalty = difference between least cost and next least cost.

2. When the total allocation of a Transportation Problem match with supply and demand values, the solution is:

(a) Non-degenerate (b) Feasible (c) Degenerate

(d) None of the above

Answer: B

3. The solution to a Transportation Problem with 'm' sources and 'n' destinations is feasible if the number of cell allocations are:

(a) m + n (b) mn (c) m – n – 1

(d) m + n – 1

Answer: D

4. To resolve Degeneracy in the solution of a Transportation Problem an infinitely small allocation is made to the solution already obtained. This allocation is known as:

(a) Dummy (b) Epsilon (c) ε – the Greek letter

(d) All of the above except (a)

Answer: D

5. Which of the following is not correct with respect to Transportation as a tool of Quantitative Technique?

(a) Transportation technique is a special case of LP.

(b) Transportation technique might give rise to solutions which are degenerate.

(c) No Transportation problem can be given with supply ≠ demand.

(d) Using Transportation technique one can maximize an Objective Function.

Answer: C

6. Which of the following method is used to test optimality of a solution in Transportation?

(a) Modified Distribution (b) Simplex (c) VAM

(d) LCM

Answer: A

7. In a solution of Transportation problem, empty cells are called:

(a) Unoccupied cells (b) Unallocated cells (c) Empty cells

(d) All of the above

Answer: D

8. The Transportation Problem deals with the transportation of:

(a) Single product from a source to several destinations

(b) Several products from a source to a destination.

(c) Single product from several sources to a destination.

(d) Single product from several sources to several destinations.

Answer: D

9. In NWCM, first allocation is made at:

(a) Upper left hand corner of the table.

(b) Lower right hand corner of the table.

(c) Upper right hand corner of the table.

(d) Lower left hand corner of the table.

Answer: A

10. One of the disadvantages of North West Corner rule for finding Initial Feasible Solution of Transportation problem is:

(a) It is complicated to use

(b) It leads to non-optimal solution

(c) It does not take into account unit cost of transportation.

(d) Generally it provides degenerate solution.

Answer: C

11. When total demand and supply are equal then the Transportation problem is said to be:

(a) A problem having multiple optimum solutions.

(b) A problem having degeneracy.

(c) A balanced one.

(d) None of the above.

Answer: C

12. Which one of the following is correct?

(a) The dummy source or destination is used in a Transportation problem to resolve degeneracy.

(b) The dummy source or destination is used in a Transportation problem to make it balanced.

(c) The dummy source or destination is used in a Transportation problem to ensure its cost effectiveness.

(d) All the above statements are correct.

Answer: B

13. For solving a maximization problem by Transportation algorithm, the very first step is to:

(a) Subtract smallest cost element of the matrix from all the other cost elements.

(b) Subtract all the cost elements of the matrix from the highest element of the same.

(c) Add smallest cost element of the matrix to all the other cost elements.

(d) Add highest cost element of the matrix to all the other elements.

Answer: B

14. Which of the following methods is used for finding an initial feasible solution of a Transportation Problem?

(a) Simplex

(b) Least Cost

(c) Hungarian

(d) Big M

Answer: B

15. Which of the following is a method for improving an initial solution of a Transportation problem?

(a) Stepping Stone

(b) North West Corner

(c) Intuitive Lowest Cost

(d) All of the above

Answer: A

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
C	B	D	D	C	A	D	D	A	C	C	B	B	B	A

State True or False

SL	STATEMENT	ANSWER
1	The first step of solving Transportation problem is to find Basic Feasible Solution.	**True**
2	Transportation algorithm is basically meant for minimization of Objective Function. So maximization problems cannot be solved using this algorithm.	**False**
3	While finding an improved solution in course of getting the optimal Transportation solution, a closed loop is drawn through even number of cells of the matrix, of which one is unallocated and the rest are allocated.	**True**
4	Basic feasible solution of a Transportation problem by VAM always gives optimal or near optimal solution.	**True**
5	LCM is the most popular method of carrying out Optimality test of Initial Solution of a Transportation problem.	**False**
6	Transportation technique was first developed while studying the problem of distribution of a product from several sources to numerous locations.	**True**
7	Using Graphical Method of Linear Programming to solve a problem of Transportation is very common.	**False**
8	Situation of Transportation under prohibition on usage of particular route occurs due to natural calamity.	**True**
9	Improved solution of a Transportation problem is obtained by creating a closed loop using at least four allocated cells of the existing solution.	**False**
10	For a (5 × 4) Transportation Matrix the Basic Feasible Solution is seen to have 6 allocations. The solution can be called non-degenerate.	**False**
11	Problems of Transportation with Degeneracy generally have multiple optimum solutions.	**True**
12	Basic feasible solution of a Transportation problem should have at least one allocation in each row and column of the matrix.	**True**
13	In NWCM if the resource availability of the 1st row is exhausted first then we move down the 2nd row and 1st column to make an allocation which either exhausts the resource availability of 2nd row or satisfies the remaining demand of the destination represented in 1st column.	**True**
14	Full form of LCM, as used in finding Basic Feasible Solution of a Transportation problem is Least Common Multiple.	**False**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	In a problem of transportation with prohibited routes, the cells having restriction are considered to have unit cost of transportation equal to _____.	**Infinity**
2	Any transportation problem can be solved as a special case of LPP using _____ method.	**Simplex**
3	Degeneracy can occur at ___ different stages while solving a Transportation Problem.	**Two**
4	By converting a given Profit Matrix to____ Matrix a problem of maximization can be solved by using Transportation algorithm.	**Relative or Opportunity Loss**
5	To resolve Degeneracy during Initial Feasible Solution of a Transportation problem an infinitely small quantity ε is allocated to the least cost _____ unoccupied cell of the solution.	**Independent**
6	From the point of view of optimality, the Transportation solution given in Q. No. 30 above is _____.	**Optimal**
7	From the point of view of degeneracy, the Transportation solution given in Q. No. 30 above is _____.	**Degenerate**
8	The objective of using Transportation algorithm is to _____ the Total Cost of Transportation.	**Minimize**
9	In a Transportation Table an ordered set of ___ or more cells is said to form a closed loop.	**Four**
10	Transportation problems are also called _____ problem after the name of its inventor F. L. Hitchcock.	**Hitchcock**
11	Chance of getting optimal solution is ___ when Basic Feasible Solution is obtained by NWCM than by VAM.	**Less**
12	MODI is a ____ time consuming method to test optimality of a solution than Stepping Stone Method.	**Less**
13	The column number v1 for the solution in Q. No. 16 above is _____.	**8**
14	To calculate Row and Column numbers (u_i & v_j) for a Transportation Table only the ____ cells are used.	**Allocated**

SL	STATEMENT	ANSWER
15	Equality of Supply and Demand is a constraint of Transportation algorithm and is called ____ condition.	**Rim**

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

"When formulated as LPP, constraints of a Transportation problem are always in the form of equations" – Explain the sentence written.

2

Why it takes more time to arrive at the optimal solution if the Initial Feasible Solution is obtained by North West Corner rule?

3

Define the term "Penalty" as applied to Vogel's Approximation Method.

4

Why alternately plus and minus signs are provided (with a plus sign at the starting point), to the corner points of a loop?

5

How the loop used for getting an alternative solution is different from the loop for improving an existing solution of a Transportation problem?

Essay Type Questions

1

Discuss on the technique of solving a maximization problem using Transportation algorithm.

2

Explain with an example how a Transportation problem can be formulated as LPP.

3

Write short notes on – (i) Least Cost Method of finding Initial Feasible Solution of a Transportation problem (ii) Use of Opportunity Cost while testing optimality of a Solution of Transportation problem (iii) Method of solving Transportation problem with Preferred Routes.

4

What are different stages of solution where Degeneracy can occur in a Transportation problem? Explain in brief how such situations can be resolved.

5

What is meant by Unbalanced problem of Transportation and how such problems are solved?

Comprehensive Numerical Problems

PRACTICE PROBLEMS WITH ANSWERS

PROBLEM 1 — ABC ENTERPRISES (PRODUCTION & DISTRIBUTION SCHEDULE)

ABC Enterprises has three Plants for manufacturing Dry Cells, located at three different states of the country. Production cost differs from Plant to Plant. There are four dedicated Sales Offices of the Company located at the four metro cities in the four regions of the country and a Head Office, in the capital of the country, which also takes care of sales. The Sales prices are different from one region to the other and also in the capital city.

PLANT NUMBER	MAXIMUM PRODUCTION CAPACITY	PRODUCTION COST (₹)
1	150 Units	20
2	200 Units	22
3	125 Units	18

FROM PLANTS	NORTHERN REGION	SOUTHERN REGION	WESTERN REGION	EASTERN REGION	CAPITAL CITY
1	1	1	5	9	4
2	9	7	8	3	6
3	4	5	3	2	7

SALES OFFICE	NORTHERN REGION	SOUTHERN REGION	WESTERN REGION	EASTERN REGION	CAPITAL CITY
Demand (Units)	80	100	75	45	125
Selling Price (₹)	30	32	31	34	29

Required: Find the Production and Distribution schedule which maximises the Profit of the company.

ANSWER

FROM PLANT	TO SALES OFFICE	NUMBER OF UNITS ALLOCATED
1	Northern Region	50
1	Southern Region	100
2	Eastern Region	25
2	Capital City	125
3	Northern Region	30
3	Western Region	75
3	Eastern Region	20

PROBLEM 2 — INVESTMENT STRATEGY

A company wishes to determine an Investment Strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years and maximum investment levels have been established for each investment type. An assumption is that amounts invested in a year will remain invested until the end of the planning horizon of four years. The following table summarises the data for this problem.

INVESTMENT MADE AT THE BEGINNING OF THE YEAR	A	B	C	D	E	AMOUNT AVAILABLE FOR INVESTMENT (₹'000)
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Maximum possible investment (₹'000)	750	600	500	800	1000	

Required: Determine the amount to be invested at the beginning of each year in different types of investment so as to maximize the net rupee return for the period of four years. Also find the maximum value of net return on investment.

ANSWER

YEAR OF INVESTMENT	INVESTMENT TYPE	AMOUNT TO BE INVESTED (₹'000)
1	E	500
2	B	600
3	D	750
4	A	250
4	C	500
4	D	50

Maximum net return on investment = ₹14,45,000

PROBLEM 3 — SCHEDULING WEEKLY PRODUCTION

Consider the problem of scheduling the weekly production of certain items for the next four weeks. The cost of production of the item is ₹10 for the first 2 weeks and ₹15 for the last 2 weeks. The weekly demands are 300, 700, 900 and 800 units, which must be met. The plant can produce maximum 700 units per week. In addition the company can employ overtime during the 2nd and 3rd week. This increases the weekly production by additional 200 units, but the production cost increases by ₹5. Excess production can be stored at a unit cost of ₹3 per week.

Required: How should the production be scheduled so as to minimize the total cost? How much production is to be carried out by overtime?

ANSWER

Optimal Schedule:

PRODUCED IN WEEK	USED IN WEEK	NUMBER OF UNITS
1	1	300
1	3	200
1	4	100
1	Dummy	100
2 (Normal time)	2	700
2 (Overtime)	Dummy	200
3 (Normal time)	3	700
3 (Overtime)	Dummy	200
4	4	700

Minimum Total Cost = ₹36,100.

From the table above it is clear that overtime productions have been allocated only to the Dummy week. Hence no production in overtime is necessary. In fact there will be spare capacity of 100 units in the 1st week because this quantity of the production of 1st week is allocated to Dummy week.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 7

ABBREVIATION	FULL FORM
NWCM	North West Corner Method
LCM	Least Cost Method
VAM	Vogel's Approximation Method
MODI	Modified Distribution Method
LP	Linear Programming
LPP	Linear Programming Problem
x_{ij}	Quantity transported from Source i to Destination j
C_{ij}	Unit transportation cost from Source i to Destination j
u_i	Row Number (Dual Variable for Supply Constraint)
v_j	Column Number (Dual Variable for Demand Constraint)
Δ_{ij}	Opportunity Cost / Improvement Index
ε	Epsilon — Infinitely small quantity used to resolve Degeneracy
M or ∞	Very high cost assigned to prohibited routes
Rim Condition	Total Supply = Total Demand

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Assignment Problem

Module 8 · Hungarian Method · Special Cases · Travelling Salesman

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	15% (with LP, Transportation, Game Theory)

✓ Website Publication · ✓ PDF Conversion · ✓ A4 Printing · ✓ Mobile & Desktop Reading · ✓ Self Study · ✓ Grand Revision · ✓ Final & Last Day Revision · ✓ Personal Reference Book

08

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 8 INCLUDES

Introduction · **Mathematical statement of the problem** · **Hungarian Method** · **Special cases in Assignment Problem** — Maximisation Problem, Unbalanced Problem, Problem having Multiple Optimum Solution, Restricted Assignment Problem · **Problem of Travelling Salesman**.

Module Learning Objectives

After studying this module, the students will be able to:

1

Recognise & Formulate

Recognise a problem of Assignment. State an Assignment problem in LP form.

2

Solve Using Hungarian Method

Solve Assignment problem using Hungarian Method.

3

Handle Special Cases

Obtain solutions for the Special cases of Assignment like Maximisation Problem, Unbalanced Problem, Problem having Multiple Optimum Solution, Restricted Assignment Problem etc.

4

Travelling Salesman

Solve the problem related to Travelling Salesman using concepts of Assignment.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Assignment to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

15%	6	8	30
GROUP WEIGHTAGE	CORE CONCEPTS	ILLUSTRATIONS	MCQS
With LP, Transportation, Game Theory	Hungarian Method, Special Cases	Worked examples	With answers

SYLLABUS STRUCTURE NOTE

Module 8 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Assignment, along with Linear Programming, Transportation and Game Theory, forms a group carrying **15% weightage** in the examination. This module introduces a special case of Linear Programming where allocation of resources to activities is done on a one-to-one basis.

📖 Table of Contents — Module 8

1. Introduction to Assignment Problem
2. Mathematical Statement of the Problem
3. Hungarian Method — Step by Step
4. Special Cases in Assignment Problem
5. Problem of Travelling Salesman
6. Case Study — Sugarcane Harvesting
7. MCQs, True/False, Fill in the Blanks

8. Numerical Problems

8.1 Introduction to Assignment Problem

CONCEPT · SPECIAL CASE OF LP · SPECIAL CASE OF TRANSPORTATION · HISTORICAL BACKGROUND

DEFINITION

Assignment is a special case of Linear Programming where allocation of various resources (items) to various activities (receivers) is done on a one-to-one basis in such a way that the effectiveness is optimised.

This can also be considered as a special case of Transportation in which the **number of sources and destinations are equal**. Also the capacity of each source as well as requirement of each destination is taken as **1**. In fact Assignment is a **degenerate problem of Transportation**.

Thus, an Assignment problem can be solved by both Simplex Method of solution of LPP and technique used for solution of Transportation problems. However both the methods are very cumbersome for solving an Assignment problem. These problems are solved by a simpler and more efficient method developed by **Harold Kuhn in 1955** who named it as **Hungarian Method** to honour two Hungarian mathematicians **D. Konig and J. Egevary** whose earlier work was the basis of Kuhn's work.

Typical Situations

There are many situations where the assignment of people or machines etc. may be called for:

- **Assignment of workers to machines**
- **Clerks to various check-out counters**
- **Salesmen to different sales areas**
- **Jobs to machines**
- **Vehicles to routes**
- **Classes to teachers**

NATURE OF THE PROBLEM

The Assignment is a problem because people possess varying abilities for performing different jobs and therefore the costs of performing jobs by different people are different. Thus, in an assignment problem, the question is **how the assignments should be made in order that the total cost involved is minimized**.

⚡ Quick Revision — Section 8.1

- **Assignment:** Special case of LPP — allocation of resources to activities on one-to-one basis.
- **Special case of Transportation:** Number of sources = Number of destinations; each capacity/requirement = 1.
- **Degenerate problem of Transportation:** Because each source and destination has capacity/demand = 1.
- **Hungarian Method:** Developed by Harold Kuhn (1955) to honour D. Konig and J. Egevary.
- **Objective:** Minimize total cost or maximize total effectiveness.

8.2 Mathematical Statement of the Problem

COST MATRIX · DECISION VARIABLES · OBJECTIVE FUNCTION · CONSTRAINTS

Suppose there are **'n' persons** and **'n' jobs** and the assignment of the jobs to be done on one-to-one basis so that the total effectiveness (Total Cost or Time taken for completion of the jobs) is optimized (minimized). This problem can be stated in the form of (n × n) matrix, as shown below, for Cost or Effectiveness (C_{ij}), where C_{ij} is the Cost or Effectiveness associated with assigning i^{th} person to the jth job.

	JOB 1	JOB 2	JOB 3	...	JOB J	...	JOB N
Person 1	C_{11}	C_{12}	C_{13}	...	C_{1j}	...	C_{1n}
Person 2	C_{21}	C_{22}	C_{23}	...	C_{2j}	...	C_{2n}
Person 3	C_{31}	C_{32}	C_{33}	...	C_{3j}	...	C_{3n}
...	...	...	...	...	...	...	...
Person i	C_{i1}	C_{i2}	C_{i3}	...	C_{ij}	...	C_{in}
...	...	...	...	...	...	...	...
Person n	C_{n1}	C_{n2}	C_{n3}	...	C_{nj}	...	C_{nn}

MATHEMATICAL FORMULATION

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}\, x_{ij}$$

Subject to the constraints: (i) $\sum_{j=1}^{n} x_{ij} = 1$ for i = 1, 2, ..., n (each person should be assigned to one and only one job); (ii) $\sum_{i=1}^{n} x_{ij} = 1$ for j = 1, 2, ..., n (each job must be assigned to one and only one person); (iii) $x_{ij} = 1$ if i^{th} person is assigned to the jth job; $x_{ij} = 0$ if i^{th} person is not assigned to the jth job.

Illustration 1 — LP Model Formulation

PROBLEM

A computer centre has three expert programmers and needs to develop three application programs. The head of the centre estimates the time required (in minutes) by the respective experts to develop the application programs are as follows:

	PROGRAMME A	PROGRAMME B	PROGRAMME C
Programmer 1	120	100	80
Programmer 2	80	90	110
Programmer 3	110	140	120

Required: Formulate an LP Model.

SOLUTION

Let x_{ij} = assignment of the ith Programmer to the jth Program (i = A, B, C and j = 1, 2, 3).

Objective: Minimize $Z = 120x_{11} + 100x_{12} + 80x_{13} + 80x_{21} + 90x_{22} + 110x_{23} + 110x_{31} + 140x_{32} + 120x_{33}$

Constraints:

- $x_{11} + x_{21} + x_{31} = 1$ (Programme A)
- $x_{12} + x_{22} + x_{32} = 1$ (Programme B)
- $x_{13} + x_{23} + x_{33} = 1$ (Programme C)
- $x_{11} + x_{12} + x_{13} = 1$ (Programmer 1)
- $x_{21} + x_{22} + x_{23} = 1$ (Programmer 2)
- $x_{31} + x_{32} + x_{33} = 1$ (Programmer 3)
- $x_{ij} = 0$ or 1 for all i and j.

⚡ Quick Revision — Section 8.2

- **Decision Variables:** $x_{ij} = 1$ if i^{th} person is assigned j^{th} job; 0 otherwise.
- **Objective Function:** Minimize $Z = \Sigma \Sigma C_{ij} x_{ij}$.
- **Constraints:** Each person gets exactly one job; each job gets exactly one person.

8.3 Hungarian Method

SIX STEPS · OPTIMALITY TEST · ASSIGNMENTS · ILLUSTRATION

Six Steps of Hungarian Method (Minimization)

Step 1 — Row Operation

Locate the **smallest cost element in each row** of the given cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of this new table, called the **reduced cost table**.

Step 2 — Column Operation

In the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest value from every entry in the column. As a consequence of this action, there would be at least one zero in each column of the reduced table. Hence in each of the rows and columns of the second reduced cost table, there would be at least one zero.

Step 3 — Optimality Test

Draw the minimum no. of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. If the no. of lines drawn is **equal to the no. of rows/columns** (i.e., order) of the given Cost Matrix, the solution is optimal and proceed to Step 6. If the no. of lines drawn is **less than the order** of the given matrix then go to Step 4.

Step 4 — Improved Matrix

Select the smallest uncovered (by the lines) cost element. Subtract this element from all uncovered elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered.

Step 5 — Repeat

Repeat Step 3 and Step 4 until an optimal solution is obtained.

Step 6 — Assignments

Given the optimal solution, make the job assignments as indicated by the 'zero' elements. This is done as follows:

- **(a)** Locate a row which contains **only one zero element**. Assign the job corresponding to this element to its corresponding person. Cross out the zeros, if any, in the column corresponding to the element, which is indicative of the fact that the particular job and person are no more available.
- **(b)** Repeat (a) for each of such rows which contain only one zero. Similarly, perform the same operation in respect of each column containing only one 'zero' element, crossing out the zero(s), if any, in the row in which the element lies.
- **(c)** If there is no row or column with only a single 'zero' element left, then select a row/column arbitrarily and choose one of the jobs (or persons) and make the assignment. Thus, in such a case, **alternative solutions exist**.

Illustration 2 — Five Mechanics and Five Repair Jobs

PROBLEM

An equipment under breakdown has five repair jobs to make it operative again. The Maintenance Manager of the organisation has assigned five mechanics of his department to do the jobs. The estimated time (hours) for each of the mechanics to carry out the jobs are given in the following table.

	A	B	C	D	E
I	7	5	9	8	11
II	9	12	7	11	10
III	8	5	4	6	9
IV	7	3	6	9	5
V	4	6	7	5	11

Required: Assuming that each mechanic can be assigned to only one job, determine the minimum time assignment.

SOLUTION — STEP BY STEP

Table 1 — Reduced Matrix after Row Operation

	A	B	C	D	E
I	2	**0**	4	3	6
II	2	5	**0**	4	3
III	4	1	**0**	2	5
IV	4	**0**	3	6	2
V	**0**	2	3	1	7

Table 2 — Reduced Matrix after Column Operation

	A	B	C	D	E
I	2	**0**	4	2	4
II	2	5	**0**	3	1
III	4	1	**0**	1	3
IV	4	**0**	3	5	**0**
V	**0**	2	3	**0**	5

Here minimum number of horizontal and vertical straight lines required to cover all the zero elements of the matrix = 4 ≠ Order (5) of the matrix. Hence the solution is **non-optimal**.

Table 3 — Improved Matrix (Optimal)

	A	B	C	D	E
I	1	0	4	1	4
II	1	5	0	2	1
III	3	1	0	0	3
IV	3	0	3	4	0
V	0	3	4	0	6

Here minimum number of horizontal or vertical straight lines required to cover all the zero elements of the matrix = 5 = Order (5) of the matrix. Hence the solution is **optimal**.

Table 4 — Optimal Assignments

	A	B	C	D	E
I	1	0	4	1	4
II	1	5	0	2	1
III	4	1	~~0~~	0	3
IV	4	~~0~~	3	4	0
V	0	3	4	~~0~~	6

Optimal Solution:

REPAIR JOB	ASSIGNED TO MECHANIC	TIME REQUIRED (HOURS)
A	V	4
B	I	5
C	II	7
D	III	6
E	IV	5
Total		**27**

Minimum time required to complete all the Repair Jobs = 27 hours

⚡ Quick Revision — Section 8.3

- **Step 1 (Row Operation):** Subtract smallest element in each row from all elements in that row.
- **Step 2 (Column Operation):** Subtract smallest element in each column from all elements in that column.
- **Step 3 (Optimality):** Draw minimum straight lines to cover all zeros. If lines = order of matrix → optimal.
- **Step 4 (Improved Matrix):** Subtract smallest uncovered element from all uncovered; add at intersections.
- **Step 6 (Assignments):** Start with rows having single zero; cross out other zeros in same row/column.
- **Illustration:** 5 mechanics, 5 jobs; minimum time = 27 hours.

8.4 Special Cases in Assignment Problem

MAXIMISATION · UNBALANCED · MULTIPLE OPTIMUM · RESTRICTED ASSIGNMENT

1. Maximisation Problem

METHODOLOGY

1. Convert the supplied Profit Matrix to a **Relative Loss or Regret Matrix** by subtracting the entry of each cell from the highest one among all the entries of the Matrix.
2. Now use **Hungarian Method** to minimise the Relative Loss.
3. Once assignments are available, find out maximum value of the total profit by considering the supplied values of Profit against the assigned cells.

Illustration 3 — Five Salesmen and Five Districts

PROBLEM

Five Salesmen are to be assigned to five Districts. Estimates of Sales Revenue (in ₹'000) for each Salesman are given in the table below.

DISTRICTS	SALESMAN A	SALESMAN B	SALESMAN C	SALESMAN D	SALESMAN E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Required: Find the assignment pattern that maximises Revenue.

SOLUTION

This is a problem of Maximisation. Convert the given matrix to a Relative Loss matrix by subtracting all the elements from the highest element 41.

Table 1 — Relative Loss Matrix

DISTRICTS	A	B	C	D	E
1	9	3	1	13	1
2	1	17	13	20	5
3	0	14	8	11	4
4	19	3	0	5	5
5	12	8	1	6	2

Table 2 — Reduced Matrix after Row Operation

DISTRICTS	A	B	C	D	E
1	8	2	**0**	12	**0**
2	**0**	16	12	19	4
3	**0**	14	8	11	4
4	19	3	**0**	5	5
5	11	7	**0**	5	1

Table 3 — Reduced Matrix after Column Operation

DISTRICTS	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

Minimum number of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. So the solution is **non-optimal**.

Table 4 — Improved Matrix (Non-optimal)

DISTRICTS	A	B	C	D	E
1	9	0	1	8	0
2	0	13	12	14	3
3	0	11	8	6	3
4	19	0	0	0	4
5	11	4	0	0	0

Here also minimum number of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. So the solution is **non-optimal**.

Table 5 — Improved Matrix (Optimal)

DISTRICTS	A	B	C	D	E
1	12	0	1	8	0
2	0	10	9	11	0
3	0	8	5	3	0
4	22	0	0	0	4
5	14	4	0	0	0

In this case minimum number of lines to cover the zeros = 5 = Order of the matrix. Hence the solution is **optimal**.

Table 6 — Optimal Assignments (Alternative 1)

DISTRICTS	A	B	C	D	E
1	12	0	1	8	~~0~~
2	0	10	9	11	~~0~~
3	~~0~~	8	5	3	0
4	22	~~0~~	0	~~0~~	4
5	14	4	~~0~~	0	~~0~~

SALESMAN	DISTRICT ASSIGNED	REVENUE (₹'000)
A	2	40
B	1	38
C	4	41
D	5	35
E	3	37
Total		**191**

Maximum Total Sales Revenue = ₹1,91,000 (Presence of not a single row or column with single zero is indicative of Multiple Optimum Solution for the problem.)

2. Unbalanced Assignment Problem

DEFINITION AND SOLUTION

An Assignment Problem is said to be **unbalanced** if the number of its rows and columns are unequal. To solve such problems additional row or column (as the case may be) has to be introduced with **all zero entries**. In fact the introduction of additional row or column is needed to make the matrix square which is the prime requirement of an Assignment Problem. This newly introduced row or column is called **Dummy Row or Dummy Column**.

Illustration 4 — Road Repairs (Unbalanced)

PROBLEM

A city corporation has decided to carry out road repairs on four main arteries of the city. The government has agreed to make a special grant of ₹50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and quickest time. Five contractors participated in bidding. One road will be awarded to one contractor. The following matrix of Cost of Repairs is prepared by the corporation on the basis of the bids submitted by the participants.

CONTRACTORS	ROAD 1	ROAD 2	ROAD 3	ROAD 4
A	9	14	19	15
B	7	17	20	19
C	9	18	21	18
D	10	12	18	19
E	10	15	21	16

Required: (i) Find the best way of assigning the repair work and total cost. (ii) If supplementary grant needed, what amount? (iii) Which contractor will be unsuccessful?

SOLUTION

This is an unbalanced problem (5 rows ≠ 4 columns). Introduce a **Dummy Column** with all zero elements.

Table 1 — Balanced Cost Matrix

CONTRACTORS	1	2	3	4	DUMMY
A	9	14	19	15	0
B	7	17	20	19	0
C	9	18	21	18	0
D	10	12	18	19	0
E	10	15	21	16	0

Table 5 — Optimal Assignments

CONTRACTORS	1	2	3	4	DUMMY
A	1	1	**0**	~~0~~	1
B	**0**	5	2	5	2
C	~~0~~	4	1	2	**0**
D	3	**0**	~~0~~	5	2
E	1	1	1	**0**	~~0~~

ROAD TO BE REPAIRED	ASSIGNED TO CONTRACTOR	COST OF REPAIR (₹ LAKHS)
1	B	7
2	D	12
3	A	19
4	E	16
Total		**₹54 Lakhs**

(ii) As the minimum total cost (₹54 Lakhs) exceeds sanctioned amount (₹50 Lakhs), a **supplementary grant of ₹4 Lakhs** is to be sought.

(iii) From the optimal table, the Dummy Road is assigned to the Contractor C. So **C will be unsuccessful** in his bid because there is no existence of the Dummy road.

3. Assignment Problem with Multiple Optimum Solution

CONCEPT

Such solutions exist if while assigning it is seen that **neither any row nor any column is having single zero**. In such case one has to firstly check the rows and then columns to find one with two zeros. One of these should be chosen arbitrarily and a square to be drawn around it to mark an assignment at that cell. The other zero in the same row or column should be crossed out. Alternatively the cell whose zero is bounded by drawing a square around it should be crossed out and the other one is assigned. This will lead to two Alternative Solutions.

Illustration 5 — Alternative Solutions

PROBLEM

Use the data of Illustration 3 above to find all the possible optimum solutions.

SOLUTION

The optimal Table 6 of Illustration 3 has multiple solutions. The alternative solutions are obtained by choosing different zeros for assignment.

Calculation of Maximum Total Sales Revenue (₹'000) for various Alternatives:

ALTERNATIVE 1	ALTERNATIVE 2	ALTERNATIVE 3	ALTERNATIVE 4
Salesman A → District 2 (₹40)	Salesman A → District 3 (₹41)	Salesman A → District 3 (₹41)	Salesman A → District 2 (₹40)
Salesman B → District 1 (₹38)	Salesman B → District 1 (₹38)	Salesman B → District 1 (₹38)	Salesman B → District 1 (₹38)
Salesman C → District 4 (₹41)	Salesman C → District 4 (₹41)	Salesman C → District 5 (₹40)	Salesman C → District 5 (₹40)
Salesman D → District 5 (₹35)	Salesman D → District 5 (₹35)	Salesman D → District 4 (₹36)	Salesman D → District 4 (₹36)
Salesman E → District 3 (₹37)	Salesman E → District 2 (₹36)	Salesman E → District 2 (₹36)	Salesman E → District 3 (₹37)
Total = 191	**Total = 191**	**Total = 191**	**Total = 191**

From above it is seen that the maximum total Sales Revenue in each case is **₹1,91,000**. Thus, each solution gives same optimum result. In other words, the problem has **multiple optimum solutions**.

4. Problems with Restriction on Assignments / Prohibited Assignment

CONCEPT

Sometimes in an Assignment Problem there may be a case when a particular resource (say, a person) cannot be assigned to a particular activity (say, a job). To handle such a problem a **very high cost (or time which is to be minimised) is assigned to that cell** of the matrix which is meant for this restricted or prohibited assignment. This automatically restricts any assignment at that cell. Very high cost is generally represented by **∞ or M**.

Illustration 6 — Restricted Assignment (Rooms to Managers)

PROBLEM

A company has taken on rent three floors (1st, 2nd and 3rd) of a multi-storied building for their city office. It has been decided to locate Managers of Marketing, Purchase, HR, Finance and Company Secretary in the office. The management has earmarked in different floors five rooms having numbers 103, 201, 205, 302 and 304 for the above mentioned Managers. But no particular room has been allotted for any particular Manager and rather they have given option to indicate their preference of rooms so that decision can be taken by the management using some scientific method. Managerial preferences are provided in the table below with 1st preference appearing in the top for each and every Manager.

MANAGER	ROOM 103	ROOM 201	ROOM 205	ROOM 302	ROOM 304
Marketing Manager	2	M	M	1	3
Purchase Manager	M	4	3	1	2
HR Manager	1	2	4	5	3
Finance Manager	4	M	2	1	3
Company Secretary	M	1	M	2	3

Required: Find out which manager should be assigned with which room to minimise the preferential measure.

SOLUTION

Preferences are quantified as 1, 2, 3, 4 and 5 with M for no preference (restricted assignment).

Table 4 — Optimal Assignments

MANAGERS	ROOM 103	ROOM 201	ROOM 205	ROOM 302	ROOM 304
Marketing Manager	1	M	M	**0**	1
Purchase Manager	M	3	1	~~0~~	**0**
HR Manager	**0**	1	2	4	1
Finance Manager	3	M	**0**	~~0~~	1
Company Secretary	M	**0**	M	1	1

Required Assignment: Room 103 to HR Manager; Room 201 to Company Secretary; Room 205 to Finance Manager; Room 302 to Marketing Manager; Room 304 to Purchase Manager.

⚡ Quick Revision — Section 8.4

- **Maximisation:** Convert to Relative Loss Matrix by subtracting all elements from highest element; then apply Hungarian Method.
- **Unbalanced:** Add Dummy row/column with zero entries to make matrix square.
- **Multiple Optimum:** When no row/column has single zero; alternative solutions possible.
- **Restricted Assignment:** Assign very high cost (M or ∞) to prohibited cells.

8.5 Problem of Travelling Salesman

CONCEPT · SPECIAL CASE OF PROHIBITED ASSIGNMENT · ILLUSTRATION

DEFINITION

This can be considered as a **special case of Prohibited Assignment**. Here a Salesman has to start his sales tour from a particular city of his territory and visit all the other cities within the territory in such a manner that he completes his tour at the same city from where he started. When the distance or time required to travel or cost of travel between the cities is known then the objective of the salesman is to schedule the tour in such a way that the **total distance travelled or total time elapsed for the travel or total cost of travel is minimised**. This type of problems can be solved by the algorithm used for Assignment.

Illustration 7 — Five Cities Tour

PROBLEM

A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in ₹'00) between any two cities is given in the table below.

TO CITY / FROM CITY	A	B	C	D	E
A	M	5	8	4	5
B	5	M	7	4	5
C	8	7	M	8	6
D	4	4	8	M	8
E	5	5	6	8	M

Required: Find the cost minimising sequence of visit.

SOLUTION

Table 1 — Reduced Matrix after Row Subtraction

	A	B	C	D	E
A	M	1	4	0	1
B	1	M	3	0	1
C	2	1	M	2	0
D	0	0	4	M	4
E	0	0	1	3	M

Table 2 — Reduced Matrix after Column Subtraction

	A	B	C	D	E
A	M	1	3	0	1
B	1	M	2	0	1
C	2	1	M	2	0
D	0	0	3	M	4
E	0	0	0	3	M

Minimum number of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. So the solution is **non-optimal**.

Table 3 — Improved Matrix (Optimal)

	A	B	C	D	E
A	M	0	2	0	1
B	0	M	1	0	1
C	1	0	M	2	0
D	0	0	3	M	5
E	0	0	0	4	M

Here the minimum number of straight lines required to cover all the zeros = 5 = Order of the matrix. So the solution is **optimal**.

Table 4 — Optimal Assignments (Alternative 1)

	A	B	C	D	E
A	M	0	2	~~0~~	1
B	~~0~~	M	1	0	1
C	1	~~0~~	M	2	0
D	0	~~0~~	3	M	5
E	~~0~~	~~0~~	0	4	M

The Salesman will travel from A to B, then B to D, then D to A. But this is not meeting the requirement of travelling through all the cities and finally returning to the starting point, i.e., A. Hence the solution is **unacceptable**.

Table 5 — Optimal Assignments (Alternative 2)

	A	B	C	D	E
A	M	~~0~~	2	**0**	1
B	**0**	M	1	~~0~~	1
C	1	**0**	M	2	~~0~~
D	~~0~~	~~0~~	3	M	5
E	~~0~~	~~0~~	**0**	4	M

The travelling route shows A to D, then D to B, then B to A. This is also not acceptable because of violation of the basic requirement of the problem.

Under the circumstances it is decided to try for the assignment at the cells which are having **next highest entry after zero**. Here next highest entry in the table after 0 is 1 and it appears at the four cells — (A, E), (B, C), (B, E) and (C, A). By arbitrarily choosing the cell (A, E) for the purpose.

Table 5 — Matrix with the Required Solution

	A	B	C	D	E
A	M	~~0~~	2	~~0~~	**1**
B	~~0~~	M	1	**0**	1
C	1	**0**	M	2	~~0~~
D	**0**	~~0~~	3	M	5
E	~~0~~	~~0~~	**0**	4	M

Required Solution: The travel route of the salesman is **A to E, E to C, C to B, B to D and D to A**.

Minimum Cost of travel = 5 + 4 + 7 + 4 + 6 = 26 ('00 ₹)

⚡ Quick Revision — Section 8.5

- **Travelling Salesman:** Special case of Prohibited Assignment.
- **Requirement:** Start from a city, visit each city once, return to starting point.
- **Diagonal:** M or ∞ (cannot travel from a city to itself).
- **Solution:** If the first optimal solution gives a sub-tour (not covering all cities), choose next highest entry after zero at appropriate cell.
- **Illustration:** Route A → E → C → B → D → A; Cost = 26 ('00 ₹).

8.6 Case Study — Sugarcane Harvesting (Generalized Assignment Problem)

THREE ACTORS · DIFFERENTIAL EVOLUTION ALGORITHM · INITIAL SOLUTION

CASE STUDY

Sugarcane Harvesting — Specialised Version of GAP

Background: Sugarcane is a globally important commercial crop that can be used to produce both direct and indirect products such as Sugar, Ethanol, Fodder etc. Traditionally hand harvesting is effective, but currently mechanical harvesters play an important role due to shortage of labour and increased labour costs. A harvester is very powerful since it can replace 200 labourers.

Problem: The price of the harvester is still high. The number of harvesters in use is limited but there are many farmers who want to use it. Therefore the scheduling of harvesters is needed to achieve the aim of the farmers or the owner of the harvester.

Complexity: One harvester can be assigned to more than one field if it has enough working time available even after serving one field. Different harvesters have to work for different hours in a field because of the variation in their harvesting capacities. Each harvester has limitation on its available working time. Therefore assigning harvesters to different fields falls under the category of **Generalized Assignment Problem (GAP)**.

Three Actors: The problem is composed of three actors — **the driver, the harvester and the sugarcane field**. A driver is to be assigned to a harvester set which is to be paired to a suitable field for harvesting so that the daily profit is maximized.

Profit: Depends on income and cost generated in the system. Income is generated from the amount of harvested sugarcane sold multiplied by the sweetness level of the harvested sugarcane. The profit is the result of total income subtracted by the cost of the harvester's fuel consumption and the corresponding driver's daily wages.

Solution Approach: A **Differential Evolution (DE) algorithm** is proposed to be used for arriving at a solution to the problem. The steps involved in the proposed DE are — (1) Generate initial solution, (2) Implement the mutation process, (3) Execute the recombination process & (4) Complete the selection process.

Data Provided: 6 Fields of sugarcane, 4 Harvesters and 5 Drivers as well as 5 sets of target vectors. The initial set of randomly generated numbers for finding an Initial Solution are given. The symbols used: A_K = Area of the sugarcane field K in Acres; B_K = Sweetness factor; E_J = Fuel consumption rate of harvester J in ₹/hour; U_J = Age factor of harvester J; T_j = Harvesting rate of harvester in Acres/hour; R_i = Experience factor of driver i (which affects fuel consumption rate); P_i = Experience level of the driver i (which affects harvesting rate).

Additional Information: Fuel cost is ₹115 per hour; Selling price of the sugarcane is ₹600 per ton; 12 tons of standard sugarcane is obtained from 1 Acre of land; All drivers are allowed to work for 9 hours a day.

Methodology: (1) Sort all the subsets of the vectors (i.e., Field, Harvester & Driver) in ascending order. (2) Assign a driver to a harvester and subsequently assign the harvester to the sugarcane field/s using the order obtained in step (1) above. Rules: (a) Assign the entity that is in position at the front of the order first; (b) If the current position violates the capacity constraint of the harvester (Maximum 9 hours of usage per day) the entity that is in the next position is allowed to be replaced.

Conclusion: From the solution, Field No. 2 could not be assigned due to non-availability of Harvester. Hence there is **insufficient resource to harvest all the six fields** as far as the initial solution is concerned.

⚡ Quick Revision — Section 8.6

- **Problem Type:** Specialised version of Generalized Assignment Problem (GAP).
- **Three Actors:** Driver, Harvester, Sugarcane Field.
- **Objective:** Maximize daily profit.
- **Solution Method:** Differential Evolution (DE) algorithm — Initial solution, Mutation, Recombination, Selection.
- **Result:** Initial solution cannot harvest all six fields — insufficient resources.

8.7 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. Which of the following methods is used to solve the Assignment problems?

(a) Stepping Stone Method (b) Hungarian Method (c) North West Corner Method

(d) Vogel's Approximation Method

Answer: B

2. Assignment of work to men and machines is known as:

(a) Scheduling (b) Loading (c) Balancing of Line

(d) None of these

Answer: B

3. In an Assignment matrix of size (5 × 5), the total number of decision variables in the objective function is:

(a) 10 (b) 5 (c) 25

(d) 15

Answer: C

4. An Assignment problem is solved to minimise the total time required to complete three jobs on three different machines. The minimum total processing time is found to be 480 minutes. Due to a design change, the processing time of the second job is increased by 15 minutes in either of the machines. The revised minimum total processing time will be:

(a) 495 minutes (b) 465 minutes (c) 480 minutes

(d) None of these

Answer: A

5. Assignment problem can be considered as a particular case of:

(a) Transportation problem (b) Sequencing problem (c) Queuing problem

(d) All of these

Answer: A

6. Dummy row or column is added in an assignment problem:

(a) To prevent a solution to become degenerate. (b) To reduce the total cost of assignment. (c) To increase the profit function.

(d) To balance total activities and total resources

Answer: D

7. While solving an assignment problem, an activity is assigned to a resource with zero opportunity cost because objective is to:

(a) Reduce total cost of assignment to zero. (b) Reduce cost of that assignment to zero. (c) Minimise total cost of assignment.

(d) Maximise total cost of assignment.

Answer: C

8. In an assignment problem:

(a) First activity is assigned to first resource (b) Any number of activities can be assigned to each resource. (c) It depends on how many resources are available.

(d) Only one activity be assigned to each resource.

Answer: D

9. An assignment problem can be viewed as a special case of transportation problem in which the capacity from each source is _____ and the demand at each destination is _____.

(a) Unlimited, unlimited (b) One, unlimited (c) One, one

(d) Unlimited, one

Answer: C

10. In marking assignments which of the following should be preferred?

(a) Only row having single zero

(b) Only column having single zero

(c) Column having more than one zero

(d) Only row/column having single zero.

Answer: D

11. The assignment matrix is always a _____:

(a) Rectangular matrix

(b) Identity matrix

(c) Square matrix

(d) None of these

Answer: C

12. Maximisation assignment problem is transformed into a minimisation problem by _____:

(a) Adding each entry of a column to the maximum value of that column

(b) Subtracting each entry in a column from maximum value in that column.

(c) Subtracting each entry of the table from the maximum value of the table.

(d) Adding each entry of the table to the maximum value in the table.

Answer: C

13. The assignment problem will have alternative solutions when it has:

(a) At least one zero in any row or column

(b) All rows have two zeros.

(c) Two diagonal elements are zeros

(d) None of the above.

Answer: D

14. In the Hungarian Method of solving Assignment problem, the row reduction is obtained by:

(a) Dividing each row by the elements of the row above it.

(b) Subtracting the elements of the row from the elements of the row above it.

(c) Subtracting the smallest element from all other elements of the row.

(d) Subtracting all the elements of the row from the highest element in the matrix.

Answer: C

15. The horizontal and vertical lines drawn to cover all zeros of the total opportunity matrix for an optimal solution must be:

(a) Equal to m x n, where m = No. of rows & n = No. of columns.

(b) Equal to each other.

(c) Equal to m + n, where m = No. of rows & n = No. of columns

(d) Equal to the Order of the matrix.

Answer: D

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
B	B	C	A	A	D	C	D	C	D	C	C	D	C	D

State True or False

SL	STATEMENT	ANSWER
1	In the optimal solution of Travelling Salesman problem all the assignments might not occur at the cells having zero entries.	**True**
2	As Assignment is a Linear Programming Problem, it can be solved using Graphical Method.	**False**
3	Square type cost matrix is the prime requirement of an Assignment Problem. Thus, an Assignment Problem with unequal number of rows and columns is not possible.	**False**
4	It is true that an Assignment problem can be solved using Transportation algorithm, but the reverse is not true.	**True**
5	To test optimality of an Assignment solution we have to cover all the zero entries of the matrix by only horizontal or only vertical straight lines.	**False**
6	While formulating an Assignment problem, described in a matrix of order 3, as a Linear Programming problem the total number of decision variables should be nine.	**True**
7	Hungarian Method is applicable for solving only the minimisation problems.	**True**
8	For problems with preferred assignment, we start with zero entries at the cells having preference.	**False**
9	If a dummy column is introduced to make an assignment problem balanced, then after row subtraction the matrix remains same as before.	**True**
10	Assignment problems are more frequently solved by Simplex Method.	**False**
11	An improvement of a non-optimal assignment solution is done by subtracting all the uncovered elements from the highest uncovered element and adding the same at the junction elements.	**False**
12	After getting an optimal solution, the first step to start assignment is to find out a row with a single zero entry.	**True**
13	While making an assignment at a cell with zero entry, the other cells with zero entry in the corresponding row or column should be crossed out because that is what we are supposed to do.	**False**
14	Assignment matrix of order 'n' yields only 'n' allocations where as a non-degenerate solution of Transportation demands (2n – 1) cell allocations. Thus, solution of Assignment problem by Transportation method is not possible.	**False**

SL	STATEMENT	ANSWER
15	In an optimal solution, while assigning cells arbitrary choice of zero entry cell is necessary when not a single row or column is available with only one zero.	**True**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	When a particular assignment in the given problem is not possible or restricted as a condition, it is called a _____ problem.	**Prohibited**
2	Assignment problem is a special case of Linear Programming and can be solved by _____ method.	**Simplex**
3	The method of solving Assignment problem is called Hungarian to honour two Hungarian mathematicians based on whose earlier work _____ developed the same in 1955.	**Kuhn**
4	If the number of rows and columns are not equal in an Assignment matrix then it is called _____ problem.	**Unbalanced**
5	The first step of solution of Assignment problem by Hungarian method is _____ subtraction operation.	**Row**
6	The _____ test of Hungarian Method refers to checking whether the minimum number of horizontal and vertical straight lines to cover all the zero entries of the matrix is equal to the order of the matrix or not.	**Optimality**
7	For a non-optimal solution of Assignment problem, the improvement of solution matrix is done by subtracting the _____ of the uncovered elements from all the uncovered elements and adding the same to the elements at the junctions of the horizontal and the vertical lines.	**Minimum**
8	Assignment problem is a degenerate type _____ problem.	**Transportation**
9	For an unbalanced Assignment problem, introduction of a ____ row or column is necessary.	**Dummy**
10	In case of Assignment problem with multiple optimum solutions, the minimum value for all the solutions is _____.	**Same**
11	All the entries in a dummy row or column are considered to be _____.	**Zero**
12	The table shows _____ solution of an Assignment problem.	**Optimal**
13	For the given Assignment Matrix, the number of zeros after row subtraction operation will be ____.	**3**
14	In the problem of Travelling Salesman, the basic requirement is a ____ trip of the person.	**Round**

SL	STATEMENT	ANSWER
15	If the matrix is for a problem of maximisation then all the elements of it should be subtracted from the element ___ to get a regret matrix.	**140**

8.8 Comprehensive Numerical Problems

PRACTICE PROBLEMS WITH ANSWERS

PROBLEM 1 — METHODS TO WORK CENTRES

A Methods Engineer wants to assign four new methods to three work centres. The assignment of the new methods will increase production and they are given in the matrix below. If only one method can be assigned to a work centre, determine the optimum assignment.

METHODS	A	B	C
1	10	7	8
2	8	9	7
3	7	12	6
4	10	10	8

Is this a problem of multiple optimum solution? If yes, then find the alternative solution/s also.

ANSWER

Assignments: 1 to A, 2 to Dummy, 3 to B and 4 to C. **Optimum assignment = 30 units**

Alternative solution: 1 to C, 2 to Dummy, 3 to B and 4 to A

PROBLEM 2 — FOUR JUNIOR ASSISTANTS TO FOUR TASKS

Consider a problem of assigning four junior assistants to four tasks. The time (hours) required to complete the tasks are given in the table below.

CLERKS	A	B	C	D
1	4	7	5	6
2	–	8	7	4
3	3	–	5	3
4	6	6	4	2

Assistant 2 cannot be assigned to task A and Assistant 3 cannot be assigned to task B. Find the optimal assignment schedule.

ANSWER

Assignments: 1 to B, 2 to D, 3 to A and 4 to C. **Minimum total time required = 18 hours**

Alternative solution: 1 to C, 2 to D, 3 to A and 4 to B

PROBLEM 3 — FOUR ZONES AND FOUR SALESMEN

A company has four zones open and four salesmen available for assignment. The zones are not equally rich in their sales potentials. It is estimated that a typical salesman operating in each zone would bring in the following annual sales: Zone A: ₹1,26,000; Zone B: ₹1,05,000; Zone C: ₹84,000; Zone D: ₹63,000.

The four salesmen are also considered to differ in ability. It is estimated that working under the same condition their sales per year would be proportionately as follows: Salesman P: 7; Salesman Q: 5; Salesman R: 5; Salesman S: 4.

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest zone, the next best to the second richest zone and so on. Verify this by the method of assignment.

ANSWER

Assignments: Salesman P to Zone A; Salesman Q to Zone B; Salesman R to Zone C & Salesman S to Zone D

Maximum Sales = ₹2,97,000

PROBLEM 4 — ALLOCATION OF OPERATORS TO MACHINES

Average time taken by an operator on a specific machine is tabulated below. The management is considering to replace one of the old machines by a new one and the estimated time (Hours) for operation by each operator on the new machine is also indicated.

OPERATOR	M1	M2	M3	M4	M5	M6	NEW
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

Required: (a) Find out an allocation of operators to the old machines to achieve a minimum operation time. (b) Reset the problem with the new machine and find out the allocation of the operators to each machine and comment on whether it is advantageous to replace an old machine to achieve a reduction in operating time only. (c) How will the operators be reallocated to the machines after replacement?

ANSWER

(a) Old Machines: 01→M3 (2); 02→M1 (4); 03→M4 (4); 04→M5 (3); 05→M2 (3); 06→M6 (1). Minimum Total Operation Time = **17 hours**

(b) With New Machine: 01→M1 (2); 02→M4 (3); 03→New (1); 04→M5 (3); 05→M2 (3); 06→M6 (1). Minimum Total Operation Time = **13 hours**. In place of M3 New machine should be installed.

(c) Reallocated Operators: As per (b) above.

PROBLEM 5 — CREW LAYOVER TIME (AIRLINE)

An airline operates seven days a week and has the timetable shown below. Crews must have a minimum layover (rest) time of 5 hrs, between flights. Obtain the pair of flights that minimizes layover time away from home. For any given pair the crews will be based at the city that result in the smaller layover.

DELHI-JAIPUR FLIGHT NO.	DEPART	ARRIVE	JAIPUR-DELHI FLIGHT NO.	DEPART	ARRIVE
1	7.00 AM	8.00 AM	101	8.00 AM	9.15 AM
2	8.00 AM	9.00 AM	102	8.30 AM	9.45 AM
3	1.30 PM	2.30 PM	103	12.00 NOON	1.15 PM
4	6.30 PM	7.30 PM	104	5.30 PM	6.45 PM

For each pair, mention the town where the crews should be based. (Assume 15 minutes = 1 unit.)

ANSWER

Optimal Assignment: Flight 1 – 103; Flight 2 – 104; Flight 3 – 101 and Flight 4 – 102.

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Assignment Problem

A special case of Linear Programming where allocation of various resources (items) to various activities (receivers) is done on a one-to-one basis in such a way that the effectiveness is optimised.

Hungarian Method

A simpler and more efficient method for solving Assignment problems, developed by Harold Kuhn in 1955 to honour D. Konig and J. Egevary.

Reduced Cost Table

The table obtained after Row Operation and Column Operation in Hungarian Method, containing at least one zero in each row and column.

Optimality Test

Drawing minimum number of horizontal and vertical lines to cover all zeros; if lines = order of matrix, solution is optimal.

Dummy Row/Column

Additional row or column introduced with all zero entries to make an unbalanced Assignment matrix square.

Relative Loss Matrix

Matrix obtained by subtracting all elements of a Profit Matrix from its highest element, used to convert Maximisation problem into Minimisation problem.

Prohibited Assignment

A situation where a particular resource cannot be assigned to a particular activity; handled by assigning very high cost (M or ∞).

Travelling Salesman Problem

A special case of Prohibited Assignment where a salesman starts from a city, visits each city once and returns to the starting point, minimising total cost/time/distance.

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

Why is it necessary to cross out the zeros of the corresponding rows and columns of a solution matrix once an assignment is made to a particular cell?

2

Where from the name Hungarian is derived for the method of solution of Assignment problems?

3

Describe the beginning step of solution of an unbalanced problem of Assignment.

4

Can an Assignment problem ever be a non-degenerate transportation problem?

5

Give examples of two areas of application of Assignment problem.

Essay Type Questions

1

Give an example to show that an Assignment problem can be formulated as a Linear Programming Problem.

2

How will you handle the following situations in an Assignment problem? Write in brief (i) Maximisation of the Objective Function, (ii) Impossible Assignment

3

What is Assignment problem? Discuss its method of solution.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 8

ABBREVIATION	FULL FORM
AP	Assignment Problem
LP	Linear Programming
LPP	Linear Programming Problem
x_{ij}	Decision variable — 1 if i^{th} person assigned j^{th} job, else 0
C_{ij}	Cost or Effectiveness of assigning i^{th} person to j^{th} job
M or ∞	Very high cost assigned to prohibited assignments
GAP	Generalized Assignment Problem
DE	Differential Evolution (Algorithm)
RFID	Radio Frequency Identification Tag
SAVE	Society of American Value Engineers
VCA	Value Chain Analysis

8.6 Case Study — Sugarcane Harvesting

8.7 MCQs, True/False, Fill in the Blanks

8.8 Numerical Problems

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Game Theory

Module 9 · Pure Strategy · Mixed Strategy · Dominance · Graphical Method

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	15% (with LP, Transportation, Assignment)

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09

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 9 INCLUDES

Introduction · **Basic Terms** · **Assumptions in a Game** · **Solution of Pure Strategy Games with Saddle Point** · **Principle of Dominance** · **Solution of Mixed Strategy Games** — Odds Method, Dominance Method, Graphical Method · **Limitations of Game Theory**.

Module Learning Objectives

After studying this module, the students will be able to:

1

Understand Conflict Situations

Understand situations where decisions are to be made under conditions of conflict.

2

Selection & Execution of Strategies

Understand the different procedures used in the selection and execution of various strategies which result in winning the Game.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Game Theory to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

15%	3	7	30
GROUP WEIGHTAGE	**SOLUTION METHODS**	**CORE CONCEPTS**	**MCQS**
With LP, Transportation, Assignment	Odds, Dominance, Graphical	Pure, Mixed, Saddle, Dominance	With answers

SYLLABUS STRUCTURE NOTE

Module 9 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Game Theory, along with Linear Programming, Transportation and Assignment, forms a group carrying **15% weightage** in the examination. This module introduces mathematical models for analysing conflicting situations where two or more rational opponents are involved.

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4. Solution of Pure Strategy Games with Saddle Point
5. Principle of Dominance
6. Solution of Mixed Strategy Games
7. Limitations of Game Theory
8. MCQs, True/False, Fill in the Blanks
9. Numerical Problems

9.1 Introduction to Game Theory

CONCEPT · HISTORICAL DEVELOPMENT · CONFLICTING SITUATIONS · APPLICATIONS

In decision making often we come across situations where two or more opposing parties are seen to have conflicting interests. Action of one depends on the action taken by the opponent. Any Military Operation is an example of such a conflicting situation. Each participant of the operation takes all possible measures to prevent the opponent from succeeding. Situations in the field of Economics, particularly when there is free competition belong to the class of conflicts, too. Firms trying to maintain their market share, work in a conflicting or competitive environment and any move by one is suitably counter moved by the other. To increase the market share if one firm takes a strategy of reducing the selling price of its product by giving some discount then the other might take a strategy to redesign its product and increase its value at a price lower than the competitor.

To analyse such conflicting situations, some special mathematical model called **"Game Theory"** is used. It was first developed, to solve problems in economics, by Hungarian born American mathematician **John von Neumann** and his Princeton University colleague **Oskar Morgenstern**, a German born American economist in the year **1944**. They observed that economics is much like a game, wherein players anticipate each other's moves. They named it Game Theory which is somewhat of a misnomer because it does not share the fun or frivolity associated with games.

DEFINITION

Game Theory may be defined as a type of Decision Making situation when two or more intelligent and rational opponents are involved under conditions of conflict and competition. It is a type of Decision Theory in which one's choice of action is determined after taking into account all possible alternatives available to the opponent participating in the same game. Game Theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected.

WHAT IS A GAME?

'Game' is defined as an activity between two or more participants according to a set of rules, at the end of which each participant either gets some benefit or suffers some loss.

⚡ Quick Revision — Section 9.1

- **Game Theory:** Mathematical model for analysing conflicting situations where two or more rational opponents are involved.
- **Developed by:** John von Neumann and Oskar Morgenstern (1944).
- **Game:** Activity between two or more participants according to a set of rules.
- **Applications:** Military operations, Economics, Market competition, Pricing strategies.

9.2 Basic Terms in Game Theory

PLAYER · PLAY · 2 PERSON GAME · N PERSON GAME · ZERO SUM · NON-ZERO SUM · STRATEGY · PAYOFF · PAYOFF MATRIX · OPTIMAL STRATEGY · VALUE OF THE GAME

A

Player

A participant in a game is called a **Player**.

B

Play

A **Play** of the game is said to occur when each Player has chosen a course of action.

C

2 Person Game

If the number of Players in a Game is two then it is called a **2 Person Game**. The term Person refers to an individual or a group aiming at a particular objective.

D

N Person Game

If the number of Players in a Game is N (where N > 2) then it is called **N Person Game**.

E

Zero Sum Game

If the sum of the amounts won by all winners is equal to that lost by all losers then the game is called **Zero Sum Game**. In other words, sum of the gains and losses in such a game is zero. When there are only two players participating in a game which has resulted in zero sum then it is called **2 Person Zero Sum Game**. For a game with zero sum and N participants, we call it as **N Person Zero Sum Game**.

An example of 2 Person Zero Sum Game is the competition of two firms who are trying to increase their share of the market. Here gain of the market share of one will almost be equal to the other's loss of the market share.

Two Person Zero Sum Games are also called **rectangular game** because their payoff matrix is in the rectangular form.

F

Non Zero Sum Game

If the sum of the gains or losses in a game is not equal to zero then it is called a **Non Zero Sum Game**. An example of such a game is the competition between two firms for increasing their respective market shares through intensive advertising campaigns. Due to the advertisement of their product, both the firms might gain market share which may not be of equal magnitude, but at the same time gain of one is not exactly equal to the loss of the other. Hence, sum of the amounts is not zero.

G

Strategy

It is the predetermined rule by which a Player while playing decides the course of action from his own list of courses of action. There are two types of strategies — **Pure** and **Mixed**.

H

Pure Strategy

If a Player knows exactly what the other Player is going to do, a deterministic situation is obtained. The objective is to maximize the gain. Thus, it is a decision in advance of all plays always to choose a particular course of action. A pure strategy is usually represented by a number with which the course of action is associated.

I

Mixed Strategy

If a Player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained. The objective in this case is to maximize the expected gain. Thus, it is a decision, in advance of all plays to choose a course of action for each play in accordance with some particular probability distribution. When a player decides in advance to use his available courses of action in some fixed proportion, he is said to use mixed strategy. In other words, the Mixed Strategy is a selection among Pure Strategies with some fixed probabilities.

J

Payoff

The outcome of playing the game is known as **Payoff**. It is the quantitative measure of satisfaction a Player gets at the end of each play. For a business situation, the measure of satisfaction mentioned above could be increase in profits, Expansion in actual market share etc. In other words, it is the net gain a course of action or strategy brings to a player for any counter course of action or strategy of the competitor.

K

Payoff Matrix

This is a tabular representation showing the outcomes or payoffs corresponding to different strategies of the participating Players. Since a Game involves at least two Players, the table referred above always forms a matrix with some rows (m, say) and columns (n, say). Rules of a Payoff Matrix are:

- Rows denote the activities or courses of action available to Player A who is considered as the maximising player.
- Columns denote the activities or courses of action available to Player B who is the minimising player.
- Figure shown in the cell x_{ij} denotes payment to A when he chooses the activity i against B's choice of activity j.
- For a Two Person Zero Sum Game, any cell entry of the Player B's payoff matrix will be negative of the corresponding cell entry in the Player A's payoff matrix, so that the sum of the payoffs of the two Players is ultimately zero.

L

Optimal Strategy

A course of action which puts the Player in the most preferred position, irrespective of the strategy of his competitors, is called **Optimal Strategy**. Not opting for this strategy will result in decreased payoff of the player.

M

Value of the Game

It is the expected payoff of play when all the players of the game follow their optimal strategies. The game is called **Fair** if the value of the game is zero and **Unfair** if it is non-zero.

Example of a Payoff Matrix (Two Person Zero Sum Game)

Two firms are competing for business with the following strategies so that one's gain is another's loss:

STRATEGIES OF FIRM A	NO ADVERTISING	MEDIUM ADVERTISING	HEAVY ADVERTISING
No advertising	10	5	–2
Heavy advertising	16	14	10

Here a positive payoff denotes gain to the maximising player i.e. Firm A (shown as Row) and loss to the minimising player i.e. Firm B (shown as Column). If Firm A chooses strategy "No advertising" and Firm B chooses strategy "Medium advertising" then gain of A will be 5 and loss of B will also be 5.

⚡ Quick Revision — Section 9.2

- **Player:** Participant in a game.
- **Play:** Occurs when each Player has chosen a course of action.
- **2 Person Game:** Two players. **N Person Game:** N > 2 players.
- **Zero Sum Game:** Sum of gains and losses = 0; also called rectangular game.
- **Non Zero Sum Game:** Sum of gains/losses ≠ 0.
- **Pure Strategy:** Deterministic; choose a particular course of action.
- **Mixed Strategy:** Probabilistic; choose strategies with fixed probabilities.
- **Payoff Matrix:** Rows = maximising player; Columns = minimising player.
- **Optimal Strategy:** Most preferred position irrespective of competitor's strategy.
- **Value of the Game:** Expected payoff when all players follow optimal strategies; Fair if zero; Unfair if non-zero.

9.3 Assumptions in a Game

SEVEN KEY ASSUMPTIONS

1

Rational and Intelligent Players

The players act rationally and intelligently.

2

Finite Set of Strategies

Each player has a finite set of strategies available to him.

3

Maximise Gains, Minimise Losses

The players attempt to maximise gains and minimise losses.

4

All Relevant Information Available

All relevant information is available to each player.

5

Individual Decisions

The players take individual decisions without direct communication with each other.

6

Simultaneous Selection

The players select their strategies simultaneously.

7

Fixed and Determined Payoff

The payoff is fixed and determined in advance.

⚡ Quick Revision — Section 9.3

- **Seven Assumptions:** Rational players, finite strategies, maximise gains/minimise losses, all information available, individual decisions, simultaneous selection, fixed payoff.

9.4 Solution of Pure Strategy Games with Saddle Point

MAXIMIN-MINIMAX CRITERIA · SADDLE POINT · ILLUSTRATION

Pure Strategy Games are solved using **Maximin – Minimax criteria**. The maximising player (whose strategies are shown along the rows of the Payoff Matrix) arrives at his optimal strategy on the basis of Maximin criteria and the minimising player (whose strategies are shown along the columns of the Payoff Matrix) follows Minimax criteria. The game is solved when Maximin and Minimax values are equal.

Maximin Value

DETERMINATION OF MAXIMIN VALUE

1. Find minimum value in each row of the given payoff matrix. This denotes minimum possible gain against each strategy of the Maximising Player.
2. Maximin value is the maximum of these minimum values.

Minimax Value

DETERMINATION OF MINIMAX VALUE

1. Find maximum value in each column of the given payoff matrix. This denotes maximum possible loss against each strategy of the Minimising Player.
2. Minimax value is the minimum of these maximum values.

SADDLE POINT

Saddle Point is said to exist when the Maximin and Minimax values are equal. Thus, Saddle Point is the position of such an element in the payoff matrix, which is **minimum in its row and maximum in its column**. The Saddle Point is the solution or Value of the game. The strategies of the two players corresponding to the Saddle Point are their optimal strategies. If there is more than one Saddle Point then more than one solution will be possible corresponding to each Saddle Point.

Illustration — Payoff Matrix with Saddle Point

STRATEGIES OF PLAYER A	B1	B2	ROW MINIMUM
A1	9	2	**2**
A2	8	6	**6 ← Maximin**
A3	6	4	**4**
Column Maximum	**9**	**6 ← Minimax**	

In the table above, A is the maximising player with strategies represented along the rows and B is the minimising player with strategies represented along the columns.

Suppose the player A starts the game knowing fully well that whatever strategy he adopts, B will select that particular counter strategy which will minimise the payoff to A. If A selects A1 then B will definitely select B2 so that A gets minimum possible gain i.e. 2 under the situation. Similarly if A chooses A2 then B will go for B2 and so on. Thus, A wants to maximise his gain which is possible by going for the maximum value among the Row minimums or the Maximin value. Similarly, B wants to minimise his loss which is the minimum among the Column maximums or the Minimax value.

RESULT

We observe here that both Maximin and Minimax values are equal to **6**. Hence there exists a **Saddle Point**. Also this value corresponds to the cell **A2B2**. That means the Optimal strategy for the Player A is **A2** and that for the Player B is **B2**. Value of the Game is **6 for A and –6 for B** which means the game is Zero Sum.

⚡ Quick Revision — Section 9.4

- **Pure Strategy Games:** Solved using Maximin-Minimax criteria.
- **Maximin:** Maximum of row minimums (for maximising player).
- **Minimax:** Minimum of column maximums (for minimising player).
- **Saddle Point:** Exists when Maximin = Minimax; element is minimum in its row and maximum in its column.
- **Value of the Game:** Determined at the Saddle Point; Zero Sum if value for one player is negative of the other.

9.5 Principle of Dominance

THREE RULES · REDUCING MATRIX SIZE · ILLUSTRATION

CONCEPT

According to the **Principle of Dominance**, if any strategy of a player dominates over another strategy in all conditions then the latter can be ignored because it will not affect the solution of the game. A strategy dominates over the other only if it is preferable over the other in all conditions. From the gainer's point of view, if a strategy gives more gain than another strategy for all strategies of the loser, then the first strategy dominates over the other and the second one can be ignored altogether. Similarly from the loser's point of view, if a strategy involves lesser loss than the other in all conditions, the second one can be omitted without affecting decision. So determination of superior or inferior strategy depends upon the objective of the player. Since each player has to select his best strategy, the inferior strategies can be eliminated. In other words, ineffective rows and columns can be deleted from the given payoff matrix so that its size is reduced.

Three Rules for Deleting Ineffective Rows and Columns

1

Rule 1 — Row Dominance

If all the elements of a row (say i^{th} row) of a payoff matrix are less than or equal to the corresponding elements of another row (say j^{th} row) then the Maximising Player will never choose the ith strategy. In other words, i^{th} strategy is dominated by the jth strategy.

2

Rule 2 — Column Dominance

If all the elements of a column (say p^{th} column) of a payoff matrix are more than or equal to the corresponding elements of another column (say q^{th} column) then the Minimising Player will never choose the pth strategy. In other words, p^{th} strategy is dominated by the qth strategy.

3

Rule 3 — Average Dominance

A pure strategy may be dominated if it is inferior to average of two or more other pure strategies. If all the elements of a row are less than or equal to the average of the corresponding elements of two or more other rows then this row is said to be dominated by the other group of rows for which average is computed. Similar concept is also applicable for column with the exception of having its elements more than the average of the corresponding elements of two or more columns.

OBJECTIVE OF DOMINANCE

Principle of Dominance can be applied to both Pure Strategy as well as Mixed Strategy problems. Its basic objective is to reduce the size of the given Payoff Matrix. Aim should always be made to get a **(2 × 2)** matrix by using this Principle.

Illustration 2 — Dominance Method (Management vs Union)

PROBLEM

The Management of a company is negotiating with its Union for revision of hourly wages of its employees. The Management deployed a Consulting Firm who has prepared a payoff matrix for the purpose which indicates the additional hourly cost (in ₹) to the company. It is shown below: you being a part of the Consulting Firm have to assist the Management in selecting the best strategy. What is the value of the game? How is it going to affect the company's cost?

MANAGEMENT'S STRATEGIES	U1	U2	U3	U4
M1	2.50	2.70	3.50	−0.20
M2	2.00	1.60	0.80	0.80
M3	1.40	1.20	1.50	1.30
M4	3.00	1.40	1.90	0

SOLUTION

As the Management's objective is to minimise the cost, they can be considered as the Minimising Player and the Union as the Maximising Player in this problem of Game. Thus, to solve the problem we have to recast the given Payoff Matrix by transposing it.

Step 1 — Transposed Matrix

STRATEGIES OF THE UNION	M1	M2	M3	M4	ROW MINIMUM
U1	2.50	2.00	1.40	3.00	1.40 = Maximin
U2	2.70	1.60	1.20	1.40	1.20
U3	3.50	0.80	1.50	1.90	0.80
U4	−0.20	0.80	1.30	0	−0.20
Column Maximum	3.50	2.00	1.50 = Minimax	3.00	

Maximin value (1.40) ≠ Minimax value (1.50). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy. Since the matrix is not a (2×2) Matrix, Dominance Rules are applied to reduce its size.

Step 2 — Applying Dominance Rules
All elements of the 3rd Row (M3) are either greater than or equal to the corresponding elements of the 4th Row (M4), the 3rd Row can be considered to dominate the 4th. So the 4th Row is ignored.

All elements of the 1st Column (M1) are greater than the corresponding elements of the 2nd Column (M2), the 1st Column is dominated by the 2nd Column. Hence the 1st Column is ignored.

All elements of the 3rd Column (M4) are more than the corresponding elements of 2nd Column (M3). Hence M4 is dominated by M3 and ignored.

All elements of the 1st Row (U1) are greater than the corresponding elements of the 2nd Row (U2). So U2 is dominated by U1 and ignored.

Step 3 — Reduced (2 × 2) Matrix

STRATEGIES OF THE UNION	M2	M3
U1	2.00 = a_1	1.40 = a_2
U3	0.80 = b_1	1.50 = b_2

This is a (2 × 2) Matrix. Now the problem of Game is solved by using **Odds Method**. Odds are calculated as below:

STRATEGIES OF THE UNION	M2	M3	ODDS
U1	2.00 = a_1	1.40 = a_2	**$b_1 - b_2$ = 0.80 – 1.50 = 0.70**
U3	0.80 = b_1	1.50 = b_2	**$a_1 - a_2$ = 2.00 – 1.40 = 0.60**
ODDs	**$a_2 - b_2$ = 1.40 – 1.50 = 0.10**	**$a_1 - b_1$ = 2.00 – 0.80 = 1.20**	

Sum of the ODDs = 1.30

Probabilities of the Union and the Management taking their different strategies:

- P(U1) = $(b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)]$ = 0.70 / 1.30 = **7/13**
- P(U3) = $(a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)]$ = 0.60 / 1.30 = **6/13**
- P(M2) = $(a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)]$ = 0.10 / 1.30 = **1/13**
- P(M3) = $(a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)]$ = 1.20 / 1.30 = **12/13**

Value of the Game = $v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)]$

= [2.00×0.70 + 0.80×0.60] ÷ [0.70 + 0.60] = [1.40 + 0.48] / 1.30 = 1.88/1.30 = **1.45**

So the Union chooses its Strategies U1, U2, U3 & U4 with probabilities (7/13, 0, 6/13, 0) and the Management chooses its Strategies M1, M2, M3 & M4 with probabilities (0, 1/13, 12/13, 0). Expected Gain to the Union is ₹1.45 and the corresponding Loss to the Management is ₹1.45. Thus, the hourly cost of the company will increase by **₹1.45**.

Illustration 3 — Dominance Principle (3 × 3 Matrix)

PROBLEM

Solve the Game using Dominance Principle: **1 3 12 / 8 6 2** (arranged as 2 rows × 3 columns)

SOLUTION

Let the given Game be played by Players A and B with A (maximising player) having strategies A1, A2 and A3 along rows and B (minimising player) having strategies B1, B2 and B3 along columns.

STRATEGIES OF A	B1	B2	B3
A1	15	2	3
A2	6	5	7
A3	–7	4	0

All elements of Row A3 are less than the corresponding elements of Row A2. So A3 is dominated by A2 and is ignored.

STRATEGIES OF A	B1	B2	B3
A1	15	2	3
A2	6	5	7

All elements of B3 are more than the corresponding elements of B2. Hence B3 is dominated by B2 and ignored.

STRATEGIES OF A	B1	B2	ROW MIN.
A1	15	2	2
A2	6	5	5
Column Max.	15	5	

Maximum among the Row minimums = 5 = Maximin value and Minimum among the Column maximums = 5 = Minimax value. As Maximin and Minimax values are equal, there exists a **Saddle Point**. It occurs at the cell **A2B2**.

Hence optimal strategies of A and B are respectively **A2** and **B2**. Also value of the Game = **5**

Illustration 4 — Dominance (Joy Givers vs Milan Toys)

PROBLEM

Joy Givers and Milan Toys are the two toy manufacturers who always compete with each other to increase their respective market shares. Payoff matrix in which the percentage increase in market share is given against different strategies of Milan Toys:

STRATEGIES OF MILAN TOYS	M1	M2	M3	M4
J1 (Working 12 hrs/day)	8	10	9	14
J2 (Working 16 hrs/day)	10	11	8	12
J3 (Working 24 hrs/day)	13	12	14	13

SOLUTION

All elements of 4th Column are either greater than or equal to the corresponding elements of the 1st Column. So 4th Column's strategy (M4) is dominated by the 1st Column's strategy (M1). Hence M4 is ignored.

All elements of 1st Row are less than the corresponding elements of the 3rd Row. Thus, strategy of 1st Row i.e. J1 is dominated by the strategy of the 3rd Row i.e. J3 and ignored.

STRATEGIES	M1	M2	M3
J2	10	11	8
J3	13	12	14

Now, if the average of the elements of the strategies M2 and M3 be taken:

STRATEGIES	M1	[M2 + M3]/2
J2	10	(11 + 8)/2 = 9.5
J3	13	(12 + 14)/2 = 13

So the elements of the strategy M1 are either more or equal to the average of the corresponding elements of M2 and M3. Hence M1 is dominated by M2 and M3. Thus, M1 is deleted.

STRATEGIES OF MILAN TOYS	M2	M3	ROW MINIMUM
J2	11	8	8
J3	12*	14	12 = Maximin
Column Maximum	12 = Minimax	14	

So Maximin value = 12 = Minimax value. Hence there exists a Saddle Point at the junction **J3M2**.

Thus, optimal strategy of Joy Giver is J3 that is "Working 24 hours/day" and that for Milan Toys is M2 that is "Working 12 hours/day". Value of the Game = **12** (which means a 12% increase in market share for Joy Givers).

⚡ Quick Revision — Section 9.5

- **Principle of Dominance:** Inferior strategies can be eliminated to reduce matrix size.
- **Rule 1 (Row):** If all elements of a row ≤ corresponding elements of another row, the first row is dominated.
- **Rule 2 (Column):** If all elements of a column ≥ corresponding elements of another column, the first column is dominated.
- **Rule 3 (Average):** A row/column may be dominated if inferior to average of two or more other rows/columns.
- **Objective:** Reduce matrix to (2 × 2) for Odds Method.
- **Illustration:** Management vs Union — Value of Game = ₹1.45; Optimal strategies: Union (U1 = 7/13, U3 = 6/13), Management (M2 = 1/13, M3 = 12/13).

9.6 Solution of Mixed Strategy Games

ODDS METHOD · DOMINANCE METHOD · GRAPHICAL METHOD

Any problem of Game without a Saddle Point is considered to be the problem of **Mixed Strategy**. In such cases both players will use various strategies with certain probabilities to optimize. Unlike Pure Strategy problems (where a single strategy will certainly be the optimum one) here we need to find out the probabilities of various strategies of both the players as well as expected value of the game. Games with Mixed Strategy are solved by the following methods depending on the size of the Payoff Matrix.

1

(2 × 2) Game — Odds Method

Odds are nothing but the magnitude (i.e., without sign or ignoring negative sign, if any) of the differences of the elements of various rows as well as columns.

2

Dominance Method

Applicable for m × n Payoff Matrix convertible to 2 × 2 Payoff Matrix by application of Rules of Dominance.

3

Graphical Method

Applicable for (2 × n) and (m × 2) Game problems, when both m and n are more than 2.

Odds Method (2 × 2 Game)

METHOD OF CALCULATING ODDS

1. Find out magnitude of difference in the values of cell (1,1) and (1,2) of the 1st Row and place it against the 2nd Row.
2. Compute magnitude of the difference in the cell entries of (2,1) and (2,2) of the 2nd Row and put it against the 1st Row.
3. Compute magnitude of the difference in the cell entries of (1,1) and (2,1) of the 1st Column and put it below the 2nd Column.
4. Compute magnitude of the difference in the cell entries of (1,2) and (2,2) of the 2nd Column and put it below the 1st Column.
5. Ensure that the sum of the differences calculated for the Rows is equal to that for the columns. In other words, Sum of the differences calculated in steps (1) and (2) should be equal to that calculated in steps (3) and (4).

STRATEGIES OF Y	Y1	Y2	ODDS
X1	a_1	a_2	$(b_1 - b_2)$
X2	b_1	b_2	$(a_1 - a_2)$
ODDs	$(a_2 - b_2)$	$(a_1 - b_1)$	

ODDS METHOD — PROBABILITY AND VALUE FORMULAE

$$P(X1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)]$$

$$P(X2) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)]$$

$$P(Y1) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)]$$

$$P(Y2) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)]$$

$$\text{Value of the Game: } v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)]$$

Note: P(X1) + P(X2) = 1 and P(Y1) + P(Y2) = 1. Once P(X1) is calculated, P(X2) can always be calculated as complement of P(X1). Similar is the case for P(Y2).

Illustration 1 — Odds Method

PROBLEM

Solve the Game with the Payoff Matrix: **1 5 / 4 2**

SOLUTION

Let the given Game be played by Players A and B with A (maximising player) having strategies A1 and A2 along rows and B (minimising player) having strategies B1 and B2 along columns.

STRATEGIES OF B	B1	B2	ROW MIN.
A1	1	5	1
A2	4	2	2 = Maximin
Column Max.	4 = Minimax	5	

Maximin value (2) ≠ Minimax value (4). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy with (2 × 2) Payoff Matrix.

STRATEGIES OF B	B1	B2	ODDS
A1	$1 = a_1$	$5 = a_2$	$\mathbf{b_1 - b_2 = 4 - 2 = 2}$
A2	$4 = b_1$	$2 = b_2$	$\mathbf{a_1 - a_2 = 1 - 5 = 4}$
ODDs	$\mathbf{a_2 - b_2 = 5 - 2 = 3}$	$\mathbf{a_1 - b_1 = 1 - 4 = 3}$	

Sum of the ODDs = 2 + 4 = 6 (also = 3 + 3 = 6)

Probabilities of A and B taking their different strategies:

- $P(A1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 2 / 6 =$ **1/3**
- $P(A2) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 4 / 6 =$ **2/3**
- $P(B1) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 3 / 6 =$ **1/2**
- $P(B2) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)] = 3 / 6 =$ **1/2**

Value of the Game = $v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)]$ = [1×2 + 4×4] ÷ [2 + 4] = 18/6 = **3**

So A chooses his strategies (A1, A2) with probabilities (1/3, 2/3) & B chooses his strategies (B1, B2) with probabilities (1/2, 1/2) and Value of the Game = **3**.

Graphical Method

Graphical Method is applied to solve (2 × n) and (m × 2) Game problems, when both m and n are more than 2. Since the optimal strategies for both the players assign non-zero probabilities to the same number of pure strategies, it is obvious that if one player has only two strategies the other will also use two strategies. Graphical method facilitates to find out which of the two strategies can be used. When Rules of Dominance cannot be applied to a payoff matrix of size (2 × n) or (m × 2) then Graphical Method is used.

Steps for Solving a (2 × n) Game

Step 1

Draw two vertical lines 1 unit apart along a horizontal line to represent the axes $x_1 = 0$ and $x_1 = 1$ & mark a suitable scale on each one.

Step 2

Take the values in the first Row of the Payoff Matrix and plot each one as a point on the scale of the vertical line $x_1 = 1$.

Step 3

Take the values in the second Row of the Payoff Matrix & plot each one as a point on the scale of the vertical line $x_1 = 0$.

Step 4

The point a_{1j} on the line $x_1 = 1$ should be joined to the point a_{2j} on the line $x_1 = 0$ to get a straight line.

Step 5

Draw n such straight lines for j = 1, 2, 3, ..., n. Each of these lines represents the expected payoff of the maximising player against n different strategies of the minimising player.

Step 6

Mark the lower envelope of the area obtained by drawing these n straight lines.

Step 7

The highest point of the lower envelope is the **Maximin point**.

Step 8

The straight lines passing through this Maximin point correspond to the optimum strategies of the minimising player. All the other strategies of the minimising player should be ignored.

Step 9

So now the desired (2 × 2) payoff matrix is obtained, the Game can be solved using the method of Odds.

Steps for Solving a (m × 2) Game

The steps for solving (m × 2) Game are almost similar to those of (2 × n) Game. The main difference is that we have to consider the values in the First and the second Columns and plot them as points on the two axes $x_1 = 0$ and $x_1 = 1$ so

that m straight lines can be drawn representing expected payoff of the minimising player against the strategies of the maximising player. Now the **upper envelope** of the common area bounded by the straight lines should be marked. **Lowest point** of this envelope represents **Minimax point**. The strategy lines which intersect to give the Minimax point are the optimum strategies of the maximising player. All the other strategies of the maximising player should be ignored to give a (2 × 2) payoff matrix which can be solved by using the method of Odds.

NOTE

Problems of Game, with Payoff Matrix of size (m × n) when both m & n > 2, are solved using **Simplex Method of Linear Programming**.

Illustration 5 — Graphical Method (2 × 3 Game)

PROBLEM

Solve the Game represented by the payoff matrix: **1 3 12 / 8 6 2**

SOLUTION

Let the given Game be played by Players A and B with A (maximising player) having strategies A1 and A2 along rows and B (minimising player) having strategies B1, B2 and B3 along columns.

STRATEGIES OF B	B1	B2	B3	ROW MIN.
A1	1	3	12	1
A2	8	6	2	2 = Maximin
Column Max.	8	6 = Minimax	12	

Maximin value (2) ≠ Minimax value (6). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy. Since the matrix is not a (2×2) Matrix, we check the possibility of applying Dominance Rules to reduce its size to (2×2) Matrix, but it is observed that the Dominance Rules are also not suitable for reducing the size of the given matrix. Hence we go for solving the problem using Graphical Method suitable for (2×n) matrix because the given matrix is (2×3).

Two vertical lines are drawn on a horizontal line 1 unit apart to represent the axes $x_1 = 0$ & $x_1 = 1$ and marked them to a scale. Now the values 1, 3 and 12 of the 1st Row of the given matrix are plotted as points on the axis $x_1 = 1$ and the values 8, 6 and 2 of the 2nd Row are plotted as points on the axis $x_1 = 0$. Then the pair of points 12 & 2, 3 & 6 and 1 & 8 are joined with the help of straight lines. These lines represent the expected payoff of Player A against the strategies B3, B2 & B1 respectively of Player B. The lower envelope of the area bounded by these lines and the axes is shaded. Highest point P of this envelope is the Maximin point. As P is the point of intersection of A's expected payoff lines against strategies B2 & B3 of B, we can say that B will opt for these two strategies and ignore strategy B1.

So the reduced payoff matrix is:

STRATEGIES OF A	B2	B3
A1	$3 = a_1$	$12 = a_2$
A2	$6 = b_1$	$2 = b_2$

STRATEGIES OF A	B2	B3	ODDS
A1	$3 = a_1$	$12 = a_2$	$\mathbf{b_1 - b_2 = 6 - 2 = 4}$
A2	$6 = b_1$	$2 = b_2$	$\mathbf{a_1 - a_2 = 3 - 12 = 9}$
ODDs	$\mathbf{a_2 - b_2 = 12 - 2 = 10}$	$\mathbf{a_1 - b_1 = 3 - 6 = 3}$	

Sum of the ODDs = 4 + 9 = 13 (also = 10 + 3 = 13)

Probabilities of A and B taking their different strategies:

- $P(A1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 4 / 13$
- $P(A2) = 1 - P(A1) = 1 - 4/13 = 9/13$
- $P(B2) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 10 / 13$
- $P(B3) = 1 - P(B2) = 1 - 10/13 = 3/13$

Value of the Game = $v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)]$ = [3×4 + 6×9] ÷ [4 + 9] = 66/13

So A chooses the strategies with probabilities (4/13, 9/13) and B chooses the strategies with probabilities (0, 10/13, 3/13).

⚡ Quick Revision — Section 9.6

- **Mixed Strategy Games:** No Saddle Point; players use strategies with certain probabilities.
- **Odds Method:** For (2 × 2) games; uses differences of elements.
- **Dominance Method:** Reduces m × n to 2 × 2.
- **Graphical Method:** For (2 × n) and (m × 2) games; lower envelope (2 × n) or upper envelope (m × 2).
- **Probability Formulae:** P(X1) = (b1 – b2) / [(b1 – b2) + (a1 – a2)]; P(Y1) = (a2 – b2) / [(a2 – b2) + (a1 – b1)].
- **Value of Game:** v = [a1(b1 – b2) + b1(a1 – a2)] / [(b1 – b2) + (a1 – a2)].

9.7 Limitations of Game Theory

FOUR KEY LIMITATIONS

1

Unrealistic Knowledge Assumption

The assumption that the players have the knowledge about their own and the opponent's payoffs is unrealistic. He can only make a guess of his own and the opponent's strategies.

2

Complexity with More Players

As the number of players increase in the game, the analysis of the strategies becomes increasingly complex and difficult. In practice there are many firms in an oligopoly situation where game theory is not useful.

3

Impractical Pure Strategy Assumptions

The assumptions of Pure Strategy game show that the players are risk averse and have complete knowledge about each other's strategies. This is impractical.

4

Collusion vs Mixed Strategy

Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out a collusion. So the mixed strategies are also not very useful.

⚡ **Quick Revision — Section 9.7**

- **Four Limitations:** Unrealistic knowledge assumption, complexity with more players, impractical pure strategy assumptions, collusion vs mixed strategy.

9.8 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. Two person zero sum game means that:

(a) The sum of losses of one player is equal to the sum of the gains of the other

(b) The sum of losses of one player may not be equal to the sum of the gains of the other

(c) No player gains or loses

(d) None of the above

Answer: A

2. Game theory models are classified by the:

(a) Number of players (b) Sum of all payoffs (c) Number of strategies

(d) All of these

Answer: D

3. A game is said to be unfair if:

(a) Upper and lower values of the game are not equal (b) Upper and lower values of the game are equal and the sum is zero (c) Option (a) is correct but not Option (b)

(d) Option (b) is correct but not Option (a)

Answer: A

4. What happens when the maximin and minimax values of the game are equal?

(a) No solution exists (b) Solution is mixed (c) Saddle point exists

(d) None of these

Answer: C

5. A mixed strategy game can be solved by:

(a) Arithmetic method (b) Graphical method (c) Dominance method

(d) All of these

Answer: D

6. The size of the payoff matrix of a game can be reduced by using the principle of:

(a) Game inversion (b) Rotation reduction (c) Dominance

(d) Game transpose

Answer: C

7. The payoff value for which each player in a game always selects the same strategy is called the:

(a) Saddle point (b) Equilibrium point (c) Both option (a) and option (b)

(d) None of the above

Answer: A

8. Games which involve more than two players are called:

(a) Conflicting games (b) Negotiable games (c) N person game

(d) All of these

Answer: C

9. When the sum of the gains of one player is equal to the sum of the losses to another player then it is called:

(a) Fair game

(b) Zero sum game

(c) Both option (a) and option (b)

(d) Only option (b) and not option (a)

Answer: D

10. When no saddle point is found in the payoff matrix of a game, the value of the game is found by:

(a) Reducing the size of the game to apply the odds method

(b) Solving any one of the (2×2) sub game

(c) Finding the average of the values of the payoff matrix

(d) None of these

Answer: A

11. A saddle point exists when:

(a) Maximin value = Maximax value

(b) Minimax value = Minimum value

(c) Minimax value = Maximin value

(d) Minimax value = Minimin value

Answer: C

12. In a pure strategy game:

(a) Any strategy can be selected arbitrarily

(b) A particular strategy is selected by each player

(c) Both players select their optimal strategy

(d) None of these

Answer: C

13. In a mixed strategy game:

(a) No saddle point exists

(b) Each player selects the same strategy without considering the choice of the other

(c) Each player always selects the same strategy

(d) None of these

Answer: A

14. Game theory is the study of:

(a) Selecting optimal strategies

(b) Resolving conflict between players

(c) Giving equal outcome to the participants

(d) None of the above

Answer: A

15. If the value of the game is zero, then the game is known as:

(a) Fair strategy

(b) Pure strategy

(c) Pure game

(d) Mixed strategy

Answer: A

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
A	D	A	C	D	C	A	C	D	A	C	C	A	A	A

State True or False

SL	STATEMENT	ANSWER
1	The name Game is derived from the fact that the end result always gives lot of fun to the players.	**False**
2	Strategies are the different courses of action of the players.	**True**
3	In Pure Strategy the objective of the maximizing player is to maximize the Gain, but in Mixed Strategy the objective is to maximize the Expected Gain.	**True**
4	Both Pure and Mixed Strategy problems can be solved by the Rules of Dominance.	**True**
5	A fair game results when the value of the game is zero.	**True**
6	Mixed Strategy Games are deterministic in nature.	**False**
7	Zero sum games always have two participants only.	**False**
8	A pure strategy may be dominated if it is inferior to average of two or more other pure strategies.	**True**
9	Columns of a payoff matrix represent the strategies of the maximising player	**False**
10	Graphical Method can be used for mixed strategy games having any size of payoff matrix.	**False**
11	In a game the players act rationally and intelligently.	**True**
12	In a (m×2) mixed strategy game the graphical method is used to find out the maximin point.	**False**
13	Equality of minimax and maximin values result in the existence of Saddle Point.	**True**
14	Optimal strategy is that course of action which puts a player in most preferred position.	**True**
15	Military operations are examples of game.	**True**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	For the Payoff Matrix **1 5 / 4 2**, the ___ strategy of the minimising player is dominated by the third.	**Second**
2	"Teesta" water distribution conflict between India and Bangladesh can be considered as a situation of _____.	**Game**
3	Arithmetic Method is used to solve _____ strategy problems of Game.	**Mixed**
4	Value of the Game **1 5 / 4 2** is ___.	**2**
5	As per the Rules of Dominance, a strategy of the maximising player is said to dominate another if all the elements of a row of the payoff matrix are ____ than or equal to the corresponding elements of the other.	**Greater**
6	Pure strategy games are ____ in nature.	**Deterministic**
7	Dominance principle has ____ rules.	**Three**
8	Strategies of maximising player of a game are represented along the ____ of the payoff matrix.	**Row**
9	For (2×n) matrix Mixed strategy Games ____ method is used to find the solution.	**Graphical**
10	Columns of the payoff matrix of a game represent the strategies of the ____ player.	**Minimising**
11	Game with payoff matrix (m×n) is solved using Simplex Method, if both m and n are greater than ___.	**2**
12	In a game, all relevant information is available to ____ player.	**Each**
13	In (2×2) mixed strategy game, sum of the probabilities of the minimising player taking its two strategies is ___.	**1**
14	Pure strategy games always have a ____ point.	**Saddle**
15	In problems of game negative payoff of the maximising player indicates a ___.	**Loss**
16	Odds Method is applicable to ____ matrix games only.	**(2×2)**
17	In practical business situations most of games have ___ than two players.	**More**

SL	STATEMENT	ANSWER
18	Multiplayer games are also known as ___ person game.	**N**
19	In graphical method for (2 × m) game the Maximin value is determined from the ____ point of the lower envelope.	**Highest**
20	A participant of any game is called a _____.	**Player**

9.9 Comprehensive Numerical Problems

PRACTICE PROBLEMS WITH ANSWERS

PROBLEM 1 — OPTIMAL STRATEGIES

Find the optimal strategies of the Players for the game having payoff matrix. What is the value of the Game?

STRATEGIES	B1	B2	B3	B4
A1	1	7	3	4
A2	5	6	4	5
A3	7	2	0	3

ANSWER

Optimal strategies A2 and B3. Value of the game = 4

PROBLEM 2 — SOLVE THE GAME

Solve the game:

	B1	B2	B3
A1	1	7	2
A2	6	2	7
A3	5	1	6

ANSWER

Strategies of the Maximising Player = (1/5, 4/5) & for the Minimising Player = (3/5, 2/5). Value of the game = 17/5

PROBLEM 3 — REDUCE BY DOMINANCE RULES AND SOLVE

Reduce the following game by Dominance Rules and Solve it.

STRATEGIES OF MAXIMISING PLAYER	P	Q	R	S	T
I	1	3	2	7	4
II	3	4	1	5	6
III	6	5	7	6	5
IV	2	0	6	3	1

ANSWER

Optimal strategy for the Maximising Player is III and that for the Minimising Player is Q. Value of the game = 5

PROBLEM 4 — GRAPHICAL METHOD

Solve the game using Graphical method:

	B1	B2	B3
A1	1	3	12
A2	8	6	2

ANSWER

Optimal strategy of the Maximising Player (4/13, 9/13) and for the Minimising Player (0, 10/13, 3/13). Value = 66/13

PROBLEM 5 — SUBURBAN STORES (LAXMI BHANDAR VS GOSWAMI STORES)

In the suburban area of a large city there are two stores Laxmi Bhandar and Goswami Stores who handle sundry goods. The total number of customers is equally divided between the two due to the fact that the price, quality of goods, services etc. of the two are at par. Assume that a gain of customers for Laxmi Bhander is a loss to Goswami Stores and vice versa. Both the stores plan to run annual sales during the festival period of the year. Sales are advertised through Social Media, Cable TV local channel and Printed Leaflets. Based on the past experience, Laxmi Bhander has prepared the following payoff matrix for the gain or loss in percentage of customers for its different strategies against various counter strategies of Goswami Stores.

STRATEGIES OF LAXMI BHANDER	PRINTED LEAFLETS	CABLE TV	SOCIAL MEDIA
Printed leaflets	30	40	–80
Cable TV	0	15	–20
Social media	90	20	50

Required: Determine the optimal strategies and worth of such strategies for the stores. What is meant by the cell entry –80 in the above payoff matrix?

ANSWER

Optimal strategies: Laxmi Bhander = (1/5, 0, 4/5) and for Goswami Stores = (0, 13/15, 2/15). Value of the game = 24

Cell entry (–80): When Laxmi Bhander takes the strategy of distributing Printed Leaflets against the counter strategy of Goswami Stores of Social Media advertisement, then they will lose 80% of their customer which will be gained by Goswami Stores.

PROBLEM 6 — FIRM A AND FIRM B (MARKET SHARE)

Two competing firms (A and B) produce consumer goods of different kind. Among the products one is considered as their bread and butter in terms of the revenue generated. Both the firms are very cautious about the market share for this particular product and keep on doing advertisement campaigns throughout the year to retain the existing customers and also to attract the new ones. For this the marketing teams of both work round the clock and that of A developed data corresponding to varying degrees of advertisement. Same is given below:

- (a) If both the firms take same strategy to counter each other then their market share will be equal.
- (b) Against firm A's strategy of "No marketing" if B goes for "Medium marketing" then A's share of the market will be 40%. For the same strategy of A the market share will be 28% if B takes the strategy "Large marketing"
- (c) Against firm A's strategy of "Medium marketing" if B goes for "No marketing" then A's share of the market will be 70%. For the same strategy of A the market share will be 45% if B takes the strategy "Large marketing"
- (d) Against firm A's strategy of "Large marketing" if B goes for "No marketing" then A's share of the market will be 75%. For the same strategy of A the market share will be 47.5% if B takes the strategy "Medium marketing"

Required: Based on the above information prepare the Payoff Matrix. Solve the game problem to get the optimal strategies of the player A. What is the value of the game?

ANSWER

Payoff Matrix Showing A's Market Share:

STRATEGIES OF B	NO ADVERTISING	MEDIUM ADVERTISING	LARGE ADVERTISING
No advertising	50	40	28
Medium advertising	70	50	45
Large advertising	75	47.5	50

Probabilities of A's strategies are (0, 1/3, 2/3). Value of the game = 145/3 = 48.3. Thus, A can expect to have 48.3% market share.

PROBLEM 7 — ADVERTISING POLICY OF FIRM A

Using the data of the above problem prepare the Payoff Matrix for A when you are supplied with the following information:

- (a) Selling price of the product = ₹4 per unit
- (b) Variable cost of the product = ₹2.50 per unit
- (c) Annual cost for Medium advertising = ₹5000
- (d) Annual cost for Large advertising = ₹15000
- (e) Annual sales volume of the product for Firm A = 30000 units

Required: What advertising policy should firm A pursue?

ANSWER

Find out the Annual sales volume, for different combination of strategies of A and B. As an example, Annual Sales volume corresponding to A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" is 45% of 30000 = 13500 units.

Calculate Annual Profit to the Firm A using the formula: **Annual Profit = (Selling price – Variable cost) × Annual Sales volume – Annual cost of advertising**

For A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" the Annual Profit of Firm A is (4 – 2.5) × 13500 – 5000 = ₹15250.

STRATEGIES OF B	NO ADVERTISING	MEDIUM ADVERTISING	LARGE ADVERTISING
No advertising	22500	18000	12600
Medium advertising	26500	17500	15250
Large advertising	18750	6375	7500

From the above matrix we find, against the various strategies of A, the minimum profit figures are as follows:

- For No advertising — ₹12600
- For Medium advertising — ₹15250
- For Large advertising — ₹6375

Thus, to maximise the minimum profit, **A should opt for Medium advertising and spend ₹5000 per annum**.

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Game Theory

A type of Decision Making situation when two or more intelligent and rational opponents are involved under conditions of conflict and competition.

Player

A participant in a game.

Play

Occurs when each Player has chosen a course of action.

2 Person Game

If the number of Players in a Game is two.

N Person Game

If the number of Players in a Game is N (where N > 2).

Zero Sum Game

If the sum of the amounts won by all winners is equal to that lost by all losers then the game is called Zero Sum Game.

Non Zero Sum Game

If the sum of the gains or losses in a game is not equal to zero.

Strategy

The predetermined rule by which a Player while playing decides the course of action from his own list of courses of action.

Pure Strategy

Decision in advance of all plays always to choose a particular course of action.

Mixed Strategy

Decision, in advance of all plays, to choose a course of action for each play in accordance with some particular probability distribution.

Payoff

The outcome of playing the game; quantitative measure of satisfaction a Player gets at the end of each play.

Payoff Matrix

Tabular representation showing the outcomes or payoffs corresponding to different strategies of the participating Players.

Optimal Strategy

A course of action which puts the Player in the most preferred position, irrespective of the strategy of his competitors.

Value of the Game

The expected payoff of play when all the players of the game follow their optimal strategies.

Saddle Point

Position of such an element in the payoff matrix, which is minimum in its row and maximum in its column; exists when Maximin = Minimax.

Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

1

What is Game Theory? Discuss its assumptions and limitations.

2

Explain the following terms: (a) Player, (b) Strategy, (c) Payoff, (d) Saddle Point, (e) Value of the Game.

3

Distinguish between Pure Strategy and Mixed Strategy games.

4

State and explain the Principle of Dominance with suitable examples.

5

Explain the Odds Method for solving a (2 × 2) game.

6

Describe the Graphical Method for solving (2 × n) and (m × 2) games.

7

What is a Two Person Zero Sum Game? How is it solved?

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 9

ABBREVIATION	FULL FORM
LP	Linear Programming
LPP	Linear Programming Problem
a_1, a_2	Elements of first row in (2×2) game
b_1, b_2	Elements of second row in (2×2) game
P(X1), P(X2)	Probabilities of Player X's strategies
P(Y1), P(Y2)	Probabilities of Player Y's strategies
v	Value of the Game
Maximin	Maximum of row minimums
Minimax	Minimum of column maximums

CMA KNOWLEDGE

Strategic Cost Management — Master Revision Handbook

Module 9 · December 2026 Edition

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MODULE COVERAGE

9.1 Introduction to Game Theory

9.2 Basic Terms in Game Theory

9.3 Assumptions in a Game

9.4 Solution of Pure Strategy Games

9.5 Principle of Dominance

9.6 Solution of Mixed Strategy Games

9.7 Limitations of Game Theory

9.8 MCQs, True/False, Fill in the Blanks

9.9 Numerical Problems

STUDY TOOLS

Payoff Matrix

Saddle Point Identification

Odds Method

Dominance Rules

Graphical Method

Formula Boxes

MCQs with Reasoning

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Simulation

Module 10 · Monte Carlo Simulation · Random Numbers

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	15% (with LP, Transportation, Assignment, Game Theory)

✓ Website Publication · ✓ PDF Conversion · ✓ A4 Printing · ✓ Mobile & Desktop Reading · ✓ Self Study · ✓ Grand Revision · ✓ Final & Last Day Revision · ✓ Personal Reference Book

10

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 10 INCLUDES

Introduction · Applications of Simulation · Advantages of Simulation · Limitations of Simulation · Types and Models of Simulation · Steps involved in constructing and using a Simulation Model · Monte Carlo Simulation · Random Numbers.

Module Learning Objectives

After studying this module, the students will be able to:

1

Recognise a Problem of Simulation

Recognise a problem of Simulation.

2

Solve using Monte Carlo

Solve problems of Simulation using Monte Carlo Simulation technique.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Simulation Techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

15%	2	4	30
GROUP WEIGHTAGE	TYPES OF SIMULATION	SIMULATION MODELS	MCQS
With LP, Transportation, Assignment, Game Theory	Analogue & Computer	Deterministic, Probabilistic, Static, Dynamic	With answers

SYLLABUS STRUCTURE NOTE

Module 10 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Simulation, along with Linear Programming, Transportation, Assignment and Game Theory, forms a group carrying **15% weightage** in the examination. This module introduces a powerful technique for analysing complex real-world problems that cannot be represented by mathematical models due to their stochastic nature.

📖 **Table of Contents — Module 10**

1. Introduction to Simulation
2. Definitions
3. Applications of Simulation
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5. Limitations of Simulation
6. Types and Models of Simulation
7. Steps in Constructing and Using a Simulation Model
8. Monte Carlo Simulation
9. Random Numbers
10. Illustrations & Numerical Problems
11. MCQs, True/False, Fill in the Blanks

10.1 Introduction to Simulation

CONCEPT · HISTORICAL DEVELOPMENT · NEED FOR SIMULATION

Simulation is the most important technique used in analyzing complex real world problems which cannot be represented by a mathematical model due to their stochastic nature, complexity in their formulation and involvement of too many unknown variables. It has become an important tool for tackling the complicated managerial decision making problems. It determines the effect of a number of alternate policies without disturbing the real system.

Simulation is a modeling and analysis tool widely used for the purpose of designing, planning, and control of manufacturing systems. Simulation in general is to pretend that one deals with a real thing while really working with an imitation. In operations research, the imitation is a computer model of the simulated reality. The task of executing simulations provides insight and a deep understanding of physical processes that are being modeled.

Simulation is generally referred to as **computer simulation**, which simulates the operation of a manufacturing system. A computer simulation or a computer model is a computer program which attempts to simulate an abstract model of a particular system. Computer simulation was developed hand-in-hand with the rapid growth of the computer, following its first large-scale deployment during the

Manhattan Project in World War II to model the process of nuclear detonation. The job was done by two mathematicians **John Von Neumann and Stanislaw Ulam**.

Computer simulation is often used as an adjunct to, or substitution for, modeling systems for which simple closed form analytic solutions are not possible. There are many different types of computer simulation; the common feature they all share is the attempt to generate a sample of representative scenarios for a model in which a complete enumeration of all possible states of the model would be prohibitive or impossible.

NEED FOR SIMULATION

With a computer simulation model, a manager or system analyst is able to observe the behaviour of a process without the necessity of experimenting with the actual system. In order to evaluate the system's performance given various disturbances, or to identify the bottlenecks, they may try out different manufacturing runs, new operational conditions, new equipment layouts or different cycle times.

SIMPLE EXAMPLE OF SIMULATION

A simple example of a simulation involves the tossing of a ball into the air. The ball can be said to "simulate" a missile, for instance. That is, by experimenting with throwing balls starting at different initial heights and initial velocity vectors, it can be said that we are simulating the trajectory of a missile. This kind of simulation is known as **analog simulation** since it involves a physical model of a ball. A flight simulator on a PC is a computer model of some aspects of the flight: it shows on the screen the controls and what the "pilot" (the youngster who operates it) is supposed to see from the "cockpit" (his armchair).

⚡ Quick Revision — Section 10.1

- **Simulation:** Technique for analyzing complex real-world problems that cannot be represented by mathematical models.
- **Historical Development:** Manhattan Project in World War II — John Von Neumann and Stanislaw Ulam.
- **Purpose:** Observe behaviour of a process without experimenting with the actual system.
- **Types:** Analog simulation (physical model) and Computer simulation (computer model).

10.2 Definitions of Simulation

STANDARD DEFINITIONS FROM VARIOUS AUTHORITIES

PRIMARY DEFINITION

Simulation can be defined as "A quantitative technique that uses a computerized symbolic model in order to represent actual decision making under uncertainty for determining alternative courses of action based upon facts and assumptions".

Other Definitions

1

Representation of Reality

Simulation is a representation of reality through the use of model or other device, which will react in the same manner as reality under a given set of conditions.

2

System Model

Simulation is the use of system model that has the designed characteristics of reality in order to produce the essence of actual operation.

3

Donald G. Malcolm

Simulation model may be defined as one which depicts the working of a large scale system consisting of men, machine, materials and information, operating over a period of time in a simulated environment of the actual real world conditions.

4

Naylor

Simulation is a numerical technique for conducting experiments on a computer, which involves certain type of mathematical and logical relationships necessary to describe the behavior and structure of a complex real world system over extended period of time.

⚡ **Quick Revision — Section 10.2**

- **Simulation:** Quantitative technique using computerized symbolic model to represent actual decision making under uncertainty.
- **Donald G. Malcolm:** Depicts working of large scale system (men, machine, materials, information).
- **Naylor:** Numerical technique for conducting experiments on a computer.

10.3 Applications of Simulation

SEVEN KEY APPLICATION AREAS

1

Scheduling Aircraft

Simulation is used to schedule aircraft operations and optimize flight schedules.

2

Job-Shop Scheduling and Personnel Scheduling

Simulation helps in scheduling jobs in job-shop production and personnel scheduling in organizations.

3

Manpower-Hiring Decisions

Simulation aids in making decisions related to hiring of manpower.

4

Traffic Light-Timing

Simulation is used to optimize traffic light timing for efficient traffic flow.

5

Transport-Scheduling

Simulation helps in scheduling transport operations and logistics.

6

Evaluating Alternative Investment Opportunities

Simulation is used to evaluate different investment opportunities and their potential outcomes.

7

Design of Parking Lots, Harbour, and Communication Systems

Simulation aids in designing parking lots, harbours, and communication systems.

⚡ Quick Revision — Section 10.3

- **Seven Applications:** Scheduling aircraft, Job-shop scheduling, Manpower-hiring, Traffic light-timing, Transport-scheduling, Investment opportunities, Design of parking lots/harbour/communication systems.

10.4 Advantages of Simulation

EIGHT KEY ADVANTAGES

a

Experiment and Study Complex Interactions

Enables to experiment and study complex interactions of a system (e.g., company operations, economic policies).

b

Study Effects of Organizational Changes

Possible to study the effects of organizational environment informational changes in the operations of a system (e.g., number of stocking points, industrial policies).

c

Better Insight and Understanding

Better insight and understanding of a complex system resulting in indication for improvement.

d

Assists in Teaching and Training

Assists in teaching and training (management games).

e

New Situations and Policies Can Be Tested

New situations policies can be tested.

f

Probabilistic Features Can Be Easily Incorporated

Probabilistic features can be easily incorporated.

g

Extended or Compressed Time

A process can be studied in extended or compressed time.

h

Risks Eliminated

Risks involved in experimenting with real problems can be eliminated.

Quick Revision — Section 10.4

- **Eight Advantages:** Complex interactions, organizational changes, better insight, teaching and training, new situations, probabilistic features, extended/compressed time, risks eliminated.

10.5 Limitations of Simulation

FIVE KEY LIMITATIONS

a

Results are Not Precise

Simulated results are not precise. Unlike mathematical models, it does not give optimum solutions. At times one may not be able to assess the extent of error in a simulated result.

b

Some Situations Not Amenable

Some situations are not amenable to simulation.

c

Expensive

Simulation may be expensive needing advanced computer supports.

d

Does Not Generate Solutions

Simulation by itself does not generate solutions, but only indicates a way of evaluating solutions.

e

Long Complicated Process

It is often a long complicated process to develop a model.

Quick Revision — Section 10.5

- **Five Limitations:** Results not precise, some situations not amenable, expensive, does not generate solutions, long complicated process.

10.6 Types and Models of Simulation

ANALOGUE SIMULATION · COMPUTER SIMULATION · FOUR MODELS

Two Types of Simulation

ANALOGUE SIMULATION

- Reality is simulated in physical form
- **Example 1:** Children's Park in a city with various signals and crossing is the simulated model of city traffic.
- **Example 2:** Planetarium is the simulated model of solar system.

VS

COMPUTER SIMULATION

- For problems of complex Managerial Decision making, it is not possible to apply the method of Analogue Simulation.
- Some mathematical model is formulated and subsequently solved using high speed computers.
- The imitation is a computer model of the simulated reality.

Four Categories of Simulation Models

1

Deterministic Models

The input and output variables are **not random**. Rather they are described by exact functional relationship.

2

Probabilistic Models

For such cases, method of **random sampling** is used. The technique used for solving these is known as **Monte Carlo technique**.

3

Static Models

These models **do not take variable time** into consideration.

4

Dynamic Models

These models deal with **time varying interaction**.

⚡ Quick Revision — Section 10.6

- **Two Types:** Analogue Simulation (physical form) and Computer Simulation (mathematical model).
- **Four Models:** Deterministic (no randomness), Probabilistic (random sampling, Monte Carlo), Static (no time), Dynamic (time-varying).

10.7 Steps in Constructing and Using a Simulation Model

NINE SEQUENTIAL STEPS

All simulation projects follow the steps given below:

(i) Define the problem

Clearly define the problem to determine the objectives and constraints.

(ii) Data collection

Collect the necessary data for the simulation.

(iii) Problem analysis

Analyse the problem systematically.

(iv) Simulation model specification

Specify the simulation model — construct the appropriate model and decide which variables are to be included. Ensure that the model is a true representative of the real life problem.

(v) Model programming

Program the model using appropriate software or programming language.

(vi) Model validation

Validate the model to ensure it is accurate and reliable.

(vii) Simulation experimentation

Run the simulation experiments.

(viii) Evaluation and interpretation of simulation results

Evaluate the results and interpret them properly.

(ix) Report generation and plans for implementation of the results

Generate the report and plan for implementation of the results.

⚡ Quick Revision — Section 10.7

- **Nine Steps:** Define problem → Data collection → Problem analysis → Simulation model specification → Model programming → Model validation → Simulation experimentation → Evaluation and interpretation → Report generation and plans for implementation.

10.8 Monte Carlo Simulation

CONCEPT · TEN STEPS · APPLICATION

DEFINITION

Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, and other forecasting models. It is a Simulation technique in which **statistical distribution functions are created using a series of random numbers**. This method is generally used to solve problems which cannot be adequately represented by mathematical models or where solution of the problem is not possible by analytical method.

This method yields a solution which is very close to the optimal but not the exact optimum one. But the solution given by this method converges to the optimum as the number of simulation trials tend to infinity. This technique has derived its name from the gambling establishments of **Monte Carlo** because the samples are selected in a purely random manner.

Ten Steps to Solve a Problem by Monte Carlo Simulation

Step 1

Clearly define the problem in order to determine the objectives and constraints.

Step 2

Construct the appropriate model and decide which variables are to be included. Ensure that the model is a true representative of the real life problem.

Step 3

Specify values of variables to be tested. Supply values of input parameters and measure the output values.

Step 4

Collect the information required. Also determine the functional relationships and the types of probability distribution which are to be applied.

Step 5

Define a coding system that will correlate the factors identified in step (1) with the random numbers that will be generated for the simulation.

Step 6

Select a random number generator and create the random numbers to be used in the simulation.

Step 7

Correlate the generated random numbers with the factors identified in steps (1) & (5) above.

Step 8

Summarise and examine the results.

Step 9

Evaluate the results of the simulation and select the best course of action.

Step 10

Formulate proposals for advising the management on new policies and modify the model if necessary.

⚡ Quick Revision — Section 10.8

- **Monte Carlo Simulation:** Technique using random numbers to create statistical distribution functions.
- **Name Origin:** Gambling establishments of Monte Carlo — samples selected in purely random manner.
- **Ten Steps:** Define problem → Construct model → Specify values → Collect information → Define coding system → Select random number generator → Correlate random numbers → Summarise results → Evaluate results → Formulate proposals.
- **Solution Quality:** Very close to optimal; converges to optimum as number of trials → infinity.

10.9 Random Numbers

CONCEPT · PSEUDO RANDOM NUMBERS · CONGRUENCE METHOD

DEFINITION

Random Number is a number in a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence. Suppose we are interested in one digit numbers, that is any number from the sequence 0, 1, 2, 3, ... 9. As there are ten numbers in all, by the above mentioned concept of random numbers we have to get equally likely number from the sequence i.e., probability of generating any number from the sequence should be 1/10.

Tables of random numbers are available in different books and literatures. Usually for Simulation on computers, random numbers are generated through a computer programme itself. Such random numbers are called **Pseudo Random Numbers**. A sequence of Pseudo Random Number is not truly random because it is generated using a deterministic mathematical process. But it satisfies the statistical test for randomness.

Process of Pseudo Random Number Generation

The process starts with an initial number called **Seed** which generates the second number and from that a third number and so on. Of the various available recursive procedures, the most common one is **Congruence Method** or the **Residue Method**. It is described with the help of the expression:

CONGRUENCE METHOD FORMULA

$$r_{i+1} = (a.r_i + b) \text{ (modulo } m)$$

Where a, b and m are constants and r_i as well as r_{i+1} are i^{th} and $(i+1)^{th}$ Random numbers. The expression implies multiplication of 'a' by r_i and subsequent addition of b with it and division of the result by m. Then r_{i+1} is the remainder or residue. To begin the process of random number generation along with the values of a, b and m the value of r_0 should also be known. It may be any random number and is called **Seed**.

Three Types of Congruential Random Number Generators

1

Mixed Type

When both a and b are non-zero constants: $\mathbf{r_{i+1} = (a.r_i + b) \text{ (modulo } m)}$

2

Additive Type

When a = 1: $\mathbf{r_{i+1} = (r_i + b) \text{ (modulo } m)}$

3

Multiplicative Type

When b = 0: $\mathbf{r_{i+1} = (a.r_i) \text{ (modulo } m)}$

Illustration 1 — Generation of Pseudo Random Numbers

PROBLEM

Generate a string of 12 pseudo random numbers by Mixed type Residue Method when it is given that the values of the constants are a = 4, b = 7, m = 9 and the Seed is 1.

SOLUTION

The expression for Random Number Generator by Mixed type Residue Method is: $r_{i+1} = (a.r_i + b)$ **(modulo m)** where a = 4, b = 7, m = 9 and Seed = $r_0 = 1$

R_I	$R_{I+1} = (A.R_I + B)$ (MODULO M)	RESULT WITH RESIDUE
r_1	$r_1 = (4.1+7)/9 = 11/9$	1 with residue 2
r_2	$r_2 = (4.2+7)/9 = 15/9$	1 with residue 6
r_3	$r_3 = (4.6+7)/9 = 31/9$	3 with residue 4
r_4	$r_4 = (4.4+7)/9 = 23/9$	2 with residue 5
r_5	$r_5 = (4.5+7)/9 = 27/9$	3 with residue 0
r_6	$r_6 = (4.0+7)/9 = 7/9$	0 with residue 7
r_7	$r_7 = (4.7+7)/9 = 35/9$	3 with residue 8
r_8	$r_8 = (4.8+7)/9 = 39/9$	4 with residue 3
r_9	$r_9 = (4.3+7)/9 = 19/9$	2 with residue 1
r_{10}	$r_{10} = (4.1+7)/9 = 11/9$	1 with residue 2
r_{11}	$r_{11} = (4.2+7)/9 = 15/9$	1 with residue 6

Required string of Pseudo Random Numbers: 1, 2, 6, 4, 5, 0, 7, 8, 3, 1, 2, 6, ...

⚡ Quick Revision — Section 10.9

- **Random Number:** Number in a sequence whose probability of occurrence is same as any other number.
- **Pseudo Random Numbers:** Generated through computer programme; deterministic mathematical process; satisfies statistical test for randomness.
- **Seed:** Initial number r_0 to begin the process.
- **Congruence Method:** $r_{i+1} = (a.r_i + b)$ (modulo m).
- **Three Types:** Mixed (a, b ≠ 0), Additive (a = 1), Multiplicative (b = 0).

10.10 Illustrations & Numerical Problems

WORKED EXAMPLES WITH SOLUTIONS

Illustration 2 — Confectionery Item Demand

PROBLEM

The past data of demand per week (in '00 kgs.) of a confectionery item is given below:

DEMAND/WEEK	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the sequence of random numbers — 35, 52, 13, 90, 23, 73, 34, 57, 35, 83, 94, 56, 67, 66 generate the demand for the next 10 weeks. Also find out the average demand per week.

SOLUTION

DEMAND/ WEEK	FREQUENCY (F)	PROBABILITY (P = F / ∑F)	CUMULATIVE PROBABILITY	RANGE
0	2	.04	.04	00-03
5	11	.22	.26	04-25
10	8	.16	.42	26-41
15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
Total	**∑f = 50**	**1.00**		

WEEK	RANDOM NO.	DEMAND IN '00 KGS.
1	35	10
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
Total = 120		

Average demand per week = Total demand / No. of weeks = 120/10 = 12 ('00) Kgs.

Illustration 3 — Book Store Inventory Problem

PROBLEM

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs ₹60 and is sold for ₹80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for ₹30. From past records, the distribution of demand for this book has been obtained as follows:

DEMAND (NO. OF COPIES)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate the demand for 20 time periods (years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

Random Numbers: 14, 02, 93, 99, 18, 71, 37, 30, 12, 10, 88, 13, 00, 57, 69, 32, 18, 08, 92, 73

SOLUTION

Cost of a book = ₹60; Selling Price = ₹80; Profit per sold book = ₹20; Selling price of each unsold book = ₹30.

DEMAND (NO. OF COPIES)	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NO. RANGE
15	.05	.05	00-04
16	.08	.13	05-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99

Simulated demand for the next 20 years and calculation of number of unsold books for different inventory levels:

YEAR	RANDOM NUMBER	EXPECTED DEMAND	NO. OF BOOKS UNSOLD IF STOCK IS 16	17	18
1	14	17	—	—	1
2	02	15	1	2	3
3	93	20	—	—	—
4	99	22	—	—	—
5	18	17	—	—	1
6	71	18	—	—	—
7	37	18	—	—	—
8	30	17	—	—	1
9	12	16	—	1	2
10	10	16	—	1	2
11	88	20	—	—	—
12	13	17	—	—	1
13	00	15	1	2	3
14	57	18	—	—	—
15	69	18	—	—	—
16	32	17	—	—	1
17	18	17	—	—	1
18	08	16	—	1	2
19	92	20	—	—	—
20	73	18	—	—	—
Total			**2**	**7**	**18**

Computation of Profit:

NO. OF BOOKS IN STOCK	NO. OF BOOKS SOLD DURING 20 YEARS	PROFIT (₹)	AVERAGE PROFIT (₹/YEAR)
15	15 × 20 = 300	6000	₹300
16	(16 × 20) – 2 = 318	6300	₹315
17	(17 × 20) – 7 = 333	6450	₹322.50
18	(18 × 20) – 18 = 342	6300	₹315

Optimal Policy: As average profit is maximum (i.e ₹322.50) when there is a stock of 17 books, the optimal policy is to place an order of **17 books**.

Illustration 4 — Retailer's Receipts and Payments

PROBLEM

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information:

WEEKLY RECEIPTS (₹)	PROBABILITY	WEEKLY PAYMENTS (₹)	PROBABILITY
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is ₹8,000. What is the estimated balance at the end of the 12 week period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

FOR RECEIPTS	03	91	38	55	17	46	32	43	69	72	24	22
For Payments	61	96	30	32	03	88	48	28	88	18	71	99

SOLUTION

Range Table:

RECEIPT (₹)	PROBABILITY	CUMULATIVE PROBABILITY	RANGE	PAYMENT (₹)	PROBABILITY	CUMULATIVE PROBABILITY	RANGE
3000	0.20	0.20	00-19	4000	0.30	0.30	00-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulated values of Receipts & Payments:

WEEK	RANDOM NO. FOR RECEIPTS	EXPECTED RECEIPTS (₹)	RANDOM NO. FOR PAYMENTS	EXPECTED PAYMENTS (₹)	END OF WEEK BALANCE (₹)
Opening balance	—	—	—	—	**8000**
1	03	3000	61	6000	5000
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	**(3000)**
Total				**45000**	

- **Estimated balance at the end of 12th week:** ₹(3,000)
- **Highest weekly balance during the quarter:** ₹7,000
- **Average weekly balance for the quarter:** 45,000/12 = **₹3,750**

Illustration 5 — Village Dispensary (Queuing Simulation)

PROBLEM

Patients arriving at a village dispensary are treated by a doctor on a first-come-first-served basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the system and determine the average time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle.

Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times and between 15 and 40 to estimate the service times required by the patients.

Series 1 (Inter-arrival): 07, 21, 12, 80, 08, 03, 32, 65, 43, 74 | **Series 2 (Service):** 23, 37, 16, 28, 30, 18, 25, 34, 19, 21

SOLUTION

PATIENT NO.	INTER ARRIVAL TIME RANDOM NO.	ENTRY TIME IN TO THE QUEUE	SERVICE TIME RANDOM NO.	SERVICE START TIME	SERVICE END TIME	WAITING TIME OF PATIENT (MINUTES)	IDLE TIME OF DOCTOR (MINUTES)
1	07	8.07 A.M	23	8.07 A.M	8.30 A.M	—	7
2	21	8.28 A.M	37	8.30 A.M	9.07 A.M	2	—
3	12	8.40 A.M	16	9.07 A.M	9.23 A.M	27	—
4	80	10.00 A.M	28	10.00 A.M	10.28 A.M	—	37
5	08	10.08 A.M	30	10.28 A.M	10.58 A.M	20	—
6	03	10.11 A.M	18	10.58 A.M	11.16 A.M	47	—
7	32	10.43 A.M	25	11.16 A.M	11.41 A.M	33	—
8	65	11.48 A.M	34	11.48 A.M	12.22 P.M	—	7
9	43	12.31 P.M	19	12.31 P.M	12.50 P.M	—	9
10	74	1.45 P.M	21	1.45 P.M	2.06 P.M	—	55
			Total			**129**	**115**

Average time a patient has to be in the queue for getting service = 129/10 = **12.9 minutes**

Doctor is there in the dispensary from 8.00 A.M to 2.06 P.M ie for 6 hours & 6 minutes = 366 minutes. During this period he is idle for 115 minutes. So proportion of time the doctor is idle = 115/366 = **0.314**

Illustration 6 — Businessman's Monthly Costs and Sales Revenues

PROBLEM

A businessman is considering taking over a certain new business. Based on past information and his own knowledge of the business, he works out the probability distribution of the monthly costs and sales revenues, as given here:

COST (₹)	PROBABILITY	SALES REVENUE (₹)	PROBABILITY
17000	0.10	19000	0.10
18000	0.10	20000	0.10
19000	0.40	21000	0.20
20000	0.20	22000	0.40
21000	0.20	23000	0.15
		24000	0.05

Use the following sequences of random numbers for estimating costs and revenues. Obtain the probability distribution of the monthly net revenue.

Sequence 1 (Cost): 82, 84, 28, 82, 36, 92, 73, 91, 63, 29, 27, 26, 92, 63, 83, 02, 10, 39, 10, 10 | **Sequence 2 (Revenue):** 39, 72, 38, 29, 71, 83, 19, 72, 92, 59, 49, 39, 72, 94, 04, 92, 72, 18, 09, 00

SOLUTION

Range Table:

COST (₹)	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NO. RANGE	REVENUE (₹)	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NO. RANGE
17000	0.1	0.1	00 - 09	19000	0.1	0.1	00 - 09
18000	0.1	0.2	10 - 19	20000	0.1	0.2	10 - 19
19000	0.4	0.6	20 - 59	21000	0.2	0.4	20 - 39
20000	0.2	0.8	60 - 79	22000	0.4	0.8	40 - 79
21000	0.2	1.0	80 - 99	23000	0.15	0.95	80 - 94
				24000	0.05	1.00	95 - 99

Simulated values of Cost and Sales for the next 20 months:

MONTH	RANDOM NO. FOR COST	COST (₹)	RANDOM NO. FOR SALES	SALES (₹)	MONTHLY NET REVENUE (₹)
1	82	21000	39	21000	0
2	84	21000	72	22000	1000
3	28	19000	38	21000	2000
4	82	21000	29	21000	0
5	36	19000	71	22000	3000
6	92	21000	83	23000	2000
7	73	20000	19	20000	0
8	91	21000	72	22000	1000
9	63	20000	92	23000	3000
10	29	19000	59	22000	3000
11	27	19000	49	22000	3000
12	26	19000	39	21000	2000
13	92	21000	72	22000	1000
14	63	20000	94	23000	3000
15	83	21000	04	19000	(2000)
16	02	17000	92	23000	6000
17	10	18000	72	22000	4000
18	39	19000	18	20000	1000
19	10	18000	09	19000	1000
20	10	18000	00	19000	1000

Probability Distribution for Monthly Net Revenue:

NET REVENUE (₹)	(2000)	0	1000	2000	3000	4000	6000	TOTAL
Probability	1/20	3/20	6/20	3/20	5/20	1/20	1/20	1

10.11

Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. Which of the following is first step for performing Simulation analysis?

(a) Choose input variables.
(b) Create entities for the simulation process.
(c) Prepare a problem statement.
(d) Determine the output variables.

Answer: C

2. Which of the following are the advantages of using Modelling and Simulation?

(a) Easy to understand.
(b) Easy to test.
(c) Easy to upgrade.
(d) All of the above.

Answer: D

3. Which one of the following is not an application area of Modelling and Simulation?

(a) Military applications
(b) Designing semiconductors
(c) Telecommunications
(d) Food industry

Answer: D

4. Which of the following is the first step for developing the Simulation Model?

(a) Design the problem
(b) Identify the problem.
(c) Collect and start processing the system data
(d) Develop the model using Network diagram.

Answer: B

5. Simulation is the process of using a model to study the performance of a system.

(a) Agreeable
(b) Not agreeable.
(c) Partly agreeable
(d) Cannot comment.

Answer: A

6. Disadvantage of using Modelling and Simulation lies in the statement –

(a) Simulation requires manpower and it is a time consuming process.
(b) Simulation results are difficult to translate and only experts can understand it.
(c) Simulation is an expensive process.
(d) All of the above.

Answer: D

7. Monte Carlo Simulation gets its name from which of the following?

(a) Data collection.
(b) Model formulation
(c) Random number assignment
(d) Analysis

Answer: C

8. Select the valid reasons for using Simulation.

(a) Relationship between the variables is non-linear.
(b) Optimized solutions are obtained.
(c) Conduct experiment without disrupting the real system.
(d) Both (a) and (c)

Answer: D

9. Simulation uses logical relationship and mathematical expressions of the –

(a) Real system
(b) Computer model
(c) Performance measures
(d) Inferences

Answer: A

10. While assigning random numbers in Monte Carlo Simulation, it is –

(a) Not necessary to assign the exact range of random number interval as the probability.

(b) Necessary to develop a cumulative probability distribution.

(c) Necessary to assign the particular appropriate random numbers.

(d) All of the above.

Answer: B

11. Random numbers are used –

(a) To give random outcomes.

(b) To describe the uncertainty of the input values.

(c) To assign values to the parameters.

(d) To change the problem solution.

Answer: B

12. Simulation can keep track of several different kinds of information, including orders, inventory, financial planning.

(a) Correct

(b) Partly correct

(c) Totally incorrect

(d) None of the above

Answer: A

13. Monte Carlo Simulation ensures that –

(a) The simulated probability distribution will be the same as the actual probability distribution.

(b) Only one uncertain decision can be taken in any simulation model.

(c) Probabilities will have at most two decimal place values.

(d) Each one of the above statement is true.

Answer: A

14. What should a Project Manager do when his preliminary task estimate differs from Monte Carlo Simulation?

(a) Perform more Monte Carlo Simulation runs.

(b) Use this information to assess project uncertainty and risk and to review and possibly change crucial variables.

(c) Scrap the project as untenable.

(d) Discuss these differences at future project team meetings.

Answer: B

15. What can be expected at the end of a Monte Carlo Simulation exercise?

(a) A conservative outcome.

(b) A decision backed by number that makes the decision maker confident about moving forward.

(c) All outcomes from all possible probabilities exercise.

(d) None of the above.

Answer: B

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
C	D	D	B	A	D	C	D	A	B	B	A	A	B	B

Fill in the Blanks

SL	STATEMENT	ANSWER
1	Congruence Method is used to generate _____ numbers.	**Pseudo Random**
2	Monte Carlo Simulation is also known as _____ Simulation.	**Probability**
3	In ____ model of simulation, input and output variables are not random.	**Deterministic**
4	The expression [r_{i+1} = (a.r_i) (modulo m)] is for _____ type Congruence Random Number generator.	**Multiplicative**
5	Planetarium is an example of _____ Simulation for the solar system.	**Analogue**
6	_____ simulation is used for solving complex problem of managerial decision making.	**Computer**
7	The ever first large scale deployment of simulation was done during World War II to model the process of _____	**Nuclear detonation**
8	Aircraft _____ is an application area of simulation.	**Scheduling**
9	____ types of Congruence Methods are used to generate Pseudo Random Numbers.	**Three**
10	___ technique can be used to solve Queuing problems.	**Monte Carlo**

State True or False

SL	STATEMENT	ANSWER
1	Simulation is a modelling and analysis tool.	**True**
2	Probability of occurrence of any single digit Random number is 1/10.	**True**
3	Simulation models are designed to generate optimal solutions which can then be applied to real world situations.	**False**
4	One of the major advantage of simulation is time compression i.e the ability to study in a relatively short period of the activities which normally take months if considered in real world terms.	**True**
5	While it is powerful, Simulation is not considered to be a flexible quantitative analysis tool.	**False**
6	Simulation can use any probability distribution that the user defines. It does not require standard distributions.	**True**
7	Simulation of a business or process is generally performed by building a mathematical model to represent the process or system.	**True**
8	A major advantage of using simulation techniques is to be able to study the interactive effect of individual components or variables.	**True**
9	To simulate is to try to duplicate the features, appearance and characteristics of a real world system.	**True**
10	Simulations are called Models because they make use of mathematics.	**False**
11	Simulations are very flexible. Thus its solutions and inferences are transferable to other problems.	**False**
12	If the Monte Carlo Simulation is used then it is expected to get same result for each set of random numbers used.	**False**
13	Simulation is a numerical technique for conducting experiments on a computer which involves certain type of mathematical and logical relationship necessary to describe the behaviour of a complex real world system.	**True**
14	Calculation of cumulative probability is a step in Monte Carlo model.	**True**
15	"Some situations are not amenable to simulation" – is a limitation of simulation.	**True**

10.12 Essay Type Questions & Numerical Problems

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

"Children's Park in a city" is the example of what type of simulation? Explain.

2

"Simulation generates a near optimal solution of a real life problem related to any process"– Discuss.

3

Name five application areas of Simulation.

Essay Type Questions

1

What are the advantages of Simulation?

2

Describe the limitations of Simulation.

3

Write short notes on the methods of generation of Pseudo Random Numbers.

Comprehensive Numerical Problems

PROBLEM 1 — AUTOMOBILE PRODUCTION LINE

An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below:

PRODUCTION/DAY	PROB.	PRODUCTION/DAY	PROB.
95	0.03	101	0.15
96	0.05	102	0.10
97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20	**Total**	**1.00**

Finished cars are transported across the bay, at the end of each day, by ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped, and what will be the average number of empty space on the boat? Random numbers are given below:

20, 63, 46, 16, 45, 41, 44, 66, 87, 26, 78, 40, 29, 92, 21

ANSWER

Average number of cars waiting to be shipped = 0.4; Average number of empty spaces = 1.2

PROBLEM 2 — PETROL STATION (QUEUING SIMULATION)

After observing heavy congestion of customers over a period of time in a petrol station, Mr. Petro has decided to set up a petrol pump facility on his own in a nearby site. He has compiled statistics relating to the potential customer arrival pattern and service pattern as given below. He has also decided to evaluate the operations by using the simulation technique.

ARRIVALS — INTER-ARRIVAL TIME (MINUTES)	PROBABILITY	SERVICES — INTER-ARRIVAL TIME (MINUTES)	PROBABILITY
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume: (i) The clock starts at 8:00 hours; (ii) Only one pump is set up; (iii) The following 12 Random Numbers are to be used to depict the customer arrival pattern: 78, 26, 94, 08, 46, 63, 18, 35, 59, 12, 97 and 82; (iv) The following 12 Random Numbers are to be used to depict the service pattern: 44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37

Required: Find out (i) probability of the pump being idle, and (ii) Average time spent by a customer waiting in queue.

ANSWER

Probability of idle time of the petrol station = 0.14; Average time spent by a customer = 11.67 mins.

PROBLEM 3 — TIT-FIT SCIENTIFIC LABORATORIES

The Tit-Fit Scientific Laboratories is engaged in producing different types of high class equipment for use in science laboratories. The company has two different assembly lines to produce its most popular product 'Pressure'. The processing time for each of the assembly lines is regarded as a random variable and is described by the following distributions:

PROCESS TIME (MINUTES)	ASSEMBLY A1	ASSEMBLY A2
10	0.10	0.20
11	0.15	0.40
12	0.40	0.20
13	0.25	0.15
14	0.10	0.05

Using the following random numbers, generate data on the process times for 15 units of the item and compute the expected process time for the product. For the purpose, read the numbers vertically taking the first two digits for the processing time on assembly A1 and the last two digits for processing time on assembly A2.

4134, 8343, 3602, 7505, 7428, 7476, 1183, 9445, 0089, 3424, 4943, 1915, 5415, 0880, 9309

ANSWER

Expected process time for the product = 23.27 minutes.

PROBLEM 4 — BOOK STORE (RAMAYANA)

A book store wishes to carry 'Ramayana' in stock. Demand is probabilistic and replenishment of stock takes 2 days (i.e. if an order is placed on March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given:

DEMAND (DAILY)	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of ₹10 per order. The store also incurs a carrying cost of ₹0.50 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day.

The manager of the bookstore wishes to compare two options for his inventory decision:

- **Option A:** Order 5 books when the inventory at the beginning of the day plus order outstanding is less than 8 books.
- **Option B:** Order 8 books when the inventory at the beginning of the day plus order outstanding is less than 8.

Currently (beginning 1st day) the store has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day.

Using Monte-Carlo Simulation for 10 cycles, recommend, which option the manager should choose. The two digit random numbers are given below: **89, 34, 70, 63, 61, 81, 39, 16, 13, 73**

ANSWER

Option B is better because it has lower inventory cost.

★ Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Simulation

A quantitative technique that uses a computerized symbolic model in order to represent actual decision making under uncertainty for determining alternative courses of action based upon facts and assumptions.

Monte Carlo Simulation

A Simulation technique in which statistical distribution functions are created using a series of random numbers.

Analogue Simulation

Reality is simulated in physical form. Example: Planetarium is the simulated model of solar system.

Computer Simulation

For complex managerial decision making problems, some mathematical model is formulated and subsequently solved using high speed computers.

Deterministic Models

Input and output variables are not random; described by exact functional relationship.

Probabilistic Models

Method of random sampling is used; solved using Monte Carlo technique.

Static Models

These models do not take variable time into consideration.

Dynamic Models

These models deal with time varying interaction.

Random Number

A number in a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence.

Pseudo Random Numbers

Random numbers generated through a computer programme using a deterministic mathematical process; satisfies statistical test for randomness.

Seed

The initial number r_0 used to begin the process of random number generation.

Congruence Method

Method of generating pseudo random numbers using the expression $r_{i+1} = (a.r_i + b)$ (modulo m).

Ab Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 10

ABBREVIATION	FULL FORM
IQF	Individual Quick Freezing
MSME	Micro, Small and Medium Enterprises
FPO	Farmer Producer Organisation
r_i	i^{th} Random Number
a, b, m	Constants in Congruence Method
Modulo	Remainder or residue after division
MT	Metric Tonne
EDI	Electronic Data Interchange
KPI	Key Performance Indicators

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Network Analysis – PERT, CPM

Module 11 · Critical Path · Floats · Crashing · Resource Levelling

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	15% (with LP, Transportation, Assignment, Game Theory, Simulation)

✓ Website Publication · ✓ PDF Conversion · ✓ A4 Printing · ✓ Mobile & Desktop Reading · ✓ Self Study · ✓ Grand Revision · ✓ Final & Last Day Revision · ✓ Personal Reference Book

11

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 11 INCLUDES

Introduction · CPM and PERT · Difference between PERT and CPM · Applications · Basic Terminology related to Network · Common Errors in drawing Network diagrams · Rules for drawing Network diagram · Procedure of drawing a Network Diagram · Time Estimates and Critical Path in Network Analysis · Three Time Estimates of PERT · Probability of Project completion within a stipulated time · Crashing (Time – Cost Trade Off) · Procedure of Crashing a Project Network.

Module Learning Objectives

After studying this module, the students will be able to:

1

Develop Network Diagrams

Develop simple Network diagrams with Activities and Events.

2

Identify Critical Path

Identify Critical Path through the calculation of Earliest expected time and Latest allowable time. Compute values of Slack of events and Float of activities.

3

CPM vs PERT

Examine the situations which suggest the use of each of CPM and PERT. Estimate the probability of completion of a project within a stipulated time period.

4

Crashing & PERT Cost

Review in detail the concepts and use of PERT Cost to plan and control projects. Show the use of Crashing in the planning and control of projects.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of analysing Network to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

15%	2	3	30
GROUP WEIGHTAGE	CORE TECHNIQUES	PERT TIME ESTIMATES	MCQS
With LP, Transportation, Assignment, Game Theory, Simulation	PERT & CPM	Optimistic, Most Likely, Pessimistic	With answers

SYLLABUS STRUCTURE NOTE

Module 11 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Network Analysis (PERT & CPM) forms a crucial part of the module group carrying **15% weightage**. It introduces project management techniques used for planning, scheduling and controlling projects across construction, manufacturing, research and development, and service industries.

📖 Table of Contents — Module 11

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7. Rules for Drawing Network Diagram
8. Procedure of Drawing a Network Diagram
9. Time Estimates and Critical Path
10. Three Time Estimates of PERT
11. Probability of Project Completion
12. Crashing (Time–Cost Trade Off)
13. Resource Levelling
14. MCQs, True/False, Fill in the Blanks

15. Numerical Problems

11.1 Introduction to Network Analysis

CONCEPT · PURPOSE · PROJECT MANAGEMENT

Network analysis is the general name given to certain specific techniques which can be used for planning, management and control of project. It often acts as a network management tool for breaking down projects into components or individual activities and recording the result on a flow chart or network diagram. These results generally reveal information that is used to determine duration, resource limitations and cost estimates associated with the project.

It offers insight into what is occurring at each critical point of the network. Project management and efficient resource allocation are two critical aspects of the production and operations managers' responsibilities. Since a project is non-repetitive and temporal in nature, the mode of management differs from the usual job shop or other related types of scheduling.

PURPOSE OF NETWORK ANALYSIS

Network analysis enables us to take a systematic quantitative structural approach to the problem of managing a project through to successful completion. Since it has a graphical representation, it can be easily understood and used also by those with less technical background.

WHAT IS A NETWORK?

Network is a graphical representation of all the Activities and Events arranged in a logical and sequential order. Network analysis plays an important role in project management. A **project** is a combination of interrelated activities all of which must be executed in a certain order for its completion. **Activity** is the actual performance of the job (consumes resources — Time, human resources, money, and material). An **event** refers to start or completion of a job (does not consume any resources).

⚡ Quick Revision — Section 11.1

- **Network Analysis:** General name for techniques used for planning, management and control of project.
- **Network:** Graphical representation of Activities and Events in logical and sequential order.
- **Project:** Combination of interrelated activities that must be executed in a certain order for completion.
- **Activity:** Actual performance of job (consumes resources).
- **Event:** Start or completion of a job (does not consume resources).

11.2 CPM and PERT

CRITICAL PATH METHOD · PROGRAM EVALUATION AND REVIEW TECHNIQUE · HISTORICAL DEVELOPMENT

CPM (Critical Path Method)

CPM is a project modeling technique developed in the late 1950s by **Morgan R. Walker** of Du Pont and **James E. Kelly, Jr.** of Remington Rand. Test of the technique was done in **1958** when it was applied for the first time in construction of a Chemical Plant. Subsequently in

March 1959 it was applied for Planning, Scheduling and Controlling the activities related to Shutdown Maintenance of Du Pont's Plant in Louisville, Kentucky and a sizeable amount of reduction in non-productive working hours was observed due to its presence.

CPM is also known as **CPA (Critical Path Analysis)**. It is commonly used with all forms of projects including Construction, Aerospace and Defense, Software Development, Research Projects, Product Development, Plant Maintenance etc. In any project with interdependent activities this technique of mathematical analysis can be successfully applied.

PERT (Program Evaluation and Review Technique)

PERT was first devised in **1958 for the Polaris Submarine Missile Program of U.S Navy**. The Navy's special Projects office, entrusted with developing the Polaris Submarine Weapon system and the Fleet Ballistic Missile system, developed a statistical technique for measuring and forecasting progress in research and development programs. This technique is applied as a decision making tool designed to save time in achieving end objectives. It is of particular interest to those who are engaged in research and development programs for which time is a critical factor. This technique takes recognition of three factors which influence successful achievement of R&D Program objectives — **time, resource and technical performance specifications**.

COMMON GROUND

Both CPM and PERT are essentially **Network oriented techniques** using the same principle. Both are basically time oriented methods and lead to determination of time schedule for a project. The significant difference between two approaches is the fact that the time estimates for the different activities in CPM are assumed to be deterministic, while in PERT they are considered as probabilistic. These techniques are referred as **Project Scheduling techniques**.

⚡ Quick Revision — Section 11.2

- **CPM:** Developed by Morgan R. Walker (Du Pont) and James E. Kelly, Jr. (Remington Rand) in late 1950s; first applied in 1958 (Chemical Plant).
- **PERT:** Devised in 1958 for the Polaris Submarine Missile Program of U.S Navy.
- **Common Feature:** Both are Network oriented techniques; both are project scheduling techniques.
- **Key Difference:** CPM — deterministic time estimates; PERT — probabilistic time estimates.

11.3 Difference between PERT and CPM

COMPARATIVE ANALYSIS OF TWO PROJECT MANAGEMENT TECHNIQUES

SL	PERT	CPM
1	It is a technique for planning, scheduling & controlling of projects whose activities are subject to **uncertainty** in the performance time. Hence it is a **probabilistic model**.	It is a technique for planning, scheduling & controlling of projects whose activities are **not subjected to any uncertainty** and the performance times are fixed. Hence it is a **deterministic model**.
2	It is an **Event oriented** system.	It is an **Activity oriented** system.
3	Basically does **not differentiate** critical and non-critical activities.	**Differentiates clearly** the critical activities from the other activities.
4	Used in projects where resources (men, materials, money) are **always available** when required.	Used in projects where **overall cost is primarily important**. Therefore better utilization of resources is noticed in it.
5	Suitable for **Research and Development** projects where times cannot be predicted.	Suitable for **civil constructions**.

PERT — KEY CHARACTERISTICS

- Probabilistic model
- Event oriented
- Three time estimates per activity (a, m, b)
- Focus on time
- Suitable for R&D, new projects
- Does not consider cost directly

VS

CPM — KEY CHARACTERISTICS

- Deterministic model
- Activity oriented
- One time estimate per activity
- Focus on time-cost trade-off
- Suitable for civil construction, maintenance
- Explicitly considers cost and crashing

⚡ Quick Revision — Section 11.3

- **PERT:** Probabilistic, Event-oriented, Three time estimates, R&D projects.
- **CPM:** Deterministic, Activity-oriented, Single time estimate, Civil constructions, Cost trade-off.

11.4 Applications of Network Analysis

EIGHT MAJOR APPLICATION AREAS

i

Construction Projects

Construction of a Residential complex, Commercial Complex, Bridges, Factories, Highways etc.

ii

Complex Equipment Installation

Installation of complex Equipment in a factory.

iii

Maintenance & Overhauling

Maintenance and overhauling complicated equipment in Petro-chemical complex, Power plants, Steel plants etc.

iv

Ship & Aircraft Building

Ship building, Aircraft building etc.

v

Satellite & Missile Development

Satellite and Missile development projects.

vi

Pipe Line Projects

Installation of a pipe line project.

vii

Organising Big Events

Organizing big events like Olympic, Soccer World Cup, Big conferences etc.

viii

Plant Relocation

Shifting of manufacturing plant from one location to other.

⚡ Quick Revision — Section 11.4

- **Eight Applications:** Construction projects, Equipment installation, Maintenance & overhauling, Ship & aircraft building, Satellite & missile development, Pipe line projects, Big events, Plant relocation.

11.5 Basic Terminology Related to Network

ACTIVITY · EVENT · DUMMY · MERGE · BURST · PREDECESSOR · SUCCESSOR · CONCURRENT

1

Activity

All projects may be viewed as being composed of operations or tasks called **Activities** which require the expenditure of time and resources for their accomplishments. An Activity is depicted by a single arrow on the project network. The activity arrow is not scaled; the length of the activity arrow is only a matter of convenience and clarity and does not represent importance of time. The head of the arrow shows the sequence or flow of activities. An activity cannot begin until the completion of the preceding activity or activities.

2

Event

An **Event** represents a specific accomplishment in the project and takes place at a particular instant of time. Therefore it does not consume time or resources. An Event in a Network is a time oriented reference point that signifies the end of one activity and the beginning of another. Events are usually represented in the Network diagram as circles. The event circles are known as **Nodes**. All activity arrows must begin and end with event nodes.

3

Predecessor Activity

Activity or Activities which must be completed immediately prior to the start of another activity or activities are called **Predecessor Activity**.

4

Successor Activity

Activities that cannot be started until one or more of the other activities are completed, but immediately succeed them are called **Successor Activities**.

5

Concurrent Activity

Activities which can be accomplished simultaneously are known as **Concurrent Activities**. An activity can be a predecessor or successor to an event or it may be concurrent with one or more of the other activities.

6

Merge Event

The Event where more than one activity ends is called the **Merge Event**. In the diagram, Event 4 is a Merge Event because both the Activities C and D1 finish here.

7

Burst Event

The Event from where more than one activity starts is called the **Burst Event**. Event 2 is a Burst Event because both the Activities B and C start from here.

8

Merge and Burst Event

The Event where more than one activity finishes and also from where more than one activity starts is called **Merge and Burst Event**.

9

Dummy Activity

Sometimes in project network it is possible that two concurrent activities have same start and end events. To avoid such situation and make activities distinguishable **Dummy Activity** is introduced. As a result, each activity can be identified by unique end event. Dummy Activities consume no time or resources. As a convention Dummy Activities are represented by dashed arrows in the Network diagram.

Purpose of Dummy Activity

1

Distinguish Activities

To make activities with common starting and finishing events distinguishable.

2

Maintain Precedence

To identify and maintain the proper precedence relationship between activities that are not connected by events.

⚡ Quick Revision — Section 11.5

- **Activity:** Task consuming time and resources; represented by arrow.
- **Event:** Specific accomplishment; consumes no time or resources; represented by circle/node.
- **Predecessor:** Activity that must be completed immediately prior to start of another.
- **Successor:** Activity that cannot start until another is completed.
- **Concurrent:** Activities that can be accomplished simultaneously.
- **Merge Event:** More than one activity ends.
- **Burst Event:** More than one activity starts.
- **Dummy Activity:** Consumes no time/resources; represented by dashed arrows; used to distinguish activities or maintain precedence.

11.6 Common Errors in Drawing Network Diagrams

DANGLING · LOOPING · REDUNDANCY

1

Dangling

To disconnect an activity before the completion of all activities in a Network diagram is known as **Dangling**. In the figure below, activities (5 – 10) and (6 – 7) are not the last activities in the Network. So the diagram is incorrect and has the error of Dangling. This is seen in both AOA and AON diagrams.

2

Looping or Cycling

Drawing an endless loop in a Network is called **Looping or Cycling** error. This is seen in both AOA and AON diagrams.

3

Redundancy

Unnecessary insertion of Dummy Activity in a Network diagram is known as the error of **Redundancy**. This type of error is seen only in AOA diagrams.

NOTE ON DIAGRAM TYPES

There can be two types of Network diagrams — **AOA (Activity on Arrow)** and **AON (Activity on Node)** diagrams. As the name implies, AOA diagrams have activities represented by arrows and AON diagrams have activities represented by nodes or circles. Though arrows are used in AON diagrams also, they are meant only for representing the logical relationships going from predecessor to successor.

⚡ Quick Revision — Section 11.6

- **Dangling:** Disconnecting an activity before completion; seen in both AOA and AON.
- **Looping/Cycling:** Drawing an endless loop; seen in both AOA and AON.
- **Redundancy:** Unnecessary insertion of Dummy Activity; seen only in AOA diagrams.

11.7 Rules for Drawing Network Diagram

FIFTEEN ESSENTIAL RULES

In a network diagram, arrows represent the activities and circles represent the events.

i

Tail and Head Events

The tail of an arrow represents the start of an activity and the head represents the completion of the activity. Event at the tail end of the activity is called **Tail Event** and the one at the head end is called **Head Event**.

ii

Numbering of Events

Head events always have number higher than the Tail events. Thus for the Activity (i – j) it goes without saying i < j always.

iii

Left-to-Right Flow

Time flows from left to right of the Network. Thus no Activity of the Network should have arrowhead pointing towards left.

iv

No Crossing

Activity arrows should not be crossed unless it is completely unavoidable.

v

Straight Arrows

Activity arrows should be kept straight and not curved or bent.

vi

Logical Precedence Only

The arrows depicting various activities are indicative of the logical precedence only. The length and bearing of the arrows are of no significance.

vii

Initial Event

The event number 1 denotes the start of the project and is called **initial event**.

viii

Terminal Event

Event carrying the highest number in the network denotes the completion of the project and is called **terminal event**.

ix

One Arrow per Activity

Each defined activity is represented by one and only one arrow in the network.

x

Unique Events

No two activities can be identified by the same beginning and end events. In such cases, a dummy activity should be introduced to resolve the issue.

xi

Immediate Predecessors

Determine which operation must be completed immediately before other can start.

xii

Following Operations

Determine which other operation must follow the other given operation.

xiii

Dependencies

The network should be developed on the basis of logical, analytical and technical dependencies between various activities of the project.

xiv

Avoid Dangling

Dangling must be avoided in a Network diagram. This happens when precedence and inter relationship of the activities are not properly identified. This is an error.

xv

Avoid Redundancy

Unnecessary Dummy Activities must not be drawn while preparing a Network diagram. This is an error known as Redundancy.

⚡ Quick Revision — Section 11.7

- **Tail and Head Events:** Start and completion of activity.
- **i < j always** for Activity (i – j).
- **Time flows left to right.**
- **Initial Event:** Number 1; **Terminal Event:** Highest number.
- **Avoid errors:** Dangling (disconnected activity), Looping (endless loop), Redundancy (unnecessary dummy).

11.8 Procedure of Drawing a Network Diagram

FIVE SEQUENTIAL STEPS

A project consists of tasks with definite starting and ultimate ending points and hence a project manager is saddled with the responsibilities of getting job done on schedule within allowable cost and time constraint specified by the management. Typically all projects can be broadly broken into:

- **(a) Separate activities** — where each activity has a duration (time from the start of the activity to its finish).
- **(b) Precedence relationships** — which govern the order in which the activities are to be performed.

The main problem is to bring all these activities together in a coherent fashion to complete the project at the stipulated time.

Five Steps of Drawing a Network Diagram

Step 1 — Specify the Individual Activities

From the work breakdown structure, a listing can be made of all the activities in the project. This listing can be used as the basis for adding sequence and duration information in later steps.

Step 2 — Determine the Sequence of the Activities

Some activities are dependent on the completion of others. A listing of the immediate predecessors of each activity is useful for constructing the CPM network diagram.

Step 3 — Draw the Network Diagram

Once the activities and their sequencing have been defined, the CPM diagram can be drawn. CPM originally was developed as an activity on node (AON) network, but some project planners prefer to specify the activities on the arrows.

Step 4 — Estimate Activity Completion Time

The time required to complete each activity can be estimated using past experience or the estimates of knowledgeable persons. CPM is a deterministic model that does not take into account variation in the completion time, so only one number is used for an activity's time estimate.

Step 5 — Identify the Critical Path

The critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning. The critical path can be identified by determining the four parameters for each activity: Earliest Start, Earliest Finish, Latest Finish and Latest Start.

⚡ Quick Revision — Section 11.8

- **Five Steps:** Specify activities → Determine sequence → Draw network diagram → Estimate time → Identify critical path.
- **Four Parameters:** Earliest Start (ES), Earliest Finish (EF), Latest Finish (LF), Latest Start (LS).

11.9 Time Estimates and Critical Path in Network Analysis

FORWARD PASS · BACKWARD PASS · SLACK · FLOAT · CRITICAL PATH

Time estimates are extremely important for planning various activities of a project. Activity time is a forecast of the time an activity is expected to take from its start to finish under normal conditions. In fact estimation of activity time is rather difficult and requires lot of hands on experience for a specific type of job. Thus an assumption is made that activity times are available in time units. These values are put in the Network diagram either on the top or the bottom of the Activity arrow.

Objectives of Time Analysis

1

Completion Time for Project

Determine the total time required to complete the project.

2

Earliest Start Time

Determine the earliest time when each activity can begin.

3

Latest Start Time

Determine the latest time when each activity can begin without delaying the project.

4

Float for Each Activity

Determine the amount of time by which the completion of an activity can be delayed without delaying the project completion.

5

Identification of Critical Activities

Identify critical activities and critical path.

Notations Used

NOTATION	MEANING
(i, j)	Activity (i, j) with tail end number i and head end number j
E_i	Earliest occurrence time of event i
L_j	Latest allowable occurrence time of event j
t_{ij}	Time estimate of activity (i, j)
ES_{ij}	Earliest Starting time of activity (i, j)
EF_{ij}	Earliest Finish time of activity (i, j)
LS_{ij}	Latest Starting time of activity (i, j)
LF_{ij}	Latest Finish time of activity (i, j)

(A) Forward Pass — For Earliest Event Times

CONCEPT

Computation using this concept yields the Earliest Start and Earliest Finish timings for each activity as well as the Earliest expected occurrence time for each event.

FORWARD PASS FORMULAE

$ES_{ij} = E_i$ (Earliest event time of Tail event)

$EF_{ij} = ES_{ij} + t_{ij}$

E_j = Maximum of all $(ES_{ij} + t_{ij})$ values (for Merge Events)

The computations begin from the START Node and move forward to complete at the END Node. An assumption is made that the earliest occurrence time for the Start Node is zero ($E_1 = 0$).

(B) Backward Pass — For Latest Allowable Event Times

CONCEPT

The Latest event time (L) means the time by which all activities merging into an event must be completed without delaying the total project. These are computed by reversing the method of calculation used for earliest event times.

BACKWARD PASS FORMULAE

$LF_{ij} = L_j$ (Latest Event time for j)

$LS_{ij} = LF_{ij} - t_{ij}$

L_i = Minimum of all $(LF_{ij} - t_{ij})$ values (for Burst Events)

The computations begin from the END Node and move backward to complete at the START Node. Assumption: L = E for the End Node.

(C) Slack of an Event

SLACK OF AN EVENT

Slack of the Event $i = L_i - E_i$

For activity (i, j): **Head Slack** = $L_j - E_j$ and **Tail Slack** = $L_i - E_i$. An Event with Zero Slack is known as **Critical Event**.

(D) Float of an Activity

There can be three types of Floats for an Activity:

1

Total Float

Amount of time by which completion of an activity can be delayed beyond the earliest expected completion time without affecting the project duration.

$$\text{Total Float } (TF_{ij}) = LS_{ij} - ES_{ij} = (L_j - E_i) - t_{ij}$$

2

Free Float

Time by which an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity.

$$\text{Free Float } (FF_{ij}) = (E_j - E_i) - t_{ij} = \text{Total Float} - \text{Head Slack}$$

3

Independent Float

Amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activity, assuming that the preceding activity has finished at its latest time.

$$\text{Independent Float } (IF_{ij}) = (E_j - L_i) - t_{ij} = \text{Free Float} - \text{Tail Slack}$$

IMPORTANT RELATIONSHIPS

1. Independent Float ≤ Free Float ≤ Total Float
2. Knowledge of Floats helps in understanding underutilized resources and flexibility of the schedule.
3. Float can be used for redeployment of resources to level the same or to reduce project duration.
4. Basic difference: **Slack** is used for **Events** and **Float** is used for **Activities**.

(E) Critical Path

A Path in a Network diagram refers to the sequence of activities such that it begins at the starting event and ends at the final event. The length or duration of a path is the sum of the durations of each individual activity lying on the path.

The sequence of critical activities in a Network diagram is called **Critical Path**. The Critical Path is the **longest path** or the path of longest duration in the network from the starting event to the ending event. In other words, it defines the minimum time required to complete the project. If the activities on Critical Path are delayed, the project will also be delayed by the same amount of time unless the durations of the future critical activities are reduced. The critical path of a network is denoted by darker or double lines to distinguish it from the non-critical paths.

KEY FEATURES OF CRITICAL PATH

1. If the duration of a project is to be shortened, duration of some of the activities on the critical path must be shortened. Induction of additional resources will not give the desired result unless the critical path is shortened first.
2. The variation in actual performance time from the expected activity duration will be completely reflected in one-to-one fashion in the anticipated completion of the project.

(F) Representation of an Event in a Network Diagram

Event or Node of a project is denoted by a circle in a Network diagram. In general, the circle is divided into three parts — a semicircle and two quadrants for the purpose of its representation in the Network. The space of the semicircle is used for writing Event or Node Number and those of the two quadrants are used for writing the Earliest and the Latest Event times for the particular event.



Node Number: 3 | **Earliest Event Time:** 14 days | **Latest Event Time:** 16 days | **Slack:** 2 days (difference between EST and LST)

⚡ Quick Revision — Section 11.9

- **Forward Pass:** Computes Earliest Event Times ($E_1 = 0$; E_j = Max of $ES_{ij} + t_{ij}$).
- **Backward Pass:** Computes Latest Allowable Event Times ($L = E$ for End Node; L_i = Min of $LF_{ij} - t_{ij}$).
- **Slack:** $L_i - E_i$; Zero Slack = Critical Event.
- **Total Float:** $LS - ES = (L_j - E_i) - t_{ij}$.
- **Free Float:** $(E_j - E_i) - t_{ij}$ = Total Float – Head Slack.
- **Independent Float:** $(E_j - L_i) - t_{ij}$ = Free Float – Tail Slack.
- **Relationships:** Independent Float ≤ Free Float ≤ Total Float.
- **Critical Path:** Longest path; determines minimum project duration.

11.10 Illustration 1 — Making of a Loaf (Critical Path Analysis)

COMPREHENSIVE EXAMPLE WITH FORWARD PASS, BACKWARD PASS AND FLOATS

PROBLEM

The following table gives the activities and other relevant information related to "Making of a loaf".

ACTIVITY	PRECEDED BY	ELAPSED TIME (MINUTES)
A - Weigh ingredients	—	1
B - Mix ingredients	A	3
C - Dough rising time	B	60
D - Prepare tins	—	1
E - Pre-heat oven	—	10
F - Knock back dough and place in tins	C&D	2
G - 2nd dough rising time	F	15
H - Cooking time	E & G	45

Required: Draw a Network diagram. Also find the Earliest and Latest Times of each Event of the Network. Identify the different paths of the Network and their corresponding durations. Which path is critical? Find the time required to complete the job.

SOLUTION

The Earliest expected Time and the Latest allowable Time of each event is determined by using the methods of Forward Pass and Backward Pass respectively.

Forward Pass — Earliest Event Times

Formula: $E_j = E_i + t_{ij}$ & for Merge Events $E_j = \text{Max}\ (E_i + t_{ij})$

- $E_1 = 0$
- $E_2 = E_1 + t_{12} = 0 + 1 = \mathbf{1}$
- $E_3 = E_2 + t_{23} = 1 + 3 = \mathbf{4}$
- $E_4 = \text{Max}\ [(E_3 + t_{34}), (E_1 + t_{14})] = \text{Max}\ [(4 + 60), (0 + 1)] = \mathbf{64}$
- $E_5 = E_4 + t_{45} = 64 + 2 = \mathbf{66}$
- $E_6 = \text{Max}\ [(E_5 + t_{56}), (E_1 + t_{16})] = \text{Max}\ [(66 + 15), (0 + 10)] = \text{Max}\ [81, 10] = \mathbf{81}$
- $E_7 = E_6 + t_{67} = 81 + 45 = \mathbf{126}$

Backward Pass — Latest Allowable Event Times

Formula: $L_i = L_j - t_{ij}$ & for Burst Events $L_i = \text{Min}\ (L_j - t_{ij})$

- $L_7 = E_7 = \mathbf{126}$
- $L_6 = L_7 - t_{67} = 126 - 45 = \mathbf{81}$
- $L_5 = L_6 - t_{56} = 81 - 15 = \mathbf{66}$
- $L_4 = L_5 - t_{45} = 66 - 2 = \mathbf{64}$
- $L_3 = L_4 - t_{34} = 64 - 60 = \mathbf{4}$
- $L_2 = L_3 - t_{23} = 4 - 3 = \mathbf{1}$
- $L_1 = \text{Min}\ [(L_2 - t_{12}), (L_4 - t_{14})\ \&\ (L_6 - t_{16})] = \text{Min}\ [(1–1), (64–1)\ \&\ (81–10)] = \mathbf{0}$

Different Paths of the Network and Their Durations

SL	PATH	DURATION (MINUTES)
1	A – B – C – F – G – H (or 1–2–3–4–5–6–7)	**1+3+60+2+15+45 = 126**
2	D – F – G – H (or 1–4–5–6–7)	1+2+15+45 = 63
3	E – H (or 1–6–7)	10+45 = 55

Of the above three, the duration of the path **A – B – C – F – G – H is maximum (126 minutes)**. So this is the **CRITICAL PATH**. Time required to make the loaf is **126 Minutes**.

Note: In this example, there is a clear sequence of events that have to happen in the right order. If any of the events on the critical path is delayed, then the bread will not be ready as soon. However, tasks D (prepare tins) and E (heat the oven) can be started at any time as long as they are done by the latest time in the following node. So, we can see that the oven could be switched on as early as time 0, but we can work out that it could be switched on at any time before 71 – any later than this and it won't be hot enough when the dough is ready for cooking. There is some 'float' available for tasks D and E as neither is on the critical path.

Illustration 2 — Floats for Making of a Loaf

PROBLEM

Using the above information, compute Total, Free and Independent Floats for each Activity.

SOLUTION

ACTIVITY (I – J)	DURATION (MINUTES)	EARLIEST START	EARLIEST FINISH	LATEST START	LATEST FINISH	HEAD SLACK	TAIL SLACK	TOTAL FLOAT	FREE FLOAT	INDEPENDENT FLOAT
A (1-2)	1	0	1	0	1	0	0	**0**	0	0
B (2-3)	3	1	4	1	4	0	0	**0**	0	0
C (3-4)	60	4	64	4	64	0	0	**0**	0	0
D (1-4)	1	0	1	63	64	0	0	63	63	63
E (1-6)	10	0	10	71	81	0	0	71	71	71
F (4-5)	2	64	66	64	66	0	0	**0**	0	0
G (5-6)	15	66	81	66	81	0	0	**0**	0	0
H (6-7)	45	81	126	81	126	0	0	**0**	0	0

Interpretation: Activities D and E are having positive Floats which outright indicate that the activities are non-critical. Thus there is some cushion for both these activities and they can be delayed without affecting the ultimate completion time of the job i.e. 126 Minutes.

Illustration 3 — XYZ Auto-Manufacturing Co.

PROBLEM

XYZ Auto-manufacturing Co. has to prepare a design of its latest model of motorcycle. The various activities to be performed to prepare design are given in the following table: Prepare a Network diagram.

ACTIVITY	DESCRIPTION OF ACTIVITY	PRECEDING ACTIVITY
A	Prepare drawing	—
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D, E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I, H
K	Testing and shipment	J

Required: Prepare a Network diagram.

SOLUTION

The Network Diagram with two Dummies D1 & D2 is required because:

- Activities B and C both have A as predecessor but are concurrent — need to be distinguishable.
- Activity E has both B and C as predecessors while D has only C as predecessor.
- Activities I and H are concurrent, and J depends on both.

Dummy activities D1 and D2 ensure proper precedence relationships and unique end events for each activity.

⚡ Quick Revision — Section 11.10

- **Forward Pass:** $E_1 = 0$; $E_j = \text{Max}\ (E_i + t_{ij})$.
- **Backward Pass:** $L_n = E_n$; $L_i = \text{Min}\ (L_j - t_{ij})$.
- **Critical Path:** Longest path; zero slack throughout.
- **Slack:** Difference between Latest and Earliest Event times.
- **Total Float:** $(L_j - E_i) - t_{ij}$.
- **Free Float:** $(E_j - E_i) - t_{ij}$.
- **Independent Float:** $(E_j - L_i) - t_{ij}$.

11.11 Three Time Estimates of PERT

OPTIMISTIC · MOST LIKELY · PESSIMISTIC · EXPECTED TIME · VARIANCE

In Critical Path Method or CPM, the Activity durations are assumed to be known with certainty. But for certain projects like R & D Projects or Projects related to New Product introduction etc. it is not possible to know the Activity durations with certainty due to involvement of various uncertainties. In such cases Program Evaluation and Review Technique or PERT is used where help of three time estimates are taken for each activity to subsequently get the Expected Time for each activity.

The Three Time Estimates

a

Optimistic Time

It is that time estimate of an activity when everything is assumed to go well as per plan. In other words it is the estimate of minimum possible time, which an activity takes for completion under ideal conditions. It is denoted as **'a'** or $\mathbf{t_o}$.

m

Most Likely Time

It is the time which the activity will take most of the time if performed number of times. In other words it is the modal value. This is denoted by **'m'** or $\mathbf{t_m}$.

b

Pessimistic Time

It is the unlikely but possible performance time if whatever could go wrong, goes wrong in series. In other words it is the maximum possible time which an activity can conceivably take under totally unfavorable conditions. It is denoted as **'b'** or $\mathbf{t_p}$.

BETA DISTRIBUTION ASSUMPTION

Above mentioned time estimates of an Activity follows **Beta (β) Probability Distribution**. It is assumed that the three time estimates form the end points and mode of Beta Distribution. It is further assumed that 'b' and 'a' are equally likely to occur whereas the probability of occurrence of 'm' is 4 times that of 'b' and 'a'.

EXPECTED TIME AND VARIANCE FORMULAE

$$\text{Expected Time: } t_E = (a + 4m + b) / 6$$

$$\text{Standard Deviation of Activity: } \sigma = (b - a) / 6$$

$$\text{Variance of Activity: } \sigma^2 = [(b - a) / 6]^2$$

The Expected Time (t_E) divides the area under the curve into two equal parts. Hence it provides the time in which there is a **50 – 50 chance** of the activity being completed. This multiple time estimate approach is better since it provides quite often the improved estimates of three expected time to complete an activity and also allows to consider the variability of the time for completion of an activity.

Formulae for Project Completion Probability

PROJECT DURATION — MEAN AND VARIANCE

$$\text{Mean of Project Time } (T_E) = \Sigma\ (t_E) \text{ of all activities on Critical Path}$$

$$\text{Variance of Project } (\sigma_T^2) = \Sigma\ (\sigma^2) \text{ of all activities on Critical Path}$$

$$\text{Standard Normal Variable: } Z = (t_S - T_E) / \sigma_T$$

Where t_S = Stipulated time. If there is more than one Critical Path in the Network, calculate Variance for each of the Critical Path and consider that value of the Variance which is **greatest**.

COMMONLY USED Z VALUES AND PROBABILITIES

- P (Z ≤ 1.645) = 95%
- P (Z ≤ 1.96) = 97.5%
- P (Z ≤ 2.33) = 99%
- P (Z ≤ 3) = 99.9%
- P (Z ≤ 0) = 50%

⚡ Quick Revision — Section 11.11

- **Three Time Estimates:** Optimistic (a), Most Likely (m), Pessimistic (b).
- **Expected Time:** $t_E = (a + 4m + b) / 6$.
- **Standard Deviation:** $\sigma = (b - a) / 6$.
- **Variance:** $\sigma^2 = [(b - a) / 6]^2$.
- **Project Mean:** Sum of Expected Times on Critical Path.
- **Project Variance:** Sum of Variances on Critical Path.
- **Z Value:** $Z = (t_S - T_E) / \sigma_T$.
- **Key Probabilities:** 95% (Z = 1.645), 97.5% (Z = 1.96), 99% (Z = 2.33).

11.12 Illustration 4 — Probability of Project Completion

CIVIL ENGINEERING FIRM — BID FOR CONSTRUCTION OF A DAM

PROBLEM

A civil engineering firm has to bid for the construction of a dam. The activities and time estimates are given below:

ACTIVITY	OPTIMISTIC	MOST LIKELY	PESSIMISTIC
1—2	14	17	25
2—3	14	18	21
2—4	13	15	18
2—8	16	19	28
3—4 (dummy)	—	—	—
3—5	15	18	27
4—6	13	17	21
5—7 (dummy)	—	—	—
5—9	14	18	20
6—7 (dummy)	—	—	—
6—8 (dummy)	—	—	—
7—9	16	20	41
8—9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% probability of at best breaking even. The fixed costs for the project are ₹8 lakhs and the variable costs are ₹9,000 every day spent working on the project. The duration is in days and the costs are in terms of rupees.

Required: What amount should the firm bid under this policy? (Perform calculations on duration etc. up to two decimal places.)

SOLUTION — EXPECTED DURATIONS AND VARIANCES

Formulae: $t_E = (a+4m+b)/6$ and $\sigma^2 = [(b - a)/6]^2$

ACTIVITY	OPTIMISTIC (A)	MOST LIKELY (M)	PESSIMISTIC (B)	EXPECTED DURATION (T_E DAYS)	VARIANCE (Σ^2 SQ. DAYS)
1—2	14	17	25	17.83	3.36
2—3	14	18	21	17.83	1.36
2—4	13	15	18	15.17	0.69
2—8	16	19	28	20	4
3—4 (dummy)	0	0	0	0	0
3—5	15	18	27	19	4
4—6	13	17	21	17	1.78
5—7 (dummy)	0	0	0	0	0
5—9	14	18	20	17.67	1
6—7 (dummy)	0	0	0	0	0
6—8 (dummy)	0	0	0	0	0
7—9	16	20	41	22.83	17.36
8—9	14	16	22	16.67	1.78

Various Paths of the Network and Their Corresponding Lengths

SL.	PATH	DURATION (DAYS)
I.	1-2-3-5-7-9	**77.49***
II.	1-2-3-5-9	72.33
III.	1-2-3-4-6-7-9	75.49
IV.	1-2-3-4-6-8-9	69.33
V.	1-2-8-9	54.50
VI.	1-2-4-6-8-9	66.67
VII.	1-2-4-6-7-9	72.83

From above, the longest path or the path of longest duration of **77.49 Days is 1-2-3-5-7-9**. So it is the **Critical Path**.

Variance of duration: σ^2 = Sum of the Variances of the Critical activities

= $(\sigma_{1\text{-}2})^2 + (\sigma_{2\text{-}3})^2 + (\sigma_{3\text{-}5})^2 + (\sigma_{5\text{-}7})^2 + (\sigma_{7\text{-}9})^2$ = 3.36 + 1.36 + 4 + 0 + 17.36 = **26.08 Sq. Days**

S.D of Project duration: $\sigma = \sqrt{26.08}$ = **5.11 Days**

To calculate the project duration which will have 95% chance of completion, we find the value of Z corresponding to 95% area under the Standard Normal curve and this value is **1.645**.

Therefore P (Project duration $\leq t_S$) = 95% Or, P [$Z \leq (t_S - 77.49)/5.11$] = P ($Z \leq 1.645$)

So, $(t_S - 77.49)/5.11 = 1.645$ Or, t_S = **86 Days**

Hence the company should bid considering project completion time to be **86 Days**.

Amount to bid = Fixed Cost + Variable Cost (@ ₹9000 per Day) = ₹8,00,000 + ₹9,000 × 86 = **₹15,74,000**

⚡ Quick Revision — Section 11.12

- **Critical Path:** 1-2-3-5-7-9 with duration 77.49 days.
- **Project Variance:** 26.08 Sq. Days; S.D = 5.11 Days.
- **95% Probability Z value:** 1.645.
- **Project duration for 95% confidence:** 86 Days.
- **Bid Amount:** ₹15,74,000.

11.13 Crashing (Time – Cost Trade Off)

DIRECT COST · INDIRECT COST · COST SLOPE · OPTIMUM DURATION

DEFINITION OF CRASHING

Crashing of a Project Network means intentionally reducing the duration of project by allocating more resources to it. Allocation of more resources apparently means increase in project cost. But this may not be true always and a detail study of relationship of Duration (or Time) and Cost of different activity is necessary to understand it.

Two Types of Costs Associated with an Activity

DIRECT COSTS

- Can be identified with the Activity
- Examples: Cost of Materials, Machinery used in the project, payments made to Labors and Sub contractors
- Whenever more resources are allocated to an Activity, the Direct Costs are going to increase
- **Crashing an Activity increases Direct Cost**

VS

INDIRECT COSTS

- Cannot be identified with the Activity
- Examples: Overheads, Office Expenses, Administrative Costs, Penalty for delay in delivery of project
- Indirect Costs decrease as project duration decreases
- **Crashing reduces Indirect Cost**

Important Elements of the Time–Cost Curve

1

Normal Time (t_n)

It is the standard time an Activity takes for its completion under normal working conditions.

2

Crash Time (t_C)

It is the minimum time that an Activity requires for its completion.

3

Normal Cost (C_n)

It is the direct cost required to complete the Activity in Normal Time.

4

Crash Cost (C_C)

It is the direct cost to be incurred by the Activity for completion in Crash Time.

COST SLOPE

$$\text{Cost Slope} = (\text{Crash cost} - \text{Normal cost}) \div (\text{Normal time} - \text{Crash time})$$

For keeping the calculations simple, linear approximation of the curve is made. Slope of the straight line is sufficient to define the curve. This slope is known as **Cost Slope**.

TOTAL PROJECT COST RELATIONSHIP

Total Direct Cost of the project can be determined by adding the Direct Costs of each individual activity. When total Indirect Cost is also known then addition of Total Direct and Total Indirect Costs lead to **Total Project Cost**. At the initial stages of Crashing there is a reduction in Total Project Cost. This happens because rate of decrease in Indirect Cost due to reduction in Project Duration is more than the rate of increase in Direct Cost at this phase. However after a certain point of time there is a reversal of scenario and the Total Project Cost is observed to be increasing again. The duration corresponding to which Total Project Cost is minimum is known as **Optimum Duration** of the Project. Crashing is always done up to this optimum point.

Procedure of Crashing a Project Network

Step 1

Draw the Network diagram with the Normal Activity Times.

Step 2

Identify the Critical Path.

Step 3

Compute Cost Slope of different critical activities using the formula written above.

Step 4

Rank the activities of the Critical Path in ascending order of Cost Slope.

Step 5

Crash the activities in the Critical Path following the rule — **"Lowest one first"** i.e., Critical Activity having lowest ranking should be crashed first to the maximum extent possible. The Activity must not be crashed to that extent which will make the Critical Path non-critical.

At the most along with the existing Critical Path some other path can be allowed to become critical. Calculate new Total Cost by adding Cost of Crashing to the cumulative Normal Cost.

Step 6

Once more than one path becomes Critical, **parallel crashing** of activities in all the critical paths will become necessary.

Step 7

Repeat the above step of Crashing until no further crashing is possible or no further reduction in Total Cost is possible.

Illustration 5 — Crashing a Project Network

PROBLEM

The following table gives data on normal time & cost as well as crash time & cost for a project. Indirect Cost is ₹100 per day.

ACTIVITY	NORMAL TIME (DAYS)	NORMAL COST (₹)	CRASH TIME (DAYS)	CRASH COST (₹)
1—2	6	600	4	1,000
1—3	4	600	2	2,000
2—4	5	500	3	1,500
2—5	3	450	1	650
3—4	6	900	4	2,000
4—6	8	800	4	3,000
5—6	4	400	2	1,000
6—7	3	450	2	800

Required: Draw the Network diagram and identify the Critical Path. Find the Normal duration of the project and the corresponding Total Cost. Crash the relevant activities systematically and determine the optimum completion time of the project.

SOLUTION — NORMAL DURATION AND COST

The network for normal activity times indicates project duration of **22 days** with critical path **1-2-4-6-7**.

Normal Direct Cost = (600 + 600 + 500 + 450 + 900 + 800 + 400 + 450) = **₹4,700**

Indirect Cost = 22 × 100 = **₹2,200**

Total Cost = 4,700 + 2,200 = ₹6,900

Cost Slopes of Critical Activities

CRITICAL ACTIVITY	COST SLOPE = (CRASH COST – NORMAL COST) ÷ (NORMAL TIME – CRASH TIME)	RANK (ASCENDING)
1 – 2	(1000 – 600) / (6 – 4) = ₹200 per day	1
2 – 4	(1500 – 500) / (5 – 3) = ₹500 per day	3
4 – 6	(3000 – 800) / (8 – 4) = ₹550 per day	4
6 – 7	(800 – 450) / (3 – 2) = ₹350 per day	2

1st Stage of Crashing: As Cost Slope of Activity 1 – 2 is minimum, crashing is started from this Activity. Activity 1 – 2 is crashed by 1 Day.

New Duration = 21 Days; Total Cost = 4700 + 21×100 + 1×200 = **₹7,000**

Now there are two Critical Paths: 1-2-4-6-7 and 1-3-4-6-7

2nd Stage of Crashing: Cost Slopes of new critical activities: 1–3 = (2000 – 600)/(4 – 2) = ₹700/day; 3–4 = (2000 – 900)/(6 – 4) = ₹550/day

Parallel Crashing options are evaluated. Lowest cost slope option is Activity (6-7) at ₹350/day. Crashed by 1 Day.

New Duration = 20 Days; Total Cost = 4700 + 20×100 + 200 + 350 = **₹7,250**

3rd Stage of Crashing: Activity (4 – 6) is having lowest Cost Slope (₹550/day). Crashed by 4 days.

New Duration = 16 Days; Total Cost = 4700 + 1600 + 200 + 350 + 550×4 = **₹9,050**

4th Stage of Crashing: Activities (1 – 2) and (3 – 4) together crashed by 1 day (cost slope ₹750/day).

New Duration = 15 Days; Total Cost = 4700 + 1500 + 200 + 350 + 2200 + 750 = **₹9,700**

5th Stage of Crashing: Activities (2 – 4) and (3 – 4) together crashed by 1 day (cost slope ₹1,050/day).

New Duration = 14 Days; Total Cost = 4700 + 1400 + 200 + 350 + 2200 + 750 + 1050 = **₹10,650**

6th Stage of Crashing: Activities (2 – 4) and (1 – 3) together crashed by 1 day (cost slope ₹1,200/day).

New Duration = 13 Days; Total Cost = 4700 + 1300 + 200 + 350 + 2200 + 750 + 1050 + 1200 = **₹11,750**

Observation: The Total Cost of the Project kept on increasing all along. This has happened due to the fact that the rate of decrease of Indirect Cost is much lower than the rate of increase of Direct Cost for Crashing. Hence optimum duration of the project cannot be obtained and rather minimum possible duration is obtained and that value is **13 Days**. Associated Total Cost of project is **₹11,750**.

⚡ Quick Revision — Section 11.13

- **Crashing:** Intentionally reducing project duration by allocating more resources.
- **Direct Costs:** Increase with crashing.
- **Indirect Costs:** Decrease with crashing.
- **Cost Slope:** (Crash cost – Normal cost) ÷ (Normal time – Crash time).
- **Optimum Duration:** Duration at which Total Project Cost is minimum.
- **Procedure:** Start with lowest cost slope critical activity; crash to maximum extent; parallel crashing when multiple critical paths emerge.

11.14 Resource Levelling

SMOOTHING MANPOWER REQUIREMENTS

CONCEPT

Resource Levelling is the process of scheduling activities in a project such that the peak manpower requirement is minimized and the period-to-period variation of manpower requirement is smoothed out. It is used to obtain a schedule which will minimize the peak manpower requirement and smooth out period to period variation of manpower requirement.

Illustration 6 — Resource Levelling

PROBLEM

Consider the following problem of project scheduling to obtain a schedule which will minimize the peak manpower requirement and smooth out period to period variation of manpower requirement.

ACTIVITY	DURATION (DAYS)	MANPOWER REQUIREMENT
1-2	6	8
1-3	10	4
1-4	6	9
2-3	10	7
2-4	4	6
3-5	6	17
4-5	6	6

SOLUTION

Step 1 — Identify the Critical Path: Critical path = 1-2-3-5

Step 2 — Compute Earliest and Latest Times: Presented in the Nodes of the Network Diagram.

Step 3 — Compute Manpower Requirement Histogram: The peak manpower requirement is **21** and it occurs between 0-6 weeks. The activities which are scheduled during this period are (1 – 2), (1 – 3) and (1 – 4).

Step 4 — Apply Resource Levelling: Activity (1 – 2) is a Critical Activity, so it should not be disturbed. Between activities (1 – 3) and (1 – 4), the activity (1 – 3) has a Total Float of 6 weeks. So it has to be delayed to the maximum possible extent (i.e., it can be started at the end of the 6th week).

Result: The manpower requirement is now balanced/smoothened throughout the project duration at **17 workers** per period.

⚡ Quick Revision — Section 11.14

- **Resource Levelling:** Scheduling activities to minimize peak manpower and smooth period-to-period variation.
- **Approach:** Delay non-critical activities (using their float) to smooth resource requirements.
- **Critical activities:** Should not be disturbed.

11.15 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. Critical Activities have:

(a) Maximum float (b) Minimum float (c) Zero float (d) Negative float

Answer: C

2. In PERT Chart, the Activity time distribution is:

(a) Normal (b) Binomial (c) Poisson (d) Beta

Answer: D

3. A PERT Network has nine activities on its Critical Path. The Standard Deviation of each activity on the Critical Path is 3. The S.D of the Critical Path is:

(a) 3 (b) 9 (c) 81 (d) 27

Answer: B

Variance of project = Sum of variances = $9 \times 3^2 = 81$; S.D = $\sqrt{81} = 9$.

4. For an activity the pessimistic, most likely and optimistic times are respectively 10, 6 and 2 days. The expected duration of the activity is:

(a) 6 days (b) 3 days (c) 2 days (d) 9 days

Answer: A

t_E = (2 + 4×6 + 10)/6 = 36/6 = 6 days.

5. The time by which the activity completion time can be delayed without affecting the start of the succeeding activities is known as:

(a) Total float (b) Free float (c) Independent float (d) Head slack

Answer: B

6. Which of the following statement is not true?

(a) PERT is deterministic in nature. (b) CPM is probabilistic in nature. (c) PERT Network can not be crashed. (d) All of the above.

Answer: D

7. Following data refers to a project Network. What will be the Critical Path?

Activity 1–2 (2 Days) 2–3 (1 Day) 3–4 (3 Days) 1–4 (3 Days)

2–5 (3 Days) 3–5 (2 Days) 4–5 (4 Days)

Answer: B

Critical Path: 1 – 2 – 3 – 4 – 5 with duration 2+1+3+4 = 10 days.

8. The amount of time by which an activity can be delayed without affecting the project completion is called:

(a) Free float (b) Total float (c) Interfering float (d) None of the above

Answer: B

9. Optimistic time and pessimistic time of an activity are respectively 4 days and 16 days. Variance of the duration of the activity will be:

(a) 4 day² (b) 2 day² (c) 3 day² (d) None of the above

Answer: A

$\sigma^2 = [(b - a)/6]^2 = [(16 - 4)/6]^2 = (12/6)^2 = 4$ day².

10. In a project planning Free float can affect which of the following?

(a) Succeeding activity (b) Only that activity (c) Preceding activity (d) All of the above

Answer: A

11. Solution of problems of Crashing has to be started by applying the technique on:

(a) Any activity of the Network. (b) Non critical activities. (c) Critical activities. (d) None of the above.

Answer: C

12. A PERT activity has an optimistic time of 3 days, pessimistic time of 15 days and an expected time of 7 days. What is the most likely time of the activity?

(a) 10 days (b) 6 days (c) 5 days (d) None of the above

Answer: B

7 = (3 + 4m + 15)/6 → 42 = 18 + 4m → m = 6 days.

13. The reduction in project time normally results in:

(a) Decrease in Direct Cost and increase in Indirect Cost

(b) Increase in Direct Cost and decrease in Indirect Cost

(c) Increase in both Direct and Indirect Costs.

(d) Decrease in both Direct and Indirect Costs.

Answer: B

14. The Normal duration and Normal cost of an activity are respectively 10 days and ₹350. The cost slope is ₹75 per day. If the Crash duration is 8 days then what is the Crash cost of the activity?

(a) ₹400

(b) ₹500

(c) ₹600

(d) ₹650

Answer: B

Crash Cost = Normal Cost + Cost Slope × (Normal Time − Crash Time) = 350 + 75 × 2 = ₹500.

15. Which of the following is incorrect?

(a) PERT is suitable for projects having probabilistic time estimates.

(b) CPM is suitable for projects having deterministic activities.

(c) Both PERT and CPM are event oriented.

(d) PERT is event oriented while CPM is activity oriented.

Answer: C

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
C	D	B	A	B	D	B	B	A	A	C	B	B	B	C

State True or False

SL	STATEMENT	ANSWER
1	Full form of PERT is Program Evaluation and Robot Technology.	**False**
2	The difference between the maximum time available and the actual time needed to perform an Activity is called the Total Float of the Activity.	**True**
3	PERT Network is Activity oriented and CPM Network is Event oriented.	**False**
4	An activity can have three different types of floats.	**True**
5	PERT and CPM are both project evaluation techniques.	**False**
6	The objective of Network Analysis is to minimize total project duration.	**True**
7	For any activity, Independent Float ≤ Free Float ≤ Total Float	**True**
8	Events neither consume any resource nor any time.	**True**
9	P (Z ≤ 2.33) = 95%, is a very important result used for finding the duration of a project which has 95% chance of completion within time.	**False**
10	In a CPM Network diagram completion of an activity is known as Event or Node or Connector	**True**
11	Project crashing is possible for both CPM and PERT.	**False**
12	As per Backward Pass technique, Latest allowable Event Time is $L_i = L_j - t_{ij}$ & for Burst Events L_i = Min. $(L_j - t_{ij})$	**True**
13	In a Network diagram Activities are represented by arrows having special importance on the lengths of the same according to the duration of the activity.	**False**
14	In a Construction project, "Curing of Concrete" is an Activity but "Concrete Cured" is an Event.	**True**
15	Critical Path of a Network helps the Project Manager to divert the resources from non-critical advanced activities to critical activities.	**True**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	The activity with minimum _____ should be crashed first.	**Cost slope**
2	____ Pass technique is used to get the Earliest Event times.	**Forward**
3	For any Merge Event, earliest time is obtained by taking ____ value of the Earliest Finish times of all the activities ending there.	**Maximum**
4	Drawing unnecessary dummy activity in a Network diagram is called _____ error.	**Redundancy**
5	Simultaneous accomplishment of those activities in a Network is possible which are _____.	**Concurrent**
6	Slack of the event at the arrowhead end of an activity is known as _____.	**Head Slack**
7	In a PERT Activity, probability of Most Likely Time is ____ times of that of Optimistic Time.	**Four**
8	Standard rule of Network drawing says that the value of i is ____ that of j always, for the Activity (i – j).	**Less than**
9	For R & D type Projects, _____ is the most suitable technique of drawing Network diagrams.	**PERT**
10	Dummy activities are drawn to make activities with ____ beginning and ending events distinguishable.	**Common**
11	An Event where more than one activity ends and also from where more than one activity begins is called _____ Event.	**Merge and Burst**
12	In a PERT Network, probability of completion of a project within a stipulated time is estimated by using ____-approximation to β Distribution.	**Normal**
13	Network is a technique, used for Planning, Scheduling and _____ of any project.	**Controlling**
14	The value of Slack of all the events in the Critical Path of a Network is ____.	**Zero**
15	Sum of the Free float and Head slack gives _____ float of an activity.	**Total**

11.16 Essay Type Questions & Numerical Problems

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

What is a Path in a Network diagram?

2

What is Crash Duration of an Activity?

3

Write short note on Expected Duration of a PERT Activity?

4

What is the difference between AOA Diagram and AON Diagram in Network?

5

What is Looping error in Network Diagram?

6

Define Cost Slope in relation to Crashing of a project Network.

7

Write short notes on Event and Merge Event in a Network.

Essay Type Questions

1

Distinguish between PERT and CPM.

2

Explain the following terms in PERT – (a) Optimistic Time, (b) Pessimistic Time, (c) Most Likely Time, (d) Variance of duration of Activity, (e) Project duration Variance, (f) Mean completion time of the project.

3

Write short notes on – Total Float, Free Float, Independent Float, Head Slack and Tail Slack.

4

Explain the meaning of Crashing in Network techniques.

5

What is Network Analysis and when it is used? What is meant by Critical Path?

Comprehensive Numerical Problems

PROBLEM 1 — PROJECT NETWORK WITH 11 ACTIVITIES

Information about a project are given as follows:

ACTIVITY	PREDECESSOR	LEAST TIME	GREATEST TIME	MOST LIKELY TIME
A	None	4	8	5
B	None	5	10	7
C	A	8	12	11
D	None	2	7	3
E	B, C	4	10	7
F	D	6	15	9
G	D	8	16	12
H	E, F	5	9	6
I	E, F	3	7	5
J	G, I	5	11	8
K	H	6	13	9

Required: Draw the Network diagram and identify the Critical Path. Find the Earliest and Latest times of each Event. Also calculate the Total Float for each activity. Consider the unit of various times given above is weeks.

ANSWER

Critical Path: A – C – E – H – K.

Values of Total Floats (in weeks) for the Activities A through K are respectively 0, 8.83, 0, 10, 0, 10, 15, 0, 2.5, 2.5, 0

PROBLEM 2 — M/S RAMSWARUP LIMITED (CASE STUDY)

M/S Ramswarup Limited is a renowned trader of Steel Structural with head office in Kolkata, has planned to diversify and decided to enter into manufacturing sector. They want to put up a Foundry with an aim to produce high quality abrasion resistant castings which has a very good export potential. After the study, Dr. Mathur suggested to go in phases. He listed the following activities with associated precedence relationships as well as Expected Durations and S.Ds in weeks for the first phase operation.

ACTIVITY	A	B	C	D	E	F	G	H	I	J
Predecessor	None	None	A	A	A	C	D	B, D, E	H	I, G, F
Duration	6	3	5	4	3	3	5	5	2	3
S.D	1	0.5	1	1	0.5	0.5	1	1	0.5	1

Required: Draw the Network diagram and identify the Critical Path. Also find the project duration. Determine 95% Confidence Interval for the Expected Project Completion. The Cost of the project is estimated to be ₹10,00,000. If it is completed within 24 weeks the expected returns should be ₹1,00,00,000. But if the deadline fails the export market will be penetrated by competitors from South Africa and Australia. That will result in a net revenue of only ₹2,00,000. Determine the expected returns.

ANSWER

Critical Path: A – D – Dummy – H – I – J; project duration = **20 weeks**.

On calculation of Total Float, the values for Activities B, E and G are respectively 7, 1 and 2 weeks. So delay of 3 weeks in B and 2 weeks in G are not going to hamper the project completion. But delay of 2 weeks in E is going to increase the project duration by 1 week.

Mean project duration = 20 weeks; **S.D of project duration = 2.06 weeks**

95% Confidence Interval: 20 ± 1.96 × 2.06 = **15.96 to 24.04 weeks**

Expected return from the project = 1,00,00,000 × P(completed within 24 weeks) + 2,00,000 × P(not completed within 24 weeks) – Initial investment

= 1,00,00,000 × (1 – 0.0262) + 2,00,000 × 0.0262 – 10,00,000 = **₹87,43,240**

PROBLEM 3 — NORMAL AND CRASH TIME & COST

The following table gives the activities and other relevant data for a project:

ACTIVITY	NORMAL TIME (DAYS)	CRASH TIME (DAYS)	NORMAL COST (₹)	CRASH COST (₹)
1 – 2	4	3	600	800
1 – 3	2	2	400	400
1 – 4	5	4	750	900
2 – 3	7	5	400	600
2 – 5	7	6	800	1000
3 – 5	2	1	500	650
4 – 5	5	4	600	850

Indirect cost for the project is ₹200 per day. Draw the Network diagram. Find Normal duration and cost of the project. Also find optimum duration and cost for it.

ANSWER

Normal project duration = 13 days and cost = ₹6,650; Optimum project duration = 10 days and cost = ₹6,450

PROBLEM 4 — MINIMUM DIRECT COST FOR TOTAL DURATION OF 21 DAYS

Activity P is followed by Activity Q which in turn, is followed by Activity R. The Direct Cost of these Activities in relation to the choice of feasible durations is given in the Table below.

ACTIVITY P	ACTIVITY Q	ACTIVITY R
Duration 7 → ₹12,000	Duration 8 → ₹20,000	Duration 9 → ₹40,000
Duration 6 → ₹14,000	Duration 7 → ₹23,000	Duration 8 → ₹42,000
Duration 5 → ₹15,000	Duration 6 → ₹27,000	Duration 7 → ₹45,000

For all the three activities taken together what is the minimum possible Direct Cost for a total duration of 21 days.

ANSWER

₹77,000

PROBLEM 5 — FOUR ACTIVITIES P, Q, R AND S

A project has four activities P, Q, R and S as shown below:

ACTIVITY	NORMAL DURATION (DAYS)	PREDECESSOR ACTIVITY	COST SLOPE (₹/DAY)
P	3	—	500
Q	7	P	100
R	4	P	400
S	5	R	200

Normal Cost of the project is ₹10,000 and the Overhead Cost is ₹200 per day. If the project duration has to be crashed down to 9 days, then what will be the Total Cost of the project?

ANSWER

₹12,500

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Network Analysis

General name for certain specific techniques which can be used for planning, management and control of project.

Network

Graphical representation of all the Activities and Events arranged in a logical and sequential order.

Activity

Actual performance of the job; consumes resources (Time, human resources, money, and material).

Event

Refers to start or completion of a job; does not consume any resources.

Dummy Activity

Consumes no time or resources; represented by dashed arrows; used to distinguish activities or maintain precedence.

Critical Path

The longest path in the network from the starting event to the ending event; defines minimum project duration.

Total Float

Amount of time by which completion of an activity can be delayed beyond the earliest expected completion time without affecting the project duration.

Free Float

Time by which an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity.

Independent Float

Amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activity, assuming that the preceding activity has finished at its latest time.

Slack

Difference between the Latest and Earliest times of an Event; Slack = $L_i - E_i$.

Crashing

Intentionally reducing the duration of project by allocating more resources to it.

Cost Slope

(Crash cost – Normal cost) ÷ (Normal time – Crash time).

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 11

ABBREVIATION	FULL FORM
PERT	Program Evaluation and Review Technique
CPM	Critical Path Method
CPA	Critical Path Analysis
AOA	Activity on Arrow
AON	Activity on Node
ES	Earliest Start
EF	Earliest Finish
LS	Latest Start
LF	Latest Finish
TF	Total Float
FF	Free Float
IF	Independent Float
a, m, b	Optimistic, Most Likely, Pessimistic time
t_E	Expected Time
σ	Standard Deviation
Z	Standard Normal Variable

11.4 Applications of Network Analysis

11.5 Basic Terminology Related to Network

11.6 Common Errors in Drawing Network Diagrams

11.7 Rules for Drawing Network Diagram

11.8 Procedure of Drawing a Network Diagram

11.9 Time Estimates and Critical Path

11.10 Illustration — Making of a Loaf

11.11 Three Time Estimates of PERT

11.12 Probability of Project Completion

11.13 Crashing (Time–Cost Trade Off)

11.14 Resource Levelling

11.15 MCQs, True/False, Fill in the Blanks

11.16 Essay Type Questions & Numerical Problems

MCQs with Reasoning

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Learning Curve

Module 12 · Concept · Phases · Ratio · Wright's Formula · Experience Curve

EDITION
December 2026

PAPER
Paper 16 · Strategic Cost Management

SECTION
Section B · Weightage 40%

MODULE GROUP
Group with PERT/CPM, Simulation, etc.

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading
✓ Self Study ✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

12

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 12 INCLUDES

Concept of Learning Curve · Significance of Learning Curve · Phases in Learning Curve · Learning Curve Ratio · Areas of Consequence · Graphical presentation of Learning Curve · Pros and cons of the Learning Curve Theory · Uses of Learning Curve · Limitations to the usefulness of the Learning Curve · Factors affecting Learning Curve · The Experience Curve.

Module Learning Objectives

After studying this module, the students will be able to:

1

Understand Learning & Curve

Understand the phenomenon of Learning and visualize the associated Curve.

2

Learning Rate & Doubling

Understand how the percentage learning rate applies to the doubling of output.

3

Apply to Business Decisions

Use Learning Curve theory successfully for Pricing decisions, Work scheduling and setting of Standards.

4

Identify Situations

Identify the situations where Learning effect should be incorporated in Industry.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Learning Curve to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

3
PHASES OF LEARNING
Initial, Intermediate, Final

80%
COMMON LEARNING RATE
Widely used benchmark

2×
DOUBLING RULE
Output doubles → Time reduces

30
MCQS
With answers

SYLLABUS STRUCTURE NOTE

Module 12 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Learning Curve introduces a mathematical technique for predicting cost behaviour in repetitive operations — widely applicable to cost estimation, pricing decisions, budgeting, make-or-buy decisions, and workforce planning. The Learning Curve phenomenon is essential for understanding how productivity improves over time in labour-intensive industries.

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8. Uses of Learning Curve
9. Limitations to the Usefulness of Learning Curve
10. Factors Affecting Learning Curve
11. The Experience Curve
12. Illustrations & Numerical Problems
13. MCQs, True/False, Fill in the Blanks

12.1 Concept of Learning Curve

LEARNING PROCESS · REPETITION · CUMULATIVE AVERAGE TIME

Learning is the process by which human beings acquire a vast variety of competencies, skills, knowledge and attitudes. In any environment if a person is assigned to do the same task again and again then after a period of time, there is an improvement in his performance. This happens due to the

fact that the phenomenon of **Learning** takes place causing increase in competency of the person carrying out the task.

Repetition of the task is likely to make the person more confident and knowledgeable and will eventually result in a more efficient and rapid operation. Ultimately the learning process will stop after continually repeating the job. As a consequence, the time to complete a task will **initially decline and then stabilize** once efficient working is achieved. The **cumulative average time per unit** is assumed to decrease by a constant percentage every time the output doubles. Cumulative average time refers to the average time per unit for all units produced so far, from and including the first one made.

WHAT IS A LEARNING CURVE?

When data of performance of such a repeated task are collected over a period of time and plotted on a graph paper we get a curve called **Learning Curve**. A Learning Curve is actually a correlation between a learner's performance on a task and the number of attempts or time required to complete the task. The curve shows that if a task is performed over and over then less time is required at each iteration. Thus, Learning Curve theory proposes that a learner's efficiency in a task improves over time the more the learner performs the task.

Historical Development

Though serious studies about the effect of learning on a repetitive task began in the twentieth century, the similar concept was first coming into the surface as early as **1885** when the German psychologist **Dr. Hermann Ebbinghaus** first conducted experimental research on human memory. He was interested in discovering why new information learnt by people tends to fade away from the memory over a period of time. The data collected by him was plotted on a graph to get a curve which was named by him as the **Forgetting Curve**. He also discovered, if learning is rehearsed and repeated at regular intervals, people forget less.

The Learning Curve (not to be confused with experience curve) is a graphical representation of the phenomenon explained by **Theodore P. Wright in his "Factors Affecting the Cost of Airplanes", 1936**. It refers to the effect that learning had on labour productivity in the aircraft industry, which translates into a relation between the cumulative number of units produced (X) and the average time (or labour cost) per unit (Y), which resulted in a convex downward slope.

SIMPLE RATIONALIZATION

The more units produced by a given worker, the less time this same worker will need to produce the following units, because he will learn how to do it faster and better. Therefore, when a firm has higher cumulative volume of production, its time (or labour cost) per unit will be lower.

⚡ Quick Revision — Section 12.1

- **Learning:** Process of acquiring competencies, skills, knowledge and attitudes; improves performance on repetitive tasks.
- **Learning Curve:** Correlation between learner's performance and number of attempts/time; shows less time required at each iteration.
- **Cumulative Average Time:** Average time per unit for all units produced so far; decreases by constant percentage every time output doubles.
- **Historical Development:** Dr. Hermann Ebbinghaus (1885) — Forgetting Curve; Theodore P. Wright (1936) — Learning Curve in aircraft industry.

12.2 Significance of Learning Curve

COST ANALYSIS · COST ESTIMATION · EFFICIENCY STUDIES

Learning Curve finds a very important role in Cost Analysis, Cost Estimation and Efficiency studies. Cost predictions especially those relating to direct labour cost must allow for the effect of learning process. This technique is a mathematical technique. It can be very much used to accurately and graphically predict cost. It is a **geometrical progression**, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation.

As the identical operation is increasingly repeated, the amount of decrease is less and less with each successive unit produced. The slope of the curve can be expressed as a percentage. **Experience curve, Improvement curve and Progress curve** are other terms which can be synonymously used.

KEY INSIGHT

Learning Curve is essentially a **measure of the experience gained** in production of an article by an individual or organisation. As more units are produced, people involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The amount of improvement will differ with each type of article produced. This improvement or experience gained is reflected in decrease in man-hours or cost.

⚡ Quick Revision — Section 12.2

- **Significance:** Role in Cost Analysis, Cost Estimation, Efficiency studies.
- **Nature:** Mathematical technique; geometrical progression; predicts cost accurately.
- **Synonyms:** Experience Curve, Improvement Curve, Progress Curve.
- **Key Insight:** Measure of experience gained; each subsequent unit takes fewer man-hours.

12.3 Phases in Learning Curve

THREE DISTINCT PHASES OF LEARNING

The Learning Curve passes through **three different phases**:

1

First Phase — Steep Learning

There will be gradual increase in production rate until the maximum expected rate is reached and this phase is generally **steep**. This is the phase where the greatest improvement happens as workers familiarize themselves with the task.

2

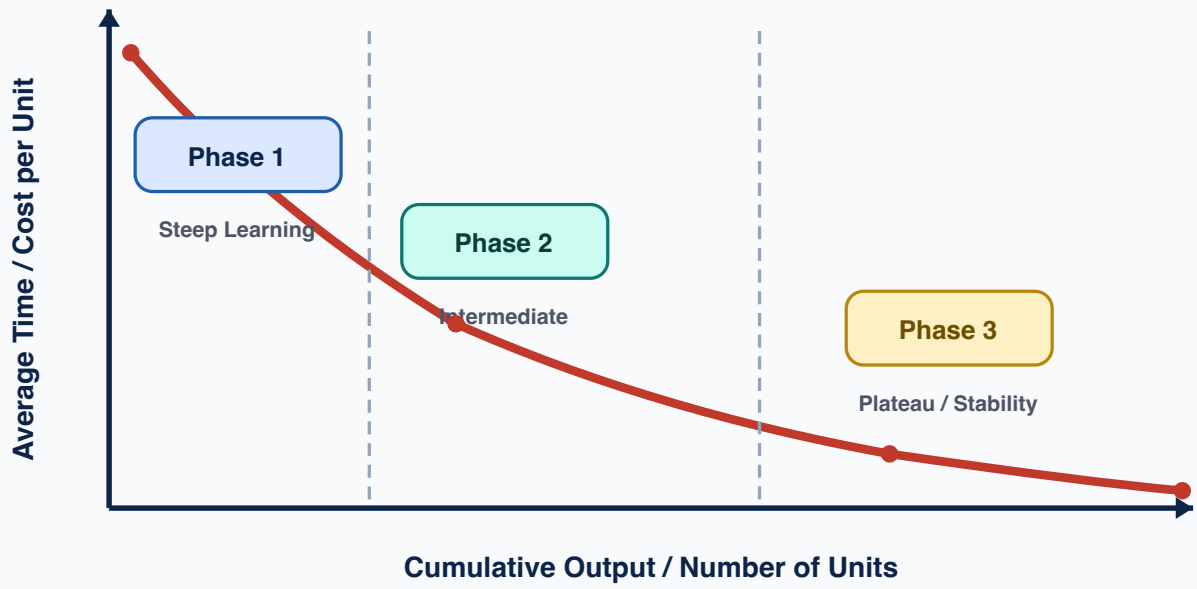
Second Phase — Intermediate Learning

The learning rate will gradually **deteriorate** because of the limitations of equipment. Improvements continue but at a slower pace as the easier gains have already been made.

3

Third Phase — Plateau

The production rate begins to **decrease** due to a reduction in customer requirements and increase in costs. This is the stabilization phase where further significant improvements are not possible without changes in technology, process, or product design.



⚡ Quick Revision — Section 12.3

- **Phase 1:** Steep learning; gradual increase in production rate until maximum expected rate is reached.
- **Phase 2:** Intermediate; learning rate deteriorates due to equipment limitations.
- **Phase 3:** Plateau; production rate decreases due to reduced customer requirements and increased costs.

12.4 Learning Curve Ratio

DOUBLING RULE · LEARNING PERCENTAGE · SLOPE OF THE CURVE

Under the Learning Curve model, the **cumulative average time per unit produced is assumed to fall by a constant percentage every time total output of the unit doubles**. Learning Curve is a geometrical progression which reveals that there is a steady decrease in cost as the identical operation is increasingly repeated.

Learning Curve is essentially a measure of the experience gained in production of an article by an organisation. As more and more units are produced, workers involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The Learning Curve exists during a worker's startup or familiarization period on a particular job. After the limits of experimental learning are reached, productivity tends to stabilize and no further improvement is possible.

LEARNING CURVE RATIO FORMULA

```
Learning Curve Ratio = (Average labour cost of first 2n units) ÷ (Average labour cost of first n units)
```

If the Average labour cost of the first 50 units of a Product is ₹40 and that of the first 100 units is ₹32, then the Learning Ratio is calculated as 32/40 = 0.8 or **80%** which means that every time the output doubles, the average labour cost is reduced to 80% of the previous amount. In this case, average labour cost of the first 200 units will be 80% of that of the first 100 units = 80% of ₹32 = **₹25.60**.

UNDERSTANDING LEARNING PERCENTAGE

Conventionally Learning Curves are referred to by complements of their decrease rates. In the above example 80% Learning Curve showed a **20% decrease** in average labour cost for every doubling of the output. Learning percentage is actually the **Slope of the Learning Curve**. It can be noted that a **100% Learning implies no decrease** of average labour cost (which means "No Learning" effect).

Wright's Learning Curve Model

WRIGHT'S FORMULA

$$Y = aX^b$$

Where: **Y** = average time (or labour cost) per unit; **a** = time (or labour cost) for the 1st unit; **X** = cumulative volume of production; **b** = learning index = [log (learning percentage)] / log 2

Example 1 — Expected Completion Times (Doubling Pattern)

PROBLEM

An operation to produce an item is known to have an 80% Learning Curve. It has taken a worker 20 hours to complete the operation for the first unit. Determine the expected completion times for the 2nd, 4th and 8th units.

SOLUTION

As per the Theory of Learning Curve, each time the cumulative output doubles, the time per unit should be equal to the previous time multiplied by the learning percentage (80% in this case):

UNIT	1ST	2ND	4TH	8TH
Expected Completion time (hours)	20	20 × 0.8 = **16**	20 × $(0.8)^2$ = **12.8**	20 × $(0.8)^3$ = **10.24**

Important point: The time reduction per unit becomes smaller and smaller as the number of units produced is increased. The reduction of time from the 1st unit to the 2nd is 4 hours whereas that from the 4th to 8th is only 2.56 hours. Also the reduction of time follows a **Geometrical Progression** with Common Ratio = Learning ratio.

Note: When the units follow a doubling pattern then the time as well as cost can be easily calculated as shown above. But when they do not follow the doubling pattern (i.e., for the 3rd, 5th, 6th etc. units) then we have to use **Wright's formula** to find the time.

⚡ Quick Revision — Section 12.4

- **Learning Curve Ratio:** Average labour cost of first 2n units ÷ Average labour cost of first n units.
- **80% Learning Curve:** 20% decrease in average labour cost for every doubling of output.
- **Learning Percentage:** Slope of the Learning Curve; 100% means no learning.
- **Wright's Formula:** $Y = aX^b$ where b = log(learning percentage)/log 2.
- **Doubling Rule:** Time reduces by constant percentage each time output doubles.

12.5 Areas of Consequence

SIX KEY AREAS WHERE LEARNING CURVE AFFECTS BUSINESS

i

Standard Costing System

A Standard Costing system would need to set standard labour times after the Learning Curve had reached a **plateau**.

ii

Budgeting

A budget will need to incorporate a **learning cost factor** until the plateau is reached.

iii

Budgetary Control & Variances

A budgetary control system incorporating labour variances will have to make **allowances for the anticipated time changes**.

iv

Business Planning

Identification of the Learning Curve will permit the company to better plan its **marketing, work scheduling, recruitment and material acquisition activities**.

v

Overhead Apportionment

The decline in labour costs will have to be considered when estimating the **overhead apportionment rate**.

vi

Material Wastage

As the employees gain experience, they are more likely to **reduce material wastage**.

⚡ Quick Revision — Section 12.5

- **Six Areas:** Standard Costing (set standards after plateau), Budgeting (include learning factor), Budgetary Control (allow for time changes), Business Planning (marketing, scheduling, recruitment), Overhead Apportionment (decline in labour costs), Material Wastage (reduced as experience grows).

12.6 Graphical Presentation of Learning Curve

CONVEX DOWNWARD SLOPE · WRIGHT'S MODEL

The Learning Curve refers to the effect that learning had on labour productivity in the aircraft industry, which translates into a relation between the cumulative number of units produced (X) and the average time (or labour cost) per unit (Y), which resulted in a **convex downward slope**.

WRIGHT'S LEARNING CURVE MODEL

Wright's Learning Curve model is defined by the following function: $\mathbf{Y = aX^b}$ where Y = average time (or labour cost) per unit; a = time (or labour cost) for the 1st unit; X = cumulative volume of production; b = learning index = [log (learning percentage)/ log 2].

MOST IMPORTANT IMPLICATION

When the average time or labour cost per unit decreases with the increase in cumulative output, then the firm which is producing more and for a longer period will **dominate the market** because of its lower average time or labour cost.

⚡ Quick Revision — Section 12.6

- **Graphical Shape:** Convex downward slope.
- **Relationship:** Cumulative output (X) vs. Average time/cost per unit (Y).
- **Wright's Model:** $Y = aX^b$.
- **Strategic Implication:** Higher cumulative production → Lower average cost → Market dominance.

12.7 Pros and Cons of the Learning Curve Theory

ADVANTAGES AND DISADVANTAGES

PROS OF THE LEARNING CURVE THEORY

- Using a Learning Curve will help a business to improve the performance and productivity of their workforce and reduce costs.
- When used to track or predict performance, it can provide psychological motivation and also facilitate in strategic planning.
- Performance improvement can't happen on its own and is connected with learning. By incorporating a learning culture within the organisation where employees are encouraged and supported to keep learning, performance levels can be expected to increase.
- The rate of learning is also considered to be consistent enough that trends can be established using the Learning Curve, enabling better forecasting and business decisions.

VS

CONS OF THE LEARNING CURVE THEORY

- A Learning Curve is dependent on the assumptions made about performance. Many variables can impact learning and future performance.
- Using a Learning Curve to predict overall performance of larger groups or processes means that many assumptions are made on variables like motivation, sociological factors, workplace dynamics, training resources and previous knowledge or experience.
- A Learning Curve that may not show expected results would need further analysis to determine the underlying variables impacting its shape, as the curve does not tell the whole story.

⚡ Quick Revision — Section 12.7

- **Pros:** Improved performance/productivity, reduced costs, psychological motivation, strategic planning, learning culture, better forecasting.
- **Cons:** Dependent on assumptions, many variables affect learning, curve doesn't tell whole story.

12.8 Uses of Learning Curve

EIGHT PRACTICAL APPLICATIONS

1

Cost Reduction Opportunities

Where applicable, the Learning Curve suggests great opportunities for cost reduction to be achieved by improving learning.

2

Staffing & Production Planning

The Learning Curve concept suggests a basis for correct staffing in continuously expanding production. The curve shows that the work force need not be increased at the same rate as the prospective output. This also helps in proper production planning through proper scheduling of work; providing manpower at the right moment permitting more accurate forecast of delivery dates.

3

Evaluating Training Programs

Learning Curve concept provides a means of evaluating the effectiveness of training programs — what level of cumulative cost reduction do they accomplish? How does the Learning Curve for this group or shop compare with others? Whether any of the employees who lack the aptitude to meet normal Learning Curve should be eliminated.

4

Establishing Bid Prices

Learning Curve is frequently used in conjunction with establishing bid prices for contracts. Usually, the bid price is based on the cumulative average unit cost for all the units to be produced for a given contract. If production is not interrupted, additional units beyond this quantity should be considered at the increment costs incurred, and not at the previous cumulative average.

5

Working Capital Requirements

The use of Learning Curve is important in finding the working capital required. If the requirement is based on average cumulative unit cost, the revenues from the first few units may not cover the actual expenditures. Provision should therefore be made to cover the deficit of working capital in the initial stages of production.

6

Inventory Planning

As employees become more efficient, the rate of production increases and so more materials are needed, the work-in-progress inventory turns over faster, and finished goods inventory grows at an accelerated rate. A knowledge of the Learning Curve assists in planning the inventories of materials, work-in-progress, and finished goods.

7

Exercising Control

Learning Curve techniques are useful in exercising control. Variable norms can be established for each situation, and a comparison between these norms and actual expenses can be made. Specific or average incremental unit cost should be used for this purpose.

8

Make-or-Buy & Wage Bargaining

The Learning Curve may be used for make-or-buy decisions especially if the outside manufacturer has reached the maximum on the Learning Curve. It also helps to calculate the sensitive rates in wage bargaining.

⚡ Quick Revision — Section 12.8

- **Eight Uses:** Cost reduction, Staffing & production planning, Evaluating training programs, Bid prices, Working capital requirements, Inventory planning, Exercising control, Make-or-buy & wage bargaining.

12.9

Limitations to the Usefulness of Learning Curve

FIVE KEY LIMITATIONS

1

Not Applicable to All Situations

The Learning Curve is useful only for new operations where machines do not constitute a major part of the production process. It is not applicable to all production situations. E.g., new and experienced workmen.

2

Assumes Continuous Production

The Learning Curve assumes that the production will continue without any major interruptions. If for any reason the work is interrupted, the curve may be deflected or assume a new slope.

3

Changes Other Than Learning

Changes other than learning may affect the Learning Curve. For example, improvement in facilities, arrangements, and equipment as well as personnel morale and performance may be the factors influencing the curve. On the other hand, negative developments in employee attitudes may also affect the curve and reverse or retard the progress of improvement.

4

No Universal Percentage

The characteristics for 80 percent Learning Curve as originally obtained in the airline industry in U.S.A. has been usually accepted as the percentage applicable to all industries. Studies show that there cannot be a unique percentage which can be universally applied.

⚡ Quick Revision — Section 12.9

- **Four Limitations:** Not applicable to all situations (machine-intensive vs. labour-intensive), assumes continuous production, changes other than learning affect the curve, no universal learning percentage.

12.10 Factors Affecting Learning Curve

FIVE KEY FACTORS

1

Bidding Strategy

While pricing for bids, general tendency is to set up a very high initial labour cost so as to show a high Learning Curve. This makes the Learning Curve **useless and misleading**.

2

Method of Production

The method of production, i.e., whether it is **labour oriented or machine oriented**, influences the slope of the learning. Labour-intensive operations exhibit a greater learning effect than machine-intensive operations.

3

Labour Turnover Rate

When labour turnover rate is high, management has to train new workers frequently. In such situations the company may never reach its maximum efficiency potential. One of the important requisites of the Learning Curve concept is that there should be **uninterrupted flow of work**. The fewer the interruptions, the greater will be the improvement in efficiency.

4

Changes in Product/Process

Changes in a product or in the methods of production, designs, machinery, or the tools used affect the slope of the Learning Curve. All these have the effect of **starting learning afresh** because of new conditions. If the changes are frequent, there may be no learning at all.

5

Labour Strikes, Lock-outs, Shutdowns

Labour strikes, lock outs and shutdowns due to other causes also affect the Learning Curve. In each such case there is **interruption in the progress of learning**.

IMPORTANT NOTE

As far as possible the effects of above factors should be carefully separated from the data used to establish the curve. The effects of these factors must also be separated from the actual costs used to measure the performance. Unless this is done, analysis of the projected cost or the actual cost will not be meaningful.

⚡ Quick Revision — Section 12.10

- **Five Factors:** Bidding strategy (high initial labour cost misleading), Method of production (labour vs machine oriented), Labour turnover rate, Changes in product/process, Labour strikes/lock-outs/shutdowns.
- **Key Requisite:** Uninterrupted flow of work.

12.11 The Experience Curve

BCG'S STRATEGIC CONCEPT · COST DECLINE WITH EXPERIENCE

CORE IDEA

The more experience a firm has in producing a particular product, the **lower is its cost**.

The **Experience Curve** is an idea developed by the **Boston Consulting Group (BCG)** in the mid-1960s. Working with a leading manufacturer of semiconductors, the consultants noticed that the company's unit cost of manufacturing fell by about **25% for each doubling of the volume** that it produced. This relationship they called the experience curve: the more experience a firm has in producing a particular product, the lower is its cost.

BRUCE HENDERSON (BCG FOUNDER)

"Costs characteristically decline by 20-30% in real terms each time accumulated experience doubles. This means that when inflation is factored out, costs should always decline. The decline is fast if growth is fast and slow if growth is slow."

Historical Context

There is no fundamental economic law that can predict the existence of the experience curve, even though it has been shown to apply to industries across the world. Its truth has been proven **inductively**, not deductively. Even when BCG first expounded the relationship, it had been known since the second world war that it applied to direct labour costs. Less labour was needed for a given output depending on the experience of that labour. In aircraft production, for instance, labour input decreased by some **10–15% for every doubling** of that labour's experience.

Strategic Implications

MARKET SHARE ADVANTAGE

If costs fell (fairly predictably) with experience, and if experience was closely related to market share (as it seemed it must be), then the competitor with the **biggest market share was going to have a big cost advantage** over its rivals. Being market leader is a valuable asset that a firm relinquishes at its peril.

This was the logical underpinning of the idea of the **growth share matrix**. The experience curve justified allocating financial resources to those businesses (out of a firm's portfolio) that were (or were going to be) market leaders in their particular sectors. This, of course, implied starvation for those businesses that were not and never would be market leaders.

Limitations of the Experience Curve

1

Imprecision

Over time, managers came to find the experience curve too imprecise to help them much with specific business plans. Different products had curves of a different slope and different sources of cost reduction. They did not all have the same downward gradient as the semiconductor industry.

2

Volume Constraints

Part of the explanation for this discrepancy was that different products provided different opportunities to gain experience. Large products (such as nuclear reactors) are inherently bound to be produced in smaller volumes than small products (such as semiconductors).

3

New Entrants

In theory, the experience curve should make it difficult for new entrants to challenge firms with a substantial market share. In practice, new firms enter old industries all the time. This is often because they have found ways of bypassing what might seem like the remorseless inevitability of the curve and its slope.

4

Leapfrogging

Experience can be gained not only first-hand, but also second-hand. Furthermore, firms can leapfrog over the experience curve by means of innovation and invention. All the experience in the world in making black and white television sets is worthless if everyone wants to buy colour ones.

⚡ Quick Revision — Section 12.11

- **Experience Curve:** Developed by Boston Consulting Group (BCG) in mid-1960s.
- **Core Idea:** Unit cost falls by 20-30% each time accumulated experience (volume) doubles.
- **Historical Example:** Semiconductor manufacturer — 25% cost decline for each doubling of volume.
- **Aircraft Industry:** Labour input decreased by 10–15% for every doubling of experience.
- **Strategic Implication:** Market leader has cost advantage → Growth Share Matrix.
- **Limitations:** Imprecise, volume constraints, new entrants, leapfrogging through innovation.

12.12 Illustrations & Numerical Problems

WORKED EXAMPLES WITH SOLUTIONS

Illustration 1 — Wright's Formula Application

PROBLEM

The usual Learning Curve model is $Y = aX^b$ where Y is the average time per unit for x units and 'a' is the time for first unit; x is the cumulative number of units; b is the learning coefficient and is equal to (log 0.8)/(log 2) = –0.322 for a learning rate of 80%. Given that a = 10 hours, you are required to Calculate:

- (i) The average time for 20 units.
- (ii) The total time for 30 units.
- (iii) The time for units 31 to 40.

Given that: log 2 = 0.301, Antilog of 0.5811 = 3.812; log 3 = 0.4771, Antilog of 0.5244 = 3.345; log 4 = 0.6021, Antilog of 0.4841 = 3.049.

SOLUTION

(i) $Y = aX^b$

$Y = 10(20)^{-0.322}$. Taking logarithm on both sides:

Log Y = log 10 + log $20^{(-0.322)}$ = log 10 – (0.322) log 20 = 1 – (0.322) × (1.3010) = 1 – 0.41892 = 0.5811

Log Y = 0.5811; Y = Anti log (0.5811) = **3.812 hrs** (average time for 20 units)

(ii) Log Y = log 10 + log $30^{(-0.322)}$ = 1 – (0.322) × (1.4771) = 1 – (0.4756) = 0.5244

Y = anti log (0.5244) = **3.345 hrs** (average time for 30 units)

Total time for 30 units = 3.345 × 30 = **100.35 hrs**

(iii) Log Y = log 10 + log $40^{(-0.322)}$ = 1 – (0.322) × (1.6021) = 0.4841

Y = anti log (0.4841) = **3.049 hrs**

Total time for 40 units = 40 × 3.049 = **121.96 hrs**

Time from 31 to 40 units = 121.96 – 100.35 = **21.61 hrs**

Illustration 2 — Expected Average Cost (Machines)

PROBLEM

The Learning Curve in management accounting has now become or is going to become an accepted tool in industry, for its applications are almost unlimited. State precisely: (i) Your understanding of the Learning Curve; (ii) The theory of Learning Curve; (iii) The areas where Learning Curves may assist in management accounting; and (iv) Illustrate the use of Learning Curves for calculating the expected average unit cost of making (a) 4 machines (b) 8 machines.

Data: Direct Labour needed to make first machine = 1000 hrs; Learning Curve = 90%; Direct Labour cost = ₹15 per hour; Direct materials cost = ₹1,50,000; Fixed cost for either size orders = ₹60,000.

SOLUTION

NO. OF MACHINES	AVERAGE TIME (HOURS)	LABOUR COST (@₹15/HR)	MATERIAL COST (₹)	FIXED COST (₹)	TOTAL COST (₹)
1	1000	15,000	1,50,000	60,000	2,25,000
2	900	13,500	1,50,000	30,000	1,93,500
4	810	12,150	1,50,000	15,000	1,77,150
8	729	10,935	1,50,000	7,500	1,68,435

Average cost of making 4 machines = ₹1,77,150; Average cost of making 8 machines = ₹1,68,435

Illustration 3 — Z.P.L.C Order Analysis

PROBLEM

Z.P.L.C experiences difficulty in its budgeting process because it finds it necessary to quantify the learning effect as new products are introduced. Substantial product changes occur and result in the need for retraining. An order for 30 units of a new product has been received by Z.P.L.C. So far, 14 have been completed; the first unit required 40 direct labour hours and a total of 240 direct labour hours has been recorded for the 14 units. The production manager expects an 80% learning effect for this type of work.

Cost data: Direct material ₹30.00 per unit; Direct Labour ₹6.00 per hour; Variable overhead ₹0.50 per direct labour hour; Fixed overhead ₹6,000 per 4 week operating period. There are ten direct employees working a five-day week, eight hours per day. Personal and other downtime allowances account for 25% of total available time. The company usually quotes a four-week delivery period for orders.

Required: (i) Determine whether the assumption of an 80% learning effect is reasonable. (ii) Calculate the number of direct labour hours likely to be required for an expected second order of 20 units. (iii) Use the cost data to produce an estimated product cost for the initial order; examine the problems which may be created for budgeting by the presence of the learning effect.

SOLUTION

(i) Reasonableness of 80% Learning Curve:

Total time taken to produce 14 units: $Y = aX^b = 40(14)^{-0.322}$

log Y = log 40 – (0.322) log 14 = 1.60221 – (0.322) × 1.1461 = 1.60221 – 0.3690 = 1.233

Y = Antilog (1.233) = 17.14; Total time = 17.14 × 14 = **239.96 ≈ 240 hours** (which matches the recorded hours). So the assumption of 80% learning ratio is **reasonable**.

(ii) Direct labour hours for second order of 20 units:

For 30 units: $Y = 40(30)^{-0.322}$ = **13.380 hours** (Average time); Total = 13.38 × 30 = 401.4 hours

For 50 units: $Y = 40(50)^{-0.322}$ = **11.35 hours** (Average time); Total = 11.35 × 50 = 567.5 hours

Time taken for 20 units from 31 to 50 units = (567.5 – 401.4) = **166.1 hours**

(iii) Total cost for the initial order (30 units):

Man hours available = 10 × 8 × 5 × 4 = 1600; Less down time (25%) = 400; Net = 1200 hours

Fixed Cost per hour = 6000/1200 = ₹5

ELEMENT	WORKINGS	AMOUNT (₹)
Material (30 × 30)	—	900.0
Labour (401.4 × 6)	—	2408.4
Variable Overheads (0.5 × 401.4)	—	200.7
Fixed Overheads (5 × 401.4)	—	2007.0
Total Cost		**₹5,516.1**

Problem for budgeting: The presence of learning effect makes budgeting difficult as the initial labour hours are high but reduce with output. Fixed cost per hour is calculated on the basis of normal working hours, but the actual hours vary due to learning. The budgeting must allow for variable labour times.

Illustration 4 — Total Time and Labour Cost (8 units & 24 units)

PROBLEM

A firm received an order to make and supply eight units of standard product which involves intricate labour operations. The first unit was made in 10 hours. It is understood that this type of operation is subject to an 80% learning rate. The workers are getting wages at the rate of ₹12 per hour.

(i) What is the total time and labour cost required to execute the above order? (ii) If a repeat order of 24 units is also received from the same customer, what is the labour cost necessary for the second order?

SOLUTION

80% Learning Curve results:

PRODUCTION (UNITS)	CUMULATIVE AVERAGE TIME (HOURS)	TOTAL TIME (HOURS)
1	10	10
2	8	16
4	6.4	25.6
8	5.12	40.96
16	4.096	65.54
32	3.2768	104.86

(i) Labour time required for first eight units = **40.96 hours**

Labour cost required for 8 units = 40.96 hours × ₹12/hr = **₹491.52**

(ii) Labour time for 32 units = 104.86 hours; Labour time for first eight units = 40.96 hours

Labour time required for 2nd order of 24 units = 104.86 – 40.96 = **63.90 hours**

Labour cost for the 2nd order of 24 units = 63.90 hours × ₹12/hr = **₹766.80**

Illustration 5 — Determining Learning Percentage

PROBLEM

A manager wants to determine an appropriate learning percentage for a new type of work his company will undertake. He has obtained unit times for the initial six repetitions which are given below:

UNIT	1	2	3	4	5	6
Completion time (Minutes)	46	39	35	33	32	30

Required: (i) The learning percentage. (ii) Estimate the average time if a total of 30 units are planned. (iii) After producing how many units, will completion time less than 15 Minutes per unit be achieved?

SOLUTION

(i) Learning Percentage:

As per the concept of doubling of Learning Curve theory:

Completion time of 2nd Unit = Completion time of 1st Unit × Learning percentage → 39 = 46 × LP → LP = 0.8478

Completion time of 4th Unit = Completion time of 2nd Unit × Learning percentage → 33 = 39 × LP → LP = 0.8461

So the Learning Percentage (LP) can be taken as **0.85 (Approx.) i.e., 85%**

(ii) Average time for 30 units:

$y_X = aX^b$ where a = 46 Mins, X = 30, b = log(0.85)/log2 = −0.2344

Average completion time of 30 units = $y_{30} = 46 \times (30^{-0.2344})$ = **20.73 Mins.**

(iii) Units required to achieve <15 Mins:

Using the above formula: $15 = 46(X^{-0.2344})$

log 15 = log 46 − 0.2344 log X; log X = 2.0762; $X = 10^{2.0762} = 119.179$

Average completion time will be less than 15 minutes if **120 units** are produced.

Illustration 6 — XYZ Co. (Batch Production)

PROBLEM

XYZ Co. does not have any costing department. You need to help them out in finding the total cost for a contract of a single order of 600 units for one of their products. The average unit variable cost data for an initial batch of 200 units are as follows:

- (i) Direct Material – 15 m² @ ₹80 per m²
- (ii) Direct Labour – 8 Hours in Department 1 @ ₹75 per hour and 100 Hours in Department 2 @ ₹100 per hour
- (iii) Variable Overhead – 25% of Direct Labour

Labour times in Departments 1 and 2 are expected to follow 80% and 70% Learning Curves respectively.

SOLUTION

From Learning Curve Theory: $y = aX^b$ where y = Cumulative average direct labour hours per unit; a = Direct labour hours for the first batch; X = Cumulative number of batches; b = Learning Index.

For Department 1: a = 200 × 8 = 1600 hours; Learning efficiency = 80%, so b = log 0.8 / log 2 = −0.322; Formula: $y = 1600(X^{-0.322})$

For Department 2: a = 200 × 100 = 20000 hours; Learning efficiency = 70%, so b = log 0.7 / log 2 = −0.515; Formula: $y = 20000(X^{-0.515})$

Calculations:

Direct Material Cost = (15 m² × ₹80 per m²) per unit × 600 units = ₹1,200 × 600 = **₹7,20,000**

Department 1: Cumulative average time to produce 600 units (3 batches) = $1600 \times (3^{-0.322})$ = 1123.277 hours per batch; Total time = 1123.277 × 3 = 3369.83 hours; Labour Cost = 3369.83 × ₹75 = **₹2,52,737.25**

Department 2: Cumulative average time to produce 600 units (3 batches) = $20000 \times (3^{-0.515})$ = 11358.279 hours per batch; Total time = 11358.279 × 3 = 34,074.84 hours; Labour Cost = 34,074.84 × ₹100 = **₹34,07,484**

Total Direct Labour Cost = ₹(2,52,737.25 + 34,07,484) = **₹36,60,221.25**

Variable Overhead = 25% of Direct Labour Cost = 0.25 × 36,60,221.25 = **₹9,15,055.31**

Total Cost for 600 units = ₹(7,20,000 + 36,60,221.25 + 9,15,055.31) = **₹52,95,276**

Illustration 7 — Automobile Manufacturer (Braking Mechanism Recall)

PROBLEM

An automobile manufacturer is conducting a product recall after it was discovered that a possible defect in the braking mechanism could cause loss of braking in certain cars. The recall covers a span of three model years. The company sent out letters to the car owners promising to repair the defect at no cost at any dealership. The company's policy is to pay the dealer a fixed amount for each repair. The repair is somewhat complicated and the company expects learning to be a factor. In order to set a reasonable rate for repairs, the company conducted a number of repairs in house. Thereafter it was decided that the appropriate pay is ₹7000 per repair based on a flat hourly rate of ₹1700 and a learning percentage of 90.

Shortly after the dealers began making the repairs, the company received complaint from several dealers that they were encountering resistance from the workers who felt the flat rate was way too low and were threatening to refuse the work on those jobs. One of the dealers collected data on task times and sent it to the company: Three mechanics each completed two repairs. Average time for the first unit was 9.6 hours and for the second unit was 7.2 hours. The dealer has suggested a rate of ₹8500 per repair.

Required: (1) Prepare a list of questions to analyze the situation. (2) Comment on the information provided. (3) What preliminary thoughts on solutions?

SOLUTION

(1) Questions for analysis:

- Is the learning percentage considered by the manufacturer correct?
- What time per repair has been allocated by the manufacturer?
- After how many repairs this time is achievable?
- What is the total number of repair jobs?
- Is the demand of the workers for higher rate justified?
- Should the rate suggested by the dealer be acceptable?

(2) Comment on information: The information provided is somewhat incomplete. Though it has been mentioned that the cars manufactured within a span of 3 model years are recalled but no numerical figure about the total number of cars recalled is clearly mentioned. In absence of this figure, it is not possible for anyone to understand exactly after how many repairs the rate suggested by the manufacturer becomes feasible. Moreover due to the very nature of learning phenomenon, the time required to complete the initial jobs of repair are bound to be more resulting in demand for higher rate by the workers.

(3) Solutions:

- From the dealer's data of repair of 1st and 2nd cars, we find the same 9.6 and 7.2 hours respectively. That means the Learning percentage is 7.2/9.6 = 0.75 or **75%** < 90%. Hence the actual learning percentage is less than that considered by the manufacturer.
- The manufacturer's rate per repair is ₹7000 @ ₹1700 per hour. Thus, time allocated per repair is 7000/1700 = **4.12 hours**
- Considering the dealer's figure of 9.6 hours for the 1st job, calculations for finding the number of repairs required to achieve 4.12 hours: $4.12 = 9.6.X^{-0.415}$; $X^{-0.415} = 4.12/9.6$; log X = 0.886; $X = 10^{0.886}$ = **7.7**. Thus the time considered by the manufacturer is achievable in the **8th repair**.
- Total number of repair jobs is not clearly spelt here, but it can be well assumed that the same would definitely be a few thousands.
- Demand of the workers is not justified because after doing only 8 repairs the rate suggested by the manufacturer is achievable and thereafter the actual labour hours will be further reduced causing the workers to earn good amount because whatever hours they take, the amount of ₹7000 they will definitely get.
- As explained above the fixed rate of ₹7000 is quite a good rate and hence the same does not need a higher revision.

⚡ Quick Revision — Section 12.12

- **Wright's Formula:** $Y = aX^b$; b = log(learning percentage)/log 2.
- **Doubling Pattern:** Time × Learning Rate for each doubling of output.
- **Non-Doubling Pattern:** Use logarithmic calculations (Log Y = log a + b log X).
- **Key Applications:** Cost estimation, bid pricing, working capital, inventory planning, make-or-buy.

12.13 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. A Learning Curve describes:

(a) The increase in number of units produced per unit time as the total number of units produced increases

(b) The rate at which an organisation acquires new information.

(c) The amount of production time per unit as the total number of units produced increases.

(d) The increase in production time as the total number of units produced increases.

Answer: C

2. Limitations of the Learning Curve approach include:

(a) Learning Curves must be redeveloped whenever the product or the production process is modified.

(b) Learning Curves are applicable when considering a highly automated process.

(c) Learning Curves are only valid when considering simple production process.

(d) Learning Curves are only valid when the total number of

units produced is relatively small.

Answer: A

3. Which of the following statements about Learning Curve is incorrect?

(a) A change in the process disrupts the Learning Curve.

(b) The rate of learning varies depending on the quality of management.

(c) The Learning Curve can be disrupted by the change in personnel.

(d) Learning Curves show that the time saved in completing each subsequent unit increases.

Answer: D

4. Which of the following is not an application of Learning Curve?

(a) Learning Curves allow a manager to predict the time required for new employee orientation on company policies and procedures.

(b) Learning Curves permit a manager to prepare a work schedule.

(c) Learning Curves allow a manager to forecast the labour requirements while preparing a departmental employee budget.

(d) Learning Curves can be employed in supply chain negotiations.

Answer: D

5. The Learning Curve remains valid:

(a) When applied to different firms in the same industry.

(b) For product modification that will simplify the product assembly.

(c) As long as process revisions involve only the addition of automated machinery.

(d) As long as production volume increases and processes are consistent.

Answer: D

6. A 100% Learning Curve implies that:

(a) Organisational learning has taken place.

(b) The Learning Curve can be applied universally across an industry.

(c) No learning has been achieved.

(d) Direct labour time is reduced 100% for each doubling of production.

Answer: C

7. Which of the following statement is true?

(a) The Learning Curve displays the relationship between process time per unit and the cumulative number of units produced.

(b) Learning Curves are most easily developed for one off type orders.

(c) Learning Curves are based on the Normal distribution.

(d) The standard time for a process is determined from the Learning Curve when cumulative production is fifty units.

Answer: A

8. Learning Curve is also known as:

(a) Growth curve

(b) Production curve

(c) Exponential curve

(d) Experience curve

Answer: D

9. For organisations with wide variety of product range which of the following statement is correct?

(a) Different product will have different Learning Curves.

(b) Learning percentage for different product will be different.

(c) Both (a) and (b)

(d) None of the above

Answer: C

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
C	A	D	D	D	C	A	D	C

State True or False

SL	STATEMENT	ANSWER
1	Learning Curves are based on the premise that employees and organisations get better at their tasks as they are repeated.	**True**
2	Learning Curves are based on the doubling of output.	**True**
3	Disruption of Learning Curve is possible if the members of a crew are changed.	**True**
4	Reduction rate of time to produce an item is higher for the curves with higher learning percentage.	**False**
5	Costs tend to decline more for standardized products and processes.	**True**
6	In Service sectors, the concept of Learning Curve is not applicable.	**False**
7	The steeper the Learning Curve, the faster is the decrease in time.	**True**
8	Time needed to complete a job is reduced by 90% when the learning percentage is 90.	**False**
9	Regardless of the industry, the Learning Curve remains same if the products or processes are same.	**False**
10	Lowering costs through a learning effect is not a natural consequence of the passage of time. These costs must be managed down.	**True**
11	If the learning percentage is 80 and the fourth unit took 16 hours to complete then the first unit should have taken 28 hours.	**False**
12	The equation log Y = log a + b log X is by no means related to the theory of Learning Curve.	**False**
13	For very large one off type jobs learning has a very minimal effect on its cost of production.	**True**
14	In case of Surgery, the effect of learning is most predominant.	**True**

SL	STATEMENT	ANSWER
15	Theory of Learning Curve is established on the assumption that no interruptions are there in the process of production.	**True**
16	Completion time of a product follows a G.P with common ratio same as learning percentage when the production process is repetitive in nature.	**True**
17	The process of learning is applicable to all the living beings of the world.	**True**
18	Consideration of Learning Curve is not important while deciding on the working capital requirement.	**False**
19	Force majeure situations like strike, lock out, shut down, natural calamities etc. have no effect on the Learning Curve.	**False**
20	Learning Curve has three phases, of which the last one is very steep.	**False**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	When the concept of learning was discovered then its curve was named as _____ curve.	**Forgetting**
2	Firms leapfrog the experience curve by means of innovation and _____	**Invention**
3	The Learning Curve is exponential in nature. In a log scale it is represented by _____ line.	**Straight**
4	Experience curve was developed by _____ Group in the year 1960.	**Boston Consulting**
5	Frequent training of new entrants is required for the industries with high ____ of employees.	**Turnover**
6	____ of the Learning Curve depends on the method of production.	**Slope**
7	Knowledge of Learning Curve assists in planning _____ of materials.	**Inventory**
8	Learning Curve shows that the crew size need not be ___at the same rate of increment of production.	**Increased**
9	Sensitive ____ bargaining with the union is possible when a clear knowledge of learning rate is there.	**Wage**
10	In the formula $Y = aX^b$ the symbol b stands for _____.	**Learning Index**
11	As per the theory of Learning Curve the time required to produce an item follows a ___ with each doubling of production.	**Geometric Progression**
12	80% Learning Curve shows a ___ reduction in average labour cost for every doubling of production.	**20%**

SL	STATEMENT	ANSWER
13	Any Learning Curve shows ____ decrease in average production time in the first phase.	**Steep**
14	Today's concept of Learning Curve is the result of a study by a German ____ named Ebbinghaus.	**Psychologist**
15	Learning effect is very predominant for _____ type task.	**Repetitive**
16	Studies show that there is no ____ accepted learning rate applicable anywhere & everywhere.	**Universally**

12.14 Essay Type Questions & Numerical Problems

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

Why should the learning percentage be higher in an Automatic process than in Manual process?

2

What would a learning of 95 percent mean?

3

If the learning phenomenon applies to all human activities, why the effect is not noticeable in short cycle production?

4

What are the phases of Learning Curve?

5

What is meant by Learning Curve ratio?

6

What do you understand by "Learning process"?

Essay Type Questions

1

What is Learning Curve and why it is important?

2

What are the limitations of Learning Curve?

3

Discuss about the various uses of Learning Curve.

Comprehensive Numerical Problems

PROBLEM 1 — MINIMUM LEARNING RATE

A tool manufacturing company wants to sell 49 units to a customer. Calculate the minimum learning rate if the 49th unit cannot take more than 5.6 hours. Given: 1st unit takes 16 hours.

ANSWER

Minimum learning rate = 83%

PROBLEM 2 — UNITS REQUIRED TO REDUCE TIME

Assume that the first unit of a very special product takes 15 labour hours to produce and the Learning rate is 74%. How many units should at least be produced to reduce the time per unit to 6 hours or less?

ANSWER

At least 9 units should be produced.

PROBLEM 3 — AVERAGE UNIT COST FOR A LOT

A lot of 20 units is to be produced. Labour cost is ₹150 per hour. Set up cost is ₹3,500 and Material cost is ₹1,500 per unit. The learning percentage is expected to be 90. Overhead is charged at the rate of 50% of total Material, Labour and Set Up cost. Determine the average unit cost for the lot when the first unit took 5 hours to complete.

ANSWER

Average unit cost = ₹3,226

PROBLEM 4 — INDIAN RAILWAYS BID (LEARNING CURVE)

The organisation to which you belong is a regular supplier of Indian Railways. Submission of bids against various tenders is a regular affair of the firm. As a part of the organisation's Costing Department your job is to help the Department who are dealing with these bids by calculating the costs.

Recently there is a limited tender for supply of 1,600 pieces of a product which is somewhat new to your organisation. Indian Railways want supply of batches of 100 at a time. It has been estimated that the average cost of the first batch of supply is ₹100 per piece. 90% Learning Curve applies to the cumulative labour cost on this contract.

(a) On the basis of the information above prepare an estimate of labour cost for the full contract. (b) As per the tender, a supplier will be awarded with another order of 800 pieces after the successful completion of supply of the first 1,600. Find out the incremental labour cost for producing an additional 800 unit?

ANSWER

(a) **Total cost of 1,600 pieces = ₹1,04,976**
(b) **Incremental cost for the additional 800 pieces = ₹43,078**

Hints: Additional 800 pieces after 1,600 means a total of 2,400 pieces which means 24 batches of 100 each. Also 2,400 pieces is not the double of 1,600. So the formula to be applied here is $Y = aX^b$ with a = ₹100, X = 24 and b = log(0.90)/log 2.

PROBLEM 5 — TRAINING PROGRAM STANDARD

A firm has a training program for an operation. The progress of the trainees is carefully monitored. An established standard requires a trainee to be able to complete the sixth repetition of the operation in 6 hours or less. Those who are unable to do this are assigned to other jobs. Currently, three trainees have each completed two repetitions: Trainee Ajay had times of 9 hours for the first and 8 hours for the second repetition; Trainee Badal had times of 10 hours and 8 hours for the first and the second repetitions and trainee Chanchal had times of 12 and 9 hours. Which trainee or trainees will achieve the standard? Explain with reasons.

ANSWER

Both Badal and Chanchal will achieve the standard because for them the time to complete the 6th repetition are respectively 5.6 hours and 5.7 hours i.e., less than 6 hours, but for Ajay, the time is **6.64 hours**.

PROBLEM 6 — VENDOR & AUTOMOBILE COMPONENT

The vendor of an automobile company is approached by the principals to build a special component for them. Based on the previous experience of similar components, the following data have been made ready by the Costing Department of the vendor: Breakeven quantity = 24 units; Time for producing the 24th unit = 161 hours; Learning rate = 80%; Direct labour cost = ₹120 per hour.

(a) Based on the above data, what quote should be given to the principals for the direct labour cost for an order of 30 units? (b) The quotation given as per above is accepted by the principals and a green signal is given to the vendor to start production. Although the first few units took longer than the estimated labour hours but thereafter it has started to stabilize. However the customer has made some changes in design after the production has started. They want the change to be implemented after the 15th unit. As per the estimation, the design change will cause 30 hours of new work for the 24th unit when the Learning Curve remains same. In fact the new work replaces the same amount of work of 30 hours for the 24th unit. What should be the revised quotation, if any, for the order of 30 units?

ANSWER

(a) **Quote for the direct labour cost for an order of 30 units = ₹5,39,421**

(b) **Revised quote for the direct labour cost to make 30 units = ₹5,83,000**

Hints: Due to design change there will be new learning effect from 16th unit onwards.

PROBLEM 7 — MULTIPLE CHOICE (LEARNING CURVE APPLICATION)

After studying the activities of the employees of one department of a large retail outlet, the manager has come out with a Learning Curve by computing the value of b using the relation b = log r / log 3 instead of the traditional way. Which statement below is best suited to this?

(a) If both the department under study and another department take 10 hours to complete a task for the first time then the time required for performing the task 30th time is lower for the department under study.

(b) If both departments have a learning rate of 90% and take 10 hours for performing a task for the first time then the time required by the department under study for performing the task 27th time is the same as the time required by the other department performing the task 8th time, given that the other department uses the traditional way of calculating b.

(c) This method is not valid since the denominator of the formula has log 3 instead of log 2.

(d) The employees of the department under study learn faster than the other employees.

ANSWER

Answer: B

PROBLEM 8 — 4TH UNIT COMPLETION TIME

How long will it take to produce the 4th unit with 80% learning if the first unit took 75 hours?

(a) 48 hours

(b) 117 hours

(c) 60 hours

(d) None of the above

ANSWER

Answer: A (48 hours)

Workings: $Y_4 = 75 \times (0.8)^2 = 75 \times 0.64 = 48$ hours.

PROBLEM 9 — COMPARATIVE LEARNING RATES (BIMAL VS KAMAL)

Bimal and Kamal are the two industrial workers engaged in doing a similar job. They have different learning rates of 80% and 90% respectively. Times taken to complete their first jobs are respectively 12 and 8 hours. If both continue with the same learning rate then after how many units Bimal will be faster than Kamal?

(a) 5th unit

(b) 7th unit

(c) 11th unit

(d) 19th unit

ANSWER

Answer: C (11th unit)

PROBLEM 10 — FIFTH UNIT WITH 85% LEARNING

How long will it take to produce the fifth unit with 85% learning rate, if the third unit took 13 hours?

(a) 10.3 hours

(b) 10.0 hours

(c) 11.4 hours

(d) 11.5 hours

ANSWER

Answer: D (11.5 hours)

PROBLEM 11 — TIME FOR 16TH UNIT (85% LEARNING)

When 24 hours is required to produce a condenser of a particular type then the time required to produce the 16th unit with 85% Learning Curve is:

(a) Between 9 and 10 hours

(b) Between 12 and 14 hours

(c) Between 15 and 17 hours

(d) Between 18 and 20 hours

ANSWER

Answer: B (Between 12 and 14 hours)

PROBLEM 12 — DIESEL ENGINE (4 ENGINES)

A diesel engine manufacturing company has an order of 4 large engines. A crew of 16 members took 4,000 hours to assemble the first engine. If 80% Learning Curve is used then what will be the labour cost of the fourth engine, assuming average labour rate to be ₹180 per hour?

(a) Between ₹4 to 4.5 Lakhs

(b) Between ₹3.5 to 4 Lakhs

(c) Between ₹3 to 3.5 Lakhs

(d) Between ₹4.5 to 5 Lakhs

ANSWER

Answer: D (Between ₹4.5 to 5 Lakhs)

PROBLEM 13 — BUILDER (10TH HOUSE)

A builder requires 4,000 hours to complete his first house. The experienced team of workers has a learning rate of 80%. How much time will be required to complete the 10th house?

(a) Between 1900 and 2000 hours

(b) Between 2000 and 2100 hours

(c) Between 2100 and 2200 hours

(d) Between 2200 and 2300 hours

ANSWER

Answer: A (Between 1900 and 2000 hours)

PROBLEM 14 — TOTAL TIME FOR FIRST 4 HOUSES

Using the data of the above question, what is the total time required to complete the first 4 houses?

(a) Between 4000 and 7000 hours

(b) Between 7000 and 9000 hours

(c) Between 9000 and 11000 hours

(d) Between 11000 and 13000 hours

ANSWER

Answer: D (Between 11000 and 13000 hours)

PROBLEM 15 — 90% LEARNING CURVE STATEMENT

For a 90% Learning Curve which one of the following statement is correct?

(a) Production time of each unit is 90% of the production time of the preceding unit.

(b) Direct labour time is reduced by 10% each time the production volume doubles.

(c) The learning process causes a 90% saving in the material content of the product.

(d) Only 10% defectives are produced during the process of learning.

ANSWER

Answer: B

PROBLEM 16 — LEARNING RATE DETERMINATION (STOP WATCH ERROR)

To determine the Learning Curve for an assembly activity in a factory, time study is carried out with the help of a stop watch which shows a minute in place of an actual time of 66 seconds. Which of the following statement suits best to describe this situation?

(a) The learning rate determined with this reading is 10% lower than the actual.

(b) The learning rate determined with this reading is 10% higher than the actual.

(c) A schedule to produce 50 units based on these calculations will have 10% excess time built into it.

(d) A schedule to produce 50 units based on these calculations will have 10% less time built into it.

ANSWER

Answer: C

PROBLEM 17 — 95% LEARNING CURVE (PROTOTYPE)

A manufacturing unit has 95% Learning Curve. To bag a huge order of a special item, the company made a prototype of the same. It took 25 hours to complete. As per the standard norms laid by the management of the company, the learning period will be over when the per unit production time is reduced by 20%. To achieve this norm for this special item, the production has to run continuously for:

(a) 736 hours

(b) 572 hours

(c) 428 hours

(d) 199 hours

ANSWER

Answer: C (428 hours)

PROBLEM	7	8	9	10	11	12	13	14	15	16	17
Answer	B	A	C	D	B	D	A	D	B	C	C

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Learning

The process by which human beings acquire a vast variety of competencies, skills, knowledge and attitudes.

Learning Curve

A correlation between a learner's performance on a task and the number of attempts or time required to complete the task; shows that less time is required at each iteration.

Cumulative Average Time

Average time per unit for all units produced so far, from and including the first one made; decreases by constant percentage every time output doubles.

Learning Curve Ratio

Average labour cost of first 2n units ÷ Average labour cost of first n units.

Learning Index (b)

b = [log (learning percentage)] / log 2; used in Wright's Formula $Y = aX^b$.

Wright's Formula

$Y = aX^b$ where Y = average time (or labour cost) per unit; a = time for the 1st unit; X = cumulative volume of production.

Experience Curve

The idea developed by the Boston Consulting Group (BCG) in the mid-1960s; the more experience a firm has in producing a particular product, the lower is its cost.

Forgetting Curve

Concept developed by Dr. Hermann Ebbinghaus (1885) showing how new information learnt tends to fade away from memory over time.

Learning Percentage

Slope of the Learning Curve; represents the rate at which cumulative average time decreases with doubling of output; 100% means no learning.

Plateau

Phase in Learning Curve where further significant improvements are not possible; stabilization of performance.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 12

ABBREVIATION	FULL FORM
LC	Learning Curve
Y	Average time (or labour cost) per unit
a	Time (or labour cost) for the first unit
X	Cumulative volume of production
b	Learning Index = log(learning percentage)/log 2
BCG	Boston Consulting Group
G.P.	Geometric Progression
LP	Learning Percentage
n	Number of units
2n	Doubling of output

STUDY TOOLS

Learning Curve Charts

Wright's Formula

Doubling Rule Tables

Formula Boxes

Quick Revision Sheets

MCQs with Reasoning

Numerical Problems

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Business Application of Maxima and Minima

Module 13 · Unconstrained Optimization · Equilibrium of a Firm · Constrained Optimization

EDITION	PAPER	SECTION	MODULE GROUP
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	Group with PERT/CPM, Learning Curve, Simulation

✓ Website Publication · ✓ PDF Conversion · ✓ A4 Printing · ✓ Mobile & Desktop Reading · ✓ Self Study · ✓ Grand Revision · ✓ Final & Last Day Revision · ✓ Personal Reference Book

13

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 13 INCLUDES

Introduction · **Unconstrained Optimization** (Single variable and Multiple variables) · **Equilibrium of a Firm** · **Constrained Optimization** (Lagrangian Multipliers).

Module Learning Objectives

After studying this module, the students will be able to:

1

Understand Applications

Understand the areas of application of Maxima and Minima with respect to business.

2

Single Variable Optimization

Solve the problems of optimization of Functions of One Variable with no constraint.

3

Multivariate Optimization

Solve the problems of optimization of Multivariate Functions with no constraint.

4

Constrained Optimization

Solve the problems of optimization of Multivariate Functions subject to equality constraint.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Differential Calculus to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

2	2	8	15
OPTIMIZATION TYPES	VARIABLES TYPES	ILLUSTRATIONS	MCQS
Unconstrained & Constrained	Single & Multiple	Worked examples	With answers

SYLLABUS STRUCTURE NOTE

Module 13 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Business Application of Maxima and Minima introduces differential calculus as a powerful tool for business decision making — finding optimal production levels, prices, cost minimization, profit maximization, and optimization under constraints.

📖 Table of Contents — Module 13

1. Introduction — Business & Profit Maximization
2. Review of Standard Formulae & Rules of Differentiation
3. Unconstrained Optimization — Single Variable
4. Unconstrained Optimization — Multiple Variables
5. Equilibrium of a Firm
6. Constrained Optimization — Lagrangian Multipliers
7. MCQs, True/False, Fill in the Blanks

8. Numerical Problems

13.1

Introduction — Business & Profit Maximization

BUSINESS OBJECTIVE · OPTIMIZATION TYPES · TRADITIONAL ECONOMIC THEORY

From the legal point of view the term **Business** is defined as "Any activity or enterprise entered into for Profit". So whenever we talk about business application of something, it boils down to profit making by an enterprise through its activities. Actually "Enterprise" is the term in the commercial world to describe a project or venture undertaken for gain. This is synonymous with the term Firm.

Thus business application of Maxima and Minima mainly refers to the process of finding the **optimal strategies** by a Firm to achieve its goal or objective. Economists believe that Business Firms always attempt to maximize their profit (or minimize their losses). They however do realise that businessmen have other goals like maximization of sales, maximization of firms' growth rate etc. for their survival which subsequently provide a quiet life. Yet the traditional economic theory assumes **profit maximization** as the sole objective of the business firms.

WHY PROFIT MAXIMIZATION ASSUMPTION WORKS

- In actual practice firms come fairly close to act like profit maximizers.
- It is helpful in correctly predicting the behaviour of the business firms with regard to the quantity of output produced and prices charged in the real world.

No wonder therefore, the whole traditional economic theory is developed on the basis of profit maximization (or loss minimization) hypothesis. As a result mathematical techniques are observed to be used to a great extent. The first condition for such techniques to be useful is that economic relationships or functions are expressed in **algebraic form** which in turn demands the objectives are well defined and transformed into quantitative statements.

Two Situations of Optimization

1

Unconstrained Optimization

Further divided into:

- **(A)** Optimization of single variable objective function
- **(B)** Optimization of objective functions having multiple variables

2

Constrained Optimization

Further divided into:

- **(A)** Equality constrained Optimization
- **(B)** Inequality constrained Optimization
- **(C)** Static Optimization
- **(D)** Dynamic Optimization

METHOD USAGE

Of all these types of optimization situations, concept of **Maxima and Minima of differential calculus** is used for both types of Unconstrained Optimization. Use of differential calculus is observed for Constrained Optimization with Equality Constraint. Though the same technique can be used for Constrained Optimization with Inequality Constraints, but in practice that has not gained much popularity.

⚡ Quick Revision — Section 13.1

- **Business:** Activity/enterprise entered into for profit.
- **Objective:** Profit maximization (or loss minimization) is assumed as the sole objective in traditional economic theory.
- **Two Optimization Types:** Unconstrained (single/multiple variables) & Constrained (equality/inequality).
- **Tool:** Differential calculus (Maxima and Minima).

13.2 Review of Standard Formulae & Rules of Differentiation

ESSENTIAL CALCULUS FORMULAE FOR OPTIMIZATION

As mentioned earlier, differential calculus is used a lot for study of the optimization in Economics. So recapitulation of it is necessitated. The standard formulae and rules are given below.

SL	FUNCTION	DERIVATIVE
1	$y = f(x) = x^n$	$dy/dx = f'(x) = n \cdot x^{n-1}$
2	$y = f(x) = e^x$	$dy/dx = f'(x) = e^x$, where e = constant = 2.718 (approx.)
3	$y = f(x) = e^{mx}$	$dy/dx = f'(x) = m \cdot e^{mx}$, where 'm' is a constant
4	$y = f(x) = \log_e x$ or ln x	$dy/dx = f'(x) = 1/x$
5	$y = f(x) = k$	$dy/dx = f'(x) = 0$, where k is a constant
6	$y = f(x) = p(x) \pm q(x)$	$dy/dx = f'(x) = p'(x) \pm q'(x)$

Partial Differentiation

In case of Partial differentiation, all the above formulae/rules hold good with the exception that all the terms present in the given function are considered as constant except the ones with the participating variable of partial differentiation.

Some examples are:

(a) If $U = xy$

$\partial U/\partial x = y$ and $\partial U/\partial y = x$

(b) If $U = x^a y^b$

$\partial U/\partial x = ax^{a-1}y^b$ and $\partial U/\partial y = bx^a y^{b-1}$ [where 'a' and 'b' are constants]

(c) If $U = px + qy$

$\partial U/\partial x = p$ and $\partial U/\partial y = q$ [where 'p' and 'q' are constants]

(d) If $U = a\sqrt{x} + b\sqrt{y}$

$\partial U/\partial x = a/2\cdot(x)^{-1/2}$ and $\partial U/\partial y = b/2\cdot(y)^{-1/2}$ [where 'a' and 'b' are constants]

⚡ Quick Revision — Section 13.2

- **Power Rule:** $d/dx(x^n) = n\cdot x^{n-1}$.
- **Exponential:** $d/dx(e^x) = e^x$; $d/dx(e^{mx}) = m\cdot e^{mx}$.
- **Logarithmic:** $d/dx(\ln x) = 1/x$.
- **Constant:** $d/dx(k) = 0$.
- **Sum/Difference:** $d/dx(p \pm q) = p' \pm q'$.
- **Partial Differentiation:** Treat other variables as constants.

13.3 Unconstrained Optimization — Single Independent Variable

NECESSARY & SUFFICIENT CONDITIONS · POINT OF INFLEXION

When the objective function is given as an Algebraic Function and no constraints are imposed, then Calculus approach of derivatives is used to optimize the function. The conditions of optimization are twofold and given as:

CONDITIONS OF OPTIMIZATION — SINGLE VARIABLE

Necessary Condition: $dy/dx = 0$ (First Order Derivative = 0)

Sufficient Condition (Maxima): $d^2y/dx^2 < 0$ at critical value

Sufficient Condition (Minima): $d^2y/dx^2 > 0$ at critical value

The value(s) of x obtained from the Necessary Condition is/are called **Critical Value(s)**. The sign of 2nd Order Derivative is checked by putting the Critical value(s) in it and subsequently decision regarding Maxima or Minima is taken as per Sufficient Condition above. When **$d^2y/dx^2 = 0$**, then there exist neither a Maxima nor a Minima. Such a point is known as **Point of Inflexion**.

Illustration 1 — Maximization Problem (Roller Coaster Ride)

PROBLEM

The demand (rides per day) of Roller Coaster Ride in an Entertainment Park in one of the metro cities is given by the equation $q = -450p + 41500$, where p = Price per ride in ₹. What price should have been charged to maximize the Total Revenue?

SOLUTION

Total Revenue is algebraically expressed as a function of price as follows:

$R(p)$ = Price per ride × Demand = $p \times q = p(-450p + 41500) =$ **$41500p - 450p^2$**

Differentiating both sides with respect to 'p': $d/dp\ [R(p)] = 41500 - 900p$ ... (i)

As per the necessary condition: $d/dp\ [R(p)] = 0 \rightarrow 41500 - 900p = 0 \rightarrow$ **$p = 46.11$**

To ascertain whether this value corresponds to a maxima, we check the sufficient condition:

Again differentiating (i): $d^2/dp^2\ [R(p)] = -900 < 0$

So there exists a **Maxima** at p = 46.11. Thus the price to be charged to maximize the Total Revenue is **₹46.11**.

Illustration 2 — Minimization Problem (Average Cost)

PROBLEM

Assume the Cost (₹) of manufacturing x numbers of a product per day is $C(x) = 14400 + 550x + 0.01x^2$. How many of the product should be manufactured per day so that the Average Cost is minimum? Also find the values of the Average Cost and the Total Cost at this level of production.

SOLUTION

Cost function is $C(x) = 14400 + 550x + 0.01x^2$

Average Cost function = $AC(x) = C(x)/x = 14400/x + 550 + 0.01x$

This is the Objective function which has to be minimized.

Differentiating: $d/dx\ [AC(x)] = -14400/x^2 + 0.01$ ... (i)

Necessary condition: $d/dx\ [AC(x)] = 0 \rightarrow -14400/x^2 + 0.01 = 0 \rightarrow 0.01x^2 = 14400 \rightarrow x^2 = 1440000 \rightarrow x = \pm 1200$

Since x (quantity) cannot be negative, **x = 1200**

Sufficient condition: $d^2/dx^2\ [AC(x)] = 28800/x^3$

For x = 1200: $d^2/dx^2\ [AC(1200)] = 28800/(1200)^3 = 1.67 \times 10^{-5} > 0$

So there exists a **Minima** at x = 1200.

At this level: Average Cost = 14400/1200 + 550 + 0.01×1200 = **₹574 per unit**

Total Cost = 14400 + 550×1200 + 0.01×1200² = **₹6,88,800**

⚡ Quick Revision — Section 13.3

- **Necessary Condition:** $dy/dx = 0$.
- **Sufficient Condition (Maxima):** $d^2y/dx^2 < 0$.
- **Sufficient Condition (Minima):** $d^2y/dx^2 > 0$.
- **Point of Inflexion:** $d^2y/dx^2 = 0$ (neither maxima nor minima).

13.4 Unconstrained Optimization — Multiple Independent Variables

HESSIAN MATRIX · SADDLE POINT · BIVARIATE FUNCTION CONDITIONS

For the situations where the Objective Function involves more than one Independent variable (say two) and the change in the dependent variable is the joint impact of changes in both the variables then the approach towards optimization of the Objective Function, though takes help of differential calculus, but is not exactly same as that used for the case of single Independent variable.

In fact the measurement of the independent impact of one variable is not possible without assuming that the other variable remains unchanged. As example we consider the case of Sales of a commodity which is a multivariate function of Price and Advertising Expenditure. Now the impact of change in Price over Sales cannot be measured if we do not assume the Advertising Expenditure to remain same. Partial Derivative of a function explains the same logic mathematically.

CONDITIONS OF OPTIMIZATION — BIVARIATE FUNCTION

Step 1: $f_x = 0$ and $f_y = 0$ (First Order Partial Derivatives = 0)

Step 2: Compute f_{xx}, f_{yy}, f_{xy} and f_{yx} at critical point (a, b)

Step 3: Let $A = f_{xx}(a,b)$, $B = f_{xy}(a,b)$, $C = f_{yy}(a,b)$

$D = AC - B^2$

If D > 0 and A, C > 0 → Local Minima at (a, b)
If D > 0 and A, C < 0 → Local Maxima at (a, b)
If D < 0 → Saddle Point at (a, b)
If D = 0 → The test fails

HESSIAN MATRIX

The expression $D = AC - B^2$ can also be represented in the form of a determinant as follows: **$D = |A\ B\ ;\ B\ C| = |f_{xx}\ f_{xy}\ ;\ f_{yx}\ f_{yy}|$**. As f_{xy} is equal to f_{yx} always, we represent both as B. This determinant actually corresponds to a (2 × 2) Matrix called **Hessian Matrix**. So Hessian Matrix of order 2 is given as ($f_{xx}\ f_{xy}$; $f_{yx}\ f_{yy}$) and the same is applicable when there are two variables in the problem. For problems with three variables, we will come across Hessian Matrix of order 3.

Illustration 3 — Maximization Problem (Two Products)

PROBLEM

A company produces two products x and y. The total Profit (in ₹ '000) earned by the company is expressed algebraically by the function $P = 100x - x^2 - 2xy + 200y - 3y^2$. Find the Profit maximizing quantities of the products. Also find out the maximum Profit.

SOLUTION

Profit function is given as: $P = 100x - x^2 - 2xy + 200y - 3y^2$

Differentiating partially with respect to x: $\mathbf{P_x = 100 - 2x - 2y}$ ... (I)

Differentiating partially with respect to y: $\mathbf{P_y = -2x + 200 - 6y}$ ... (II)

To determine the Critical Point: $P_x = 0$ and $P_y = 0$

$100 - 2x - 2y = 0 \rightarrow x + y = 50$ ... (1)

$-2x + 200 - 6y = 0 \rightarrow x + 3y = 100$ ... (2)

(2) – (1): $2y = 50 \rightarrow \mathbf{y = 25}$

Putting y = 25 in (1): **x = 25**. Thus Critical Point = (25, 25)

To check whether this is a local Maxima:

$P_{xx} = -2 \rightarrow A = -2 (< 0)$

$P_{yy} = -6 \rightarrow C = -6 (< 0)$

$P_{xy} = -2 \rightarrow B = -2$

$D = AC - B^2 = (-2)(-6) - (-2)^2 = 12 - 4 = \mathbf{8 > 0}$

Since D > 0 and A, C < 0, there is a **local Maxima** at (25, 25). Required Profit maximizing quantities: x = 25 units and y = 25 units. Maximum Profit = $100(25) - 25^2 - 2(25)(25) + 200(25) - 3(25)^2$ = **₹3,750 ('000)**

Illustration 4 — Critical Points & Saddle Point Analysis

PROBLEM

Find the Critical Points for the function $f(x,y) = x^2y - 2xy^2 + 3xy + 4$. Examine the presence of Local Maxima and Minima among the Critical Points.

SOLUTION

$f(x,y) = x^2y - 2xy^2 + 3xy + 4$

$f_x = 2xy - 2y^2 + 3y$ ... (1); $f_y = x^2 - 4xy + 3x$ ... (2)

Setting $f_x = 0$: $y(2x - 2y + 3) = 0$ ... (3)

Setting $f_y = 0$: $x(x - 4y + 3) = 0$ ... (4)

Solving (3) and (4): Critical Points are (0,0), (–3, 0), (0, 3/2) and (–1, 1/2)

Second Order Partial Derivatives: $f_{xx} = 2y$, $f_{xy} = 2x - 4y + 3$, $f_{yy} = -4x$

Evaluation at each Critical Point

CRITICAL POINT	$A = F_{XX}$	$B = F_{XY}$	$C = F_{YY}$	$D = AC - B^2$	CONCLUSION
(0, 0)	0	3	0	$-9 < 0$	**Saddle Point**
(–3, 0)	0	–3	12	$-9 < 0$	**Saddle Point**
(0, 3/2)	3	–3	0	$-9 < 0$	**Saddle Point**
(–1, 1/2)	1	–1	4	$3 > 0$	**Local Minima**

Since $D > 0$ and $A, C > 0$ at (–1, 1/2), there exists a **local Minima** at that point.

⚡ Quick Revision — Section 13.4

- **Step 1:** Set $f_x = 0$ and $f_y = 0$ to find Critical Point(s).
- **Step 2:** Compute $A = f_{xx}$, $B = f_{xy}$, $C = f_{yy}$.
- **Step 3:** $D = AC - B^2$.
- **D > 0 and A, C > 0:** Local Minima.
- **D > 0 and A, C < 0:** Local Maxima.
- **D < 0:** Saddle Point.
- **D = 0:** Test fails.

13.5 Equilibrium of a Firm

REVENUE · COST · PROFIT · MR = MC CONDITION · MULTI-PLANT · PRICE DISCRIMINATION

Equilibrium means a "State of Rest". In this State the forces working in opposite directions are exactly in balance so that there is no tendency to move in any direction. A firm is said to be in equilibrium when it selects a particular level of output at which it would like to "Stay" or "Rest". There is no incentive for the firm to increase or decrease output from that level. In other words, a firm is in equilibrium when, given the demand and cost conditions, it produces that level of output at which the Profit is maximised.

NEO-CLASSICAL THEORY

From some other point of view, a firm is supposed to be in equilibrium when its objective is optimised. The objective of a firm may be many, but the Neo Classical theory of Economics assumes maximization of Profit is the sole objective of it. The level of output and the price charged corresponding to the equilibrium are called the **Equilibrium Output** and the **Equilibrium Price** respectively.

Important Terminology

TERM	DEFINITION	FORMULA
Total Revenue (TR)	Product of Price (P) and Quantity Sold (x)	$TR = P \cdot x$
Average Revenue (AR)	Revenue earned per unit sale; equals Price	$AR = TR/x = P$
Marginal Revenue (MR)	Revenue earned by selling an additional unit of output	$MR = d(TR)/dx$
Total Cost (TC)	Sum of Fixed and Variable Costs	$TC = TFC + TVC$
Average Cost (AC)	Cost per unit of output	$AC = TC/x = AFC + AVC$
Marginal Cost (MC)	Cost of producing an additional unit of output	$MC = d(TC)/dx$
Total Profit (Tπ)	Residual after deducting Cost from Revenue	$T\pi = TR - TC = x(AR - AC)$
Average Profit (Aπ)	Profit per unit of output	$A\pi = T\pi/x = AR - AC$
Marginal Profit (Mπ)	Profit from an additional unit	$M\pi = MR - MC$

Condition for Firm's Equilibrium

EQUILIBRIUM CONDITION

$$\text{Necessary Condition: } d(T\pi)/dx = 0 \rightarrow MR = MC$$

$$\text{Sufficient Condition: } d^2(T\pi)/dx^2 < 0 \rightarrow d(MR)/dx < d(MC)/dx$$

The firm maximizes profit where Marginal Revenue equals Marginal Cost, and the Marginal Cost curve cuts the Marginal Revenue curve from below.

Conditions for Special Types of Firms

For Multi-Plant Monopolist Firm

Necessary Condition: $MR = MC_1 = MC_2$

Sufficient Condition: $d^2(TR)/dx^2 < d^2(TC_1)/dx_1^2$ and $d^2(TR)/dx^2 < d^2(TC_2)/dx_2^2$

$x = x_1 + x_2$; MC_1 and MC_2 are Marginal Costs of Plants 1 & 2

For Price Discriminating Monopolist Firm

Necessary Condition: $MC = MR_1 = MR_2$

Sufficient Condition: $d^2(TR_1)/dx_1^2 < d^2(TC)/dx^2$ and $d^2(TR_2)/dx_2^2 < d^2(TC)/dx^2$

MR_1 and MR_2 are Marginal Revenues for two market segments

Illustration 5 — Equilibrium Output, Price and Profit

PROBLEM

A firm has the Cost function $C = x^3/3 - 7x^2 + 111x + 50$ and Demand function $x = 100 - p$. Determine the Equilibrium Output, Price and Profit earned.

SOLUTION

Demand function: $x = 100 - p \rightarrow p = 100 - x$

Total Revenue: $TR = p \cdot x = (100 - x)x = 100x - x^2$

Profit: $\pi = TR - C = (100x - x^2) - (x^3/3 - 7x^2 + 111x + 50) = \mathbf{-x^3/3 + 6x^2 - 11x - 50}$

Differentiating: $d(\pi)/dx = -x^2 + 12x - 11$ ... (1)

Necessary condition: $-x^2 + 12x - 11 = 0 \rightarrow (x - 1)(x - 11) = 0 \rightarrow x = 1$ or $\mathbf{x = 11}$

Differentiating again: $d^2(\pi)/dx^2 = -2x + 12$

At $x = 1$: $-2(1) + 12 = 10 > 0 \rightarrow$ Minima

At $x = 11$: $-2(11) + 12 = -10 < 0 \rightarrow$ **Maxima**

Thus Profit is Maximum at **x = 11 units**. This is the required Equilibrium Output.

Equilibrium Price = $p = 100 - 11 =$ **₹89**

Equilibrium Profit = $-(11)^3/3 + 6(11)^2 - 11(11) - 50 =$ **₹111.33**

Illustration 6 — Multi-Plant Monopolist Firm

PROBLEM

A manufacturer produces a liquid commodity at two different Plants located in the two regions of the country. The selling price (in ₹/litre) of the product is given by the equation $p = 200 - 0.8x$, where $x = x_1 + x_2$ = Total production of the two Plants together. The Cost functions of the 2 Plants are given as $C_1 = 0.3(x_1)^2 + 60x_1 + 5000$ and $C_2 = 0.5(x_2)^2 + 30x_2 + 8000$. Determine the quantities produced by the two Plants which will put the manufacturer into an equilibrium condition.

SOLUTION

Total Revenue: $TR = p \cdot x = (200 - 0.8x)x = 200x - 0.8x^2$

Marginal Revenue: $MR = d(TR)/dx = 200 - 1.6x$ ... (1)

Marginal Cost for Plant 1: $MC_1 = dC_1/dx_1 = 0.6x_1 + 60$ ... (2)

Marginal Cost for Plant 2: $MC_2 = dC_2/dx_2 = 1.0x_2 + 30$ ... (3)

Necessary Condition of Equilibrium for Multi-Plant Firm: $MR = MC_1 = MC_2$

From (1) & (2): $200 - 1.6x = 0.6x_1 + 60 \rightarrow 1.1x_1 + 0.8x_2 = 70$ ... (4)

From (2) & (3): $0.6x_1 + 60 = 1.0x_2 + 30 \rightarrow x_2 = 0.6x_1 + 30$ ... (5)

Substituting (5) in (4): $1.1x_1 + 0.8(0.6x_1 + 30) = 70 \rightarrow$ **$x_1 = 29.1$**

Substituting $x_1 = 29.1$ in (5): **$x_2 = 47.5$**

Thus required equilibrium outputs are **29.1 litres from Plant 1** and **47.5 litres from Plant 2**.

Illustration 7 — Discriminating Monopolist Firm

PROBLEM

A discriminating Monopolist is able to separate its customers into two markets with respective Demand functions as $x_1 = 16 - 0.2p_1$ and $x_2 = 9 - 0.05p_2$. Total Cost function is $TC = 20 + 20x$, where $x = x_1 + x_2$. Determine the Equilibrium Price of the product in the two markets. Also determine the Equilibrium Profit.

SOLUTION

Demand function 1: $x_1 = 16 - 0.2p_1 \rightarrow p_1 = 80 - 5x_1$

Demand function 2: $x_2 = 9 - 0.05p_2 \rightarrow p_2 = 180 - 20x_2$

Revenue from Market 1: $TR_1 = p_1 \cdot x_1 = 80x_1 - 5x_1^2 \rightarrow MR_1 = 80 - 10x_1$ ... (1)

Revenue from Market 2: $TR_2 = p_2 \cdot x_2 = 180x_2 - 20x_2^2 \rightarrow MR_2 = 180 - 40x_2$ ... (2)

Marginal Cost: $MC = d(TC)/dx = 20$ ... (3)

Necessary Condition: $MC = MR_1 = MR_2$

From (1) & (3): $80 - 10x_1 = 20 \rightarrow \mathbf{x_1 = 6}$

From (2) & (3): $180 - 40x_2 = 20 \rightarrow \mathbf{x_2 = 4}$

Equilibrium Price in Market 1: $p_1 = 80 - 5(6) = \mathbf{₹50}$

Equilibrium Price in Market 2: $p_2 = 180 - 20(4) = \mathbf{₹100}$

Equilibrium Profit: $\pi = [80(6) - 5(6)^2 + 180(4) - 20(4)^2] - [20 + 20(6+4)] = \mathbf{₹480}$

⚡ Quick Revision — Section 13.5

- **Equilibrium:** State of Rest; firm maximizes profit.
- **MR = MC:** Necessary condition for equilibrium.
- **Sufficient Condition:** MC curve cuts MR curve from below (d(MR)/dx < d(MC)/dx).
- **Multi-Plant Monopolist:** $MR = MC_1 = MC_2$.
- **Price Discriminating Monopolist:** $MC = MR_1 = MR_2$.
- **Formulae:** $TR = P \cdot x$; $MR = d(TR)/dx$; $MC = d(TC)/dx$; $T\pi = TR - TC$.

13.6 Constrained Optimization

LAGRANGIAN MULTIPLIERS · BORDERED HESSIAN MATRIX · EQUALITY CONSTRAINTS

The Optimization technique discussed earlier is an example of Unbounded Maxima. It assumed that the Firm is capable of not only finding its Equilibrium Output but also producing it without any restrictions. All the resources required to produce the given level of output are at its command. It has no shortage of inputs, energy, labour, transport, liquidity etc. The economic actor under Neo Classical theory thus happened to be an unbounded maximizer.

In practical life the conditions are not exactly like the ones assumed in case of Unbounded Maxima. Firms need to compete for the procurement of inputs in the market. Scarcity of good quality energy is a very regular affair in our

country. Availability of labours of desired skill is another questionable area. Transportation of goods from one place to the other, particularly from production centres to the warehouses, is troublesome. Credits of required amount may not be available at the time when it is actually required or the rate of interest may be exorbitantly high. Thus working under restrictions is the most common phenomenon as far as operations of a Firm is concerned.

Mathematically the constraints are expressed either in the form of **Equations** (also known as Equalities) or in the form of **Inequalities**. Problems of equality constraints are dealt with **Lagrangian Multipliers** and those involving inequality constraints are solved by **Linear Programming** techniques.

Types of Constrained Optimization Situations

1

Utility Maximization

A consumer has to choose how much to buy of each product such that it satisfies the budget constraint.

2

Cost Minimization

A firm would look to minimize its cost of production subject to a given output level.

3

Output Maximization

A manufacturer would try to maximize its production with whatever quantity of a scarce material available.

Lagrangian Multiplier Method

To explain the steps involved, we take help of the following example: A consumer has a choice of two commodities X & Y. Prices of these are p and q respectively. The person has limited money (M) and wants to procure maximum possible quantities of X & Y with the amount he/she has. The problem can be rewritten as: **Maximize Utility function U = f(X,Y) subject to the constraint pX + qY = M**

Step 1 — Transform the Constraint Equation

Transform the constraint equation to a form with 0 on the R.H.S. In this case: $pX + qY - M = 0$

Step 2 — Multiply by Lagrange's Multiplier (λ)

Multiply L.H.S of the transformed Constraint Equation by Lagrange's Multiplier (λ). In this case it takes the form $\lambda(pX + qY - M)$

Step 3 — Form the Lagrangian Function

From the original Objective Function subtract the one obtained in the previous step to form Lagrangian function given by: **$L(X, Y, \lambda) = f(X,Y) - \lambda(pX + qY - M)$**. It can be noted that $L(X, Y, \lambda) = f(X,Y)$ when the constraint holds i.e., $pX + qY = M$. Hence maximization of Lagrangian function ultimately maximises the original objective function f(X,Y).

Step 4 — Find Critical Values (1st Order Conditions)

Find the critical values of the unknowns (X, Y and λ) using the 1st Order conditions i.e., all the partial derivatives are equal to zero: $\partial L/\partial X = 0 \rightarrow f_X - \lambda p = 0$; $\partial L/\partial Y = 0 \rightarrow f_Y - \lambda q = 0$; $\partial L/\partial \lambda = 0 \rightarrow pX + qY - M = 0$. Solving the above three equations one can get the values of three unknowns X, Y and λ.

Step 5 — Form the Bordered Hessian Matrix

Now find a Bordered Hessian Matrix (H_B) given as: H_B = (0 g_x g_y ; g_x L_{xx} L_{yx} ; g_y L_{xy} L_{yy}) where g_x and g_y are partial derivatives with respect to x and y respectively for the function given by g(X,Y) = pX + qY – M i.e., the given constraint expressed as a function.

Step 6 — Evaluate the Determinant

Find the value of the Determinant corresponding to the matrix H_B i.e., evaluate Det. H_B.

Step 7 — Interpret the Result

If Det.H_B > 0 then the critical values of X and Y obtained in step (4) corresponds to a **Maxima**. If Det.H_B < 0 then the critical values of X and Y obtained in step (4) corresponds to a **Minima**.

Illustration 8 — Constrained Optimization (TV Sets)

PROBLEM

Suppose a firm produces TV Sets at two different locations which produced x_1 and x_2 sets respectively. The joint cost function is given as C = $0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$. If the firm has to supply 1000 sets of TV then find the number of sets to be produced in the two plants so that the joint cost is minimum.

SOLUTION

The firm has to supply 1000 sets in total. So $x_1 + x_2 = 1000$

Thus the problem becomes: Minimize $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$ subject to $x_1 + x_2 = 1000$

Constraint expressed as: $g(x_1, x_2) = x_1 + x_2 - 1000 = 0$

Lagrangian Function: $\mathbf{L(x_1, x_2, \lambda) = C - \lambda(x_1 + x_2 - 1000)}$

Differentiating partially:

$L_{x_1} = 0.2x_1 + 0.2x_2 + 180 - \lambda = 0$ ... (1)

$L_{x_2} = 0.4x_2 + 0.2x_1 + 60 - \lambda = 0$ ... (2)

$L_\lambda = -(x_1 + x_2 - 1000) = 0 \rightarrow x_1 + x_2 = 1000$ ... (3)

From (1): $0.2(x_1 + x_2) + 180 - \lambda = 0 \rightarrow 0.2(1000) + 180 - \lambda = 0 \rightarrow \mathbf{\lambda = 380}$

From (2): $0.2x_2 + 0.2(1000) + 60 - 380 = 0 \rightarrow 0.2x_2 = 120 \rightarrow \mathbf{x_2 = 600}$

Putting $x_2 = 600$ in (3): $\mathbf{x_1 = 400}$

Now $g(x_1, x_2) = x_1 + x_2 - 1000$; $g_{x_1} = 1$, $g_{x_2} = 1$

Second Order Partial Derivatives: $L_{x_1x_1} = 0.2$, $L_{x_1x_2} = 0.2$, $L_{x_2x_2} = 0.4$

Bordered Hessian Matrix: $H_B = (0\ 1\ 1 ; 1\ 0.2\ 0.2 ; 1\ 0.2\ 0.4)$

Determinant: Det. $H_B = 0(0.2\times0.4 - 0.2\times0.2) - 1(1\times0.4 - 0.2\times1) + 1(1\times0.2 - 0.2\times1) = -0.2 < 0$

As Det. $H_B < 0$, there exists a **Minima** at $x_1 = 400$ & $x_2 = 600$. Hence Cost minimisation is possible by producing **400 TV Sets in 1st Plant** & **600 Sets in 2nd Plant**.

⚡ Quick Revision — Section 13.6

- **Constrained Optimization:** Optimization with restrictions; constraints expressed as equations or inequalities.
- **Lagrangian Multiplier Method:** Used for equality-constrained optimization.
- **Lagrangian Function:** $L = f(X,Y) - \lambda \cdot g(X,Y)$.
- **1st Order Conditions:** $\partial L/\partial X = 0$, $\partial L/\partial Y = 0$, $\partial L/\partial \lambda = 0$.
- **Bordered Hessian Matrix:** Used to test for Maxima/Minima.
- **Det. $H_B > 0$:** Maxima; **Det. $H_B < 0$:** Minima.

13.7 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. Optimization is the method of finding:

(a) The maximum point (b) The minimum point (c) The critical point

(d) All of the above

Answer: D

2. Choose the correct answer:

(a) Optimization problems should have only one objective function (b) Constraint functions are compulsory for any optimization problem. (c) Objective function must be a continuous function

(d) None of the above

Answer: A

3. The process of finding relative maximum or minimum of a function is known as:

(a) Optimization (b) Maximization (c) Minimization

(d) Any of these

Answer: A

4. For a Cost Function $TC = 3Q^2 + 7Q + 12$, MC is:

(a) 6Q (b) 6Q + 7 (c) 3Q + 12

(d) None of the above

Answer: B

MC = d(TC)/dQ = 6Q + 7.

5. MR is:

(a) First order derivative of TC (b) Second order derivative of TR (c) First order derivative of TR

(d) Second order derivative of TC

Answer: C

6. In unconstrained optimization with single variable the sufficient condition for maximization is:

(a) Second order derivative of the objective function must be zero.

(b) Second order derivative of the objective function must be less than zero

(c) Second order derivative of the objective function must be less than zero.

(d) None of the above

Answer: B

7. In case of unconstrained optimization involving two variables the necessary condition is:

(a) First order derivative of the objective function with respect to the variables should be zero.

(b) First order partial derivative of the objective function with respect to the variables should be zero.

(c) Either one of (a) and (b)

(d) Both (a) and (b)

Answer: B

8. A Firm is said to achieve Condition of equilibrium when:

(a) Its objective is optimized.

(b) Its profit is maximized.

(c) Its loss is minimized

(d) All of the above.

Answer: D

9. In the expression $D = AC - B^2$ used for describing the sufficient conditions for unconstrained optimization involving two variables (x and y), the meaning of A and C are:

(a) 2nd order partial derivative of the objective function (f) with respect to x and y respectively.

(b) 2nd order partial derivative of $\partial f/\partial x$ with respect to y

(c) Both (a) and (b)

(d) Only (a) but not (b)

Answer: D

10. A price discriminating Monopolist Firm operates in:

(a) Such a Market where it is the sole supplier.

(b) More than one Market.

(c) Markets where it sells same product but in different prices.

(d) All of the above.

Answer: D

11. In the expression $D = AC - B^2$ used for describing the sufficient conditions for a dual variable unconstrained optimization the term D is known as:

(a) Hessian Matrix of order 2

(b) Determinant for Hessian Matrix of order 2.

(c) Matrix of partial derivatives of order 2.

(d) Determinant of the Matrix of partial derivatives

Answer: B

12. For a dual plant Monopolist Firm with respective production costs C_1 & C_2 in the two plants, the necessary condition of equilibrium is:

(a) $MC_1 = MC_2 \neq MR$

(b) $MC = MR$

(c) $MC_1 = MC_2 = MR$

(d) $MC_1 = MR_1$ & $MC_2 = MR_2$

Answer: C

13. Use of Lagrange's Multiplier is seen while:

(a) Solving a problem of unconstrained optimization with single variable.

(b) Solving a problem of optimization with inequality constraints.

(c) Solving a problem of optimization with one equality constraint.

(d) Solving a problem of optimization having no constraint.

Answer: C

14. For a firm the total cost function is $C(x) = -0.5x^2 + 11x + 600$. Which of the following statement is incorrect?

(a) Average variable cost function is $AVC(x) = -0.5x + 11$

(b) Marginal cost function is $MC(x) = -x + 11$

(c) Cost of producing 10 units is ₹710/-

(d) Average cost function is $AC(x) = -0.5x + 11 + 600/x$

Answer: C

Cost of producing 10 units = $-0.5(100) + 11(10) + 600 = -50 + 110 + 600 = ₹660$ (not ₹710).

15. Which one of the following statement is not correct?

(a) Average Revenue of a Firm is same as the price at which its product is sold.

(b) Total Profit is the product of quantity sold and the difference of Average Revenue and Average Cost.

(c) When Marginal Revenue is zero then Total Revenue is maximum

(d) None of the above.

Answer: D

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
D	A	A	B	C	B	B	D	D	D	B	C	C	C	D

State True or False

SL	STATEMENT	ANSWER
1	Profit maximization is not the sole objective of a Firm.	**True**
2	Static Optimization & Dynamic Optimization are two types of Unconstrained Optimization.	**False**
3	For unconstrained optimization involving single variable the sufficient condition to get a Minima is $d^2y/dx^2 < 0$.	**False**
4	Bordered Hessian Matrix is a square matrix of order 3.	**True**
5	If for a firm the Cost function is C = wl + rk. To minimize Cost subject to the constraint $y = k^{\alpha}.l^{\beta}$, the Lagrangian Function is given by $L = C - \lambda(k^{\alpha}.l^{\beta} - y)$	**True**
6	The critical values of x and y for optimization of the function $z = x^2 + y^2 + 0.5xy$ subject to $y = 90 - 2x$ are respectively x = 39.375 and y = 11.25	**True**
7	For multivariate unconstrained optimization involving two variables x and y, the necessary conditions require df/dx as well as df/dy are not equal.	**False**
8	The value of Hessian Determinant to be zero is the condition of getting a Saddle Point in the unconstrained optimization involving variables x and y.	**False**
9	Lagrangian Method can be used for constrained optimization problems involving two equality constraints.	**True**
10	The problem, Minimize $Z = -7x^2 + 6xy - 9y^2$ subject to $2x + y = 165$ can be solved only by using Lagrangian Method.	**False**
11	If the Lagrangian function is $L = 4x^2 - 5xy + 6y^2 + \lambda(x + y - 30)$ then the critical values of x and y are 17 & 13 respectively.	**True**
12	Problems of constrained optimization with multiple equality constraints cannot be solved by Lagrangian Method.	**False**
13	A multiproduct firm has profit function given by $\pi = 100x - x^2 - 2xy + 200y - 3y^2$. The value of Hessian determinant will be (–8).	**False**
14	D = 0 gives inconclusive result for a problem of dual variable unconstrained optimization, where D is the Hessian Determinant.	**True**
15	Price discriminating Monopolists operate in different markets with different products.	**False**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	Business is an activity or enterprise entered into for _____.	**Profit**
2	Unconstrained and _____ optimization are the two situations under which optimization is carried out.	**Constrained**
3	Unconstrained optimization is also known as ____ Maxima technique.	**Unbounded**
4	Calculus approach of _____ is used to optimize Objective Functions not subjected to any restriction.	**Derivatives**
5	$d^2y/dx^2 = 0$ indicates existence of a _____ for the curve of the Objective Function.	**Point of Inflexion**
6	(f_{xx} f_{xy} ; f_{yx} f_{yy}) is known as _____ determinant and used in unconstrained optimization of multivariate function.	**Hessian**
7	To optimize an objective function with single equality constraint _____ method is used.	**Lagrangian**

13.8 Comprehensive Numerical Problems

PRACTICE PROBLEMS WITH ANSWERS

PROBLEM 1 — TWO WHEELERS & THREE WHEELERS

A manufacturing company selling two wheelers (x) and three wheelers (y) has the following Demand functions: $p_x = 40 - 0.02x - 0.01y$ and $p_y = 80 - 0.06y - 0.01x$. Find the Revenue maximizing levels of output and price of the Two Wheelers as well as Three Wheelers. What is the maximum Total Revenue? The prices are in ₹'000. Ensure the Total Revenue obtained is maximum by 2nd Order derivative test.

ANSWER

Revenue maximizing output of two and three wheelers are respectively **727 and 545 numbers** and the corresponding prices are **₹20,000 and ₹40,000**. Maximum Total Revenue = **₹3,63,40,000**

PROBLEM 2 — PRODUCTION FUNCTION (COBB-DOUGLAS)

The Production function of a Firm is given as $Q = f(K,L) = K^{0.5}L^{0.25}$ and the prices of Capital (K) and Labour (L) are respectively w and r. Derive the Cost function of the Firm. Find the cost minimizing combination of Capital and Labour.

ANSWER

Cost function is C = rk + wl & Cost minimizing combination of Labour and Capital is given as: **$K = \sqrt[3]{[(2w/r)Q^4]}$ and $L = \sqrt[3]{[(r^2/4w^2)Q^4]}$**

PROBLEM 3 — SOCIAL MEDIA & TV ADVERTISING

A manufacturer estimates his Annual Sales (in units) as a function of the expenditure made for Social Media and TV Advertising as $Z = 50000x + 40000y - 10x^2 - 20y^2 - 10xy$, where Z denotes Number of TVs sold per year and x & y denote Amounts spent on TV & Social Media Advertising in ₹'000. Determine how much amounts should be spent on the two types of Advertising in order to maximize the number of TVs sold.

ANSWER

Amounts to be spent on TV and Social Media Advertising are respectively **₹22,85,720 and ₹4,28,570**

PROBLEM 4 — MONOPOLIST (TWO PRODUCTS)

A monopolist offers 2 products which have the demand functions given as $q_1 = 14 - p_1/4$ & $q_2 = 24 - p_2/2$. The monopolist's joint cost function is $C(q_1, q_2) = (q_1)^2 + 5q_1q_2 + (q_2)^2$. Determine the quantities to be sold in order to maximize the Total Profit.

ANSWER

The quantities to be sold are **$q_1 = 2.75$ and $q_2 = 5.7$**.

PROBLEM 5 — MARGINAL COST FALLING CONTINUOUSLY

If the relation between Total Cost (y) and Output (x) is $y = 3x(x + 7)/(x + 5) + 5$, prove that the Marginal Cost falls continuously as the Output increases.

ANSWER

Differentiating the Total Cost function with respect to x and showing that the derivative is decreasing (i.e., $d^2y/dx^2 < 0$) proves that Marginal Cost falls continuously as Output increases.

PROBLEM 6 — OPTIMIZATION WITH CONSTRAINT (TV SETS)

Suppose a firm produces TV Sets at two different locations which produced x_1 and x_2 sets respectively. The joint cost function is given as $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$. If the firm has to supply 1000 sets of TV then find the number of sets to be produced in the two plants so that the joint cost is minimum.

ANSWER

As Det. $H_B < 0$, there exists a Minima at **$x_1 = 400$ & $x_2 = 600$**. Hence Cost minimisation is possible by producing **400 TV Sets in 1st Plant & 600 Sets in 2nd Plant**.

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Business

Any activity or enterprise entered into for Profit; synonymous with Firm or Enterprise.

Total Revenue (TR)

The product of Price (P) and the Quantity Sold (x); TR = P·x.

Average Revenue (AR)

Revenue earned per unit sale; AR = TR/x = P (Price).

Marginal Revenue (MR)

Revenue earned by selling an additional unit of output; MR = d(TR)/dx.

Marginal Cost (MC)

Cost of producing an additional unit of output; MC = d(TC)/dx.

Equilibrium of a Firm

State of Rest; firm maximizes profit at the level where MR = MC.

Point of Inflexion

Point where $d^2y/dx^2 = 0$; neither a Maxima nor a Minima exists.

Saddle Point

Critical point where $D = AC - B^2 < 0$; neither maximum nor minimum.

Hessian Matrix

Matrix of second-order partial derivatives used to test optimization conditions.

Lagrangian Multiplier

Method for solving constrained optimization with equality constraints; $L = f(X,Y) - \lambda \cdot g(X,Y)$.

Bordered Hessian Matrix

Matrix used in constrained optimization to test for maxima/minima; Det. $H_B > 0 \rightarrow$ Maxima; $< 0 \rightarrow$ Minima.

Price Discriminating Monopolist

Firm selling same product at different prices in different markets; equilibrium condition $MC = MR_1 = MR_2$.

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 13

ABBREVIATION	FULL FORM
TR	Total Revenue
AR	Average Revenue
MR	Marginal Revenue
TC	Total Cost
AC	Average Cost
MC	Marginal Cost
Tπ	Total Profit
Aπ	Average Profit
Mπ	Marginal Profit
H_B	Bordered Hessian Matrix
λ	Lagrange's Multiplier

13.2 Review of Differentiation

13.3 Unconstrained Optimization — Single Variable

13.4 Unconstrained Optimization — Multiple Variables

13.5 Equilibrium of a Firm

13.6 Constrained Optimization

13.7 MCQs, True/False, Fill in the Blanks

13.8 Numerical Problems

Hessian Matrix

Lagrangian Method Steps

Formula Boxes

MCQs with Reasoning

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Business Forecasting Models

Module 14 · Time Series and Regression Analysis

EDITION	PAPER	SECTION	MODULE WEIGHTAGE
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	10% (Business Forecasting)

✓ Website Publication ✓ PDF Conversion ✓ A4 Printing ✓ Mobile & Desktop Reading ✓ Self Study
✓ Grand Revision ✓ Final & Last Day Revision ✓ Personal Reference Book

14

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 14 INCLUDES

Introduction — Importance of Business Forecasting in different areas of an organisation · **Pros and Cons of Business Forecasting** · **Limitations of Business Forecasting** · **Types of Business Forecasts** · **Methods or models of Business Forecasting** · **Quantitative Methods** — Regression Analysis, Time Series Analysis, Exponential Smoothing, Input-Output Models · **Qualitative Methods** — Historical Analogy Method, Executive Opinion Method, Survey Techniques, Barometric Techniques, Delphi Technique.

Module Learning Objectives

After studying this module, the students will be able to:

1

Basics of Business Forecasting

Acquire knowledge on the basics of Business Forecasting.

2

Importance

Understand importance of Business Forecasting in different areas of an organisation.

3

Pros & Cons

Understand the pros and cons of Business Forecasting as well as its limitations.

4

Types & Methods

Have idea about different types as well as methods of Business Forecasting.

5

Time Series

Understand how concept of Time Series is used for Business Forecasting.

6

Regression Analysis

Understand how concept of Regression Analysis is used for Business Forecasting.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Business Forecasting techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

10%	2	4	30
MODULE WEIGHTAGE	CORE METHODS	TIME SERIES COMPONENTS	MCQS
Of Section B total	Quantitative & Qualitative	Trend, Seasonal, Cyclical, Irregular	With answers

SYLLABUS STRUCTURE NOTE

Module 14 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. Business Forecasting Models covers a weightage of **10%** in the examination. This module is highly practical and requires strong understanding of statistical tools — Regression Analysis, Time Series Analysis, Moving Averages, Exponential Smoothing, and various qualitative forecasting methods.

📖 Table of Contents — Module 14

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2. Importance of Business Forecasting
3. Pros and Cons of Business Forecasting

4. Limitations of Business Forecasting
5. Types of Business Forecasts
6. Methods or Models of Business Forecasting
7. Regression Analysis
8. Time Series Analysis
9. Exponential Smoothing
10. Input–Output Model
11. Qualitative Methods of Forecasting
12. MCQs, True/False, Fill in the Blanks
13. Numerical Problems

14.1

Introduction to Business Forecasting

CONCEPT · PURPOSE · STEPS IN FORECASTING

Business forecasting is very popular in today's world because any management is interested to know if the organisation to which they belong is financially on track. The more clarity they have over financial situation and future sales, the more empowered are they to take intelligent business decisions and ensure success.

There are many ways to judge the financial situation of a business, but **forecasting is the best one for predicting the future events and trends**. If the management of a company wants to know the sales figures of a future date, forecasting technique can help them analyse past and present data to generate answers. Actually Forecasting is a technique of a business's future developments based on trends, patterns and analysis of data — both current and historical. Business forecasting allows a company to make long term plans and prepare for any changes in the market.

Based on the unique needs of a business, the approach to business forecasting must be targeted and tweaked to be useful. The business forecasting models typically involves the following steps:

Collecting Primary & Secondary Data
Sources of information

→

Analysing the Datasets
Statistical tools

→

Creating Strategies for Projections
Model selection

→

Comparing Forecast with Realised Outcomes
Validation

PURPOSE OF BUSINESS FORECASTING

An accurate business forecast helps to create business budgets, allocate funds, make decisions about cash flow and credit needs and to create timelines for new initiatives or acquisitions. There are different forecasting methods one can use to make predictions. What will work best depends on the circumstances. A start-up business with almost no past data would not use the same method as an already established organisation.

KEY INSIGHT

One should know why forecasting is important and what approaches are worthwhile for his company before putting enough efforts for forecasting.

⚡ Quick Revision — Section 14.1

- **Business Forecasting:** Technique of a business's future developments based on trends, patterns and analysis of data — both current and historical.
- **Purpose:** Create business budgets, allocate funds, decide cash flow and credit needs, create timelines for new initiatives.
- **Four Steps:** Collecting data → Analysing datasets → Creating projection strategies → Comparing forecast with realised outcomes.

14.2 Importance of Business Forecasting

DIFFERENT AREAS OF AN ORGANISATION · BEYOND INTER-BUSINESS BENEFITS

Business forecasting is one of the most vital business tools been used by organisations towards future projections. Its importance is manifested in the following areas:

1

Valuable Insight

Management can set smarter goals, make informed business decisions and create strategies that reflect the insights they can gain from forecasted data.

2

Resource Allocation & Control

Facilitates allocation of resources among the functional areas of the organisation as well as control the operations of the entity.

3

Purchasing & Supply

Helps purchasing and supply department towards planning and procurement of raw materials as well as scheduling their delivery time.

4

Finance Department

Useful for the finance department towards planning for fund sourcing to forestall cash flow constraint.

5

Human Resources

Helps human resources department to plan the manpower requirement as the time it is required.

6

Production Scheduling

Plays a deciding role for a manufacturing entity to schedule its production — when to produce at full capacity and when not to.

7

Return on Investment

Useful to determine well in advance expected return on investment fairly correct.

8

Learning from Past

Facilitates learning from past mistakes causing eradication of weaknesses in the current system.

9

Cost Reduction & Profit Increase

Useful for cost reduction and profit increase. The management can stick to an accurate budget based on current market conditions and expected future outcomes.

10

Foresee Market Trends

Helps to foresee market trends causing the management to take measures proactively.

BEYOND INTER-BUSINESS BENEFITS

Any management can also use forecasting to keep customers happy by providing services and the products they are looking for. This leads to greater brand loyalty and of course better profits in the future. A forecast can also help businesses apply for loans or new lines of credit since many financial institutions will request a report before approving a commercial application.

⚡ Quick Revision — Section 14.2

- **Ten Areas:** Valuable insight, Resource allocation, Purchasing & supply, Finance, HR, Production scheduling, ROI, Learning from past, Cost reduction, Market trends.
- **Beyond Business:** Customer satisfaction, Brand loyalty, Loan applications.

14.3

Pros and Cons of Business Forecasting

ADVANTAGES AND CHALLENGES

Forecasting relies heavily on the past data. Although past events are analysed as a guide to the future, a question is always raised as to the accuracy of these recorded events. In fact the predictions are never 100% accurate, even if they are based on actual numbers of the past.

PROS OF BUSINESS FORECASTING

- Results from committing to forecasting as a business activity are usually worth the effort.
- One can gain a lot of clarity or even debunk some of the assumptions which are holding back from realizing greater profits.
- Predicting future trends and events enables informed business decisions.
- Helps create business budgets, allocate funds, make decisions about cash flow and credit needs.
- Allows a company to make long term plans and prepare for any changes in the market.

VS

CONS OF BUSINESS FORECASTING

- Any management strives for accuracy which needs more thorough collection and analysis of data.
- Thorough data collection and analysis can be time consuming, expensive and resource intensive.
- Not every business can handle the cost and effort.
- That's why many companies don't use forecasting as regularly or consistently as they should.

BOTTOM LINE

However, the results that come from committing to forecasting as a business activity are usually worth the effort because one can gain a lot of clarity or even debunk some of the assumptions which are holding back from realizing greater profits. **The bottom line is that predicting future trends and events enables informed business decisions.**

⚡ Quick Revision — Section 14.3

- **Pros:** Worth the effort, gain clarity, debunk assumptions, informed decisions, budgets, long term plans.
- **Cons:** Time consuming, expensive, resource intensive, not all businesses can handle.
- **Bottom Line:** Predicting future trends enables informed business decisions.

14.4 Limitations of Business Forecasting

SIX KEY LIMITATIONS

Business Forecasting, though a very useful tool for present day businesses, has the following limitations:

1

Unreliable Past Data

The past data used for forecasting cannot be relied upon to a great extent.

2

Bias in Judgement

Accuracy in judgement is almost impossible due to presence of bias.

3

Inconsistent Measurements

Inconsistencies exist in the measurement of forecasts.

4

Tedious Methods

Some methods of forecasting are tedious and cumbersome.

5

Expensive

In most cases forecasting is expensive. It increases the price of the products while failing to reduce the uncertainty attached to the business.

6

Doubtful Data Veracity

The veracity of data may be doubtful as they can be wrongly compiled and recorded.

⚡ Quick Revision — Section 14.4

- **Six Limitations:** Unreliable past data, Bias in judgement, Inconsistent measurements, Tedious methods, Expensive, Doubtful data veracity.

14.5 Types of Business Forecasts

SIX KEY TYPES OF FORECASTS

There can be various types of Business forecasts depending on what the company wants to know or predict. It can range from the general (Sales next quarter) to the incredibly specific (Demand for a specific product during rainy season). Most common types of business forecasts are as follows:

1

General Business Forecasting

Used to determine the overall business climate for a future date and can be widely applied for many different businesses and industries. Used for determining overall market conditions and the impact of the environmental factors in which the business operates. Best suited for businesses operating in influential environments, such as countries experiencing political upheavals, major technological advancements, dramatic seasonal shifts etc.

2

Financial Forecasting

About getting a clear picture of where a company is headed for. It includes weighing assets and liabilities, accounts payable and receivable, operating costs, capital structure and cash flow and general market conditions. Used for tracking the future trajectory of a company as a whole. Best suited for any business looking to stay on top of its business's health through financial projections.

3

Accounting Forecast

The practice of predicting the future costs which will be incurred by a company, using past and present data to estimate how much the business will pay for raw materials, inventory, man hours, utilities and rent, insurance and more. Used for determining the future operating costs of a business and suitable for any business concerned with covering future costs.

4

Demand Forecasting

Goes hand in hand with a sales forecast. Demand forecast will predict what the market needs and the sales forecast will predict how a business will be able to capitalize with those needs. Used for determining market and customer demand for any goods or services in the future. Best suited for planning how much to invest in raw materials or inventory, deciding if a new product will perform well.

5

Sales Forecasting

Used to estimate future sales of a specific product or service within the range of offerings of a business, using the available sales data. It allows the management to anticipate the future needs of workforce, resources, cash flow, inventory and capital investment. It provides the management with the figures of expected revenue over the next month or next quarter or next year of a sales cycle. Best suited for businesses relying solely on sales history or looking to project sales for investors and funding.

6

Capital Forecasting

Based on current and future assets and liabilities as well as predictions for liquid capital and cash flow estimates. It is tricky and not as reliable as the other types of forecasts because it involves a lot of factors like Cash and Savings, Assets, Accounts receivable, Revenue, Investment funding, Lines of credit etc. Mainly used for predicting available capital for a future date or event and best suited for companies preparing for investment, growth, hiring, acquisitions or other changes that will require cash.

⚡ Quick Revision — Section 14.5

- **Six Types:** General Business, Financial, Accounting, Demand, Sales, Capital Forecasting.
- **Demand vs Sales:** Demand forecast predicts market needs; Sales forecast predicts business's ability to capitalize on those needs.
- **Capital Forecasting:** Most tricky and least reliable due to multiple factors.

14.6 Methods or Models of Business Forecasting

QUANTITATIVE & QUALITATIVE METHODS

A **Forecasting Type** identifies the target a company is pursuing (like sales, cash flow etc.) whereas **Forecasting Method** deals with the method of gathering and identifying data (qualitative versus quantitative). In fact the Forecasting Method is the tool a company uses to gather and evaluate the relevant data for its Forecasting Type.

While there are several forecasting methods or models, they all fall within two general categories — **QUANTITATIVE** and **QUALITATIVE**. Depending on the available data and the age of the business, one approach will be more beneficial than the other.

QUANTITATIVE METHODS

- Focus on numbers and measurable data
- Focus on existing data, numbers and formulae
- Very little human interfacing
- Analysis is statistical
- Outcomes are conclusive
- Patterns observed provide a straightforward course of action
- Helpful when businesses are adding new services or products or making adjustments to existing ones
- Beneficial for predicting sales figures from one year to the next

VS

QUALITATIVE METHODS

- Aim to gain a qualitative understanding of a given subject, problem or point of interest
- Focus on human feedback from experts and customers
- Unstructured, broad, non-statistical approach
- Subjective interpretations
- Outcome helps management develop further understanding of a previously posed question
- Best for businesses that don't have enough raw data to reach an accurate quantitative forecast
- Used by companies who don't need numbers to lead them in the right direction but want to know what brings the most value to their customers

Quantitative Methods

Regression Analysis | Time Series Analysis | Exponential Smoothing | Input–Output Models

Qualitative Methods

Historical Analogy Method | Executive Opinion Method | Survey Techniques | Barometric Techniques | Delphi Technique

⚡ Quick Revision — Section 14.6

- **Quantitative Methods:** Regression Analysis, Time Series Analysis, Exponential Smoothing, Input-Output Models.
- **Qualitative Methods:** Historical Analogy, Executive Opinion, Survey, Barometric, Delphi.
- **Key Difference:** Quantitative — numbers and measurable data; Qualitative — human feedback from experts and customers.

14.7 Regression Analysis

SIMPLE LINEAR REGRESSION · MULTIPLE LINEAR REGRESSION · OLS METHOD

In one of the methods of Quantitative Forecasting (also known as Casual Method) a study on relationship between two (or more) variables is conducted where one variable affects the other/s. In other words two or more variables have a **cause-effect relationship**.

In general the dependent variable represents the prediction. An example is the Sales Growth of certain products. The independent variable might describe the issue or point of interest, for example, short staffed warehouse crew. The Casual Model then asks how the shortage of staff going to affect future sales of the product.

DEFINITION (M.M. BLAIR)

"Regression Analysis is a mathematical measure of the average relationship between two or more variables in terms of the original units of data."

The answer to the question mentioned above can be obtained if it is possible to establish certain relationship between the variables by means of mathematical or statistical equations. Simple Correlation technique is useful for establishing associative relationship between two variables, while the Regression technique (Simple or Multiple) is meant to isolate the casual relationship between them. Several Regression Models are available to test and establish a statistically satisfactory fit between the dependent variable and a specific range of independent variables. Forecasts are made by substituting values of the independent variables in the equation and then computing the value of the dependent

variable. These methods are useful for long term forecasting and are relatively more sophisticated and expensive to use.

The Regression Analysis confined to the study of only two variables at a time is called **Simple Regression** and it is called **Multiple Regression** when the study involves more than two variables.

Simple Linear Regression (SLR)

We confine our study to the linear models of Regression. Simple Regression mentioned above is also called Simple Linear Regression or SLR. It involves two variables — one **Explanatory** (or Independent) and one **Response** (or Dependent). The Explanatory variable is manipulated by the investigator while the Response variable responds to the manipulation of the Explanatory one. When "Time spent studying for an examination" is the Explanatory variable then the Response variable could be "Examination score".

SLR Analysis yields a regression model between two variables that can be used to make predictions or estimations about observations inside and outside the range of the data used to make the regression model. One important component of conducting a SLR Analysis is **Ordinary Least Squares** (which is also termed OLS in short). OLS is a method for estimating the unknown parameters of a linear regression model.

OLS REGRESSION FORMULAE

Model: $y = \alpha + \beta x + u$ (where u indicates the amount of Error)

Slope: $b = r(\sigma_y / \sigma_x)$ — Slope of the Regression model

y intercept: $a = \bar{y} - b \cdot \bar{x}$ — y intercept of the Regression model

Residual: $[(y_i) - (\hat{y}_i)]$ — Difference between observed and predicted values of y

Where r = Correlation Coefficient; σ = Standard Deviation. The reason behind the use of OLS Regression model is the fact that the line obtained by this method has least value of the sum of the squares of all the residuals.

Illustration 1 — Simple Linear Regression

PROBLEM

There are two variables that need to be studied — Exports of raw cotton and Imports of manufactured goods into India. Following dataset for 7 years is provided. What kind of regression model should be used here? What are the results of this regression? Interpret the model estimators.

₹ IN CRORES	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6	YEAR 7
Exports	42	44	58	55	89	98	60
Imports	56	49	53	58	67	76	58

SOLUTION

As only two variables are involved, **Simple Linear Regression (SLR)** model should be used.

Let the Regression model be $y = a + bx$ where x = Exports and y = Imports

Using the concept of Least Squares the values of a and b can be calculated. For that the Normal Equations are:

$\Sigma y = a \cdot N + b \cdot \Sigma x$ (where N = No. of pairs of observations) and $\Sigma xy = a \cdot \Sigma x + b \cdot \Sigma x^2$

SL. NO.	X (₹ CRORES)	Y (₹ CRORES)	X^2	XY
1	42	56	1764	2352
2	44	49	1936	2156
3	58	53	3364	3074
4	55	58	3025	3190
5	89	67	7921	5963
6	98	76	9604	7448
7	60	58	3600	3480
Total	**446**	**417**	**31214**	**27663**

Putting the values of Σy, Σx and N in the 1st Normal Equation we get: $417 = a \cdot 7 + b \cdot 446$ ... (1)

Similarly putting the values of Σxy, Σx and Σx^2 in the 2nd Normal Equation we get: $27663 = a \cdot 446 + b \cdot 31214$ ... (2)

From (1) we get $a = (417 - 446b)/7$

Substituting the above expression for 'a' in (2) we get: $27663 = 446(417 - 446b)/7 + 31214b$

$27663 = 26568.86 - 28416.57b + 31214b$

$1094.14 = 2797.43b \rightarrow$ **b = 0.39**

Putting b = 0.39 in the expression for 'a' we have $a = (417 - 446 \times 0.39)/7 =$ **34.72**

Substituting the values of a and b in the assumed model we get: **y = 34.72 + 0.39x**

Interpretation:

- **Slope (b = 0.39):** Indicates the amount of change in y (Imports) per unit change in the value of x (Exports).
- **y intercept (a = 34.72):** Value of y when x = 0 i.e., even for no Exports there will be Imports worth ₹34.72 Crores of Rupees.

Important Notes on SLR

1

Two Types of SLR

SLR deals with only two variables x and y of which x is conventionally considered as the Independent one, but on a number of occasions y is considered as Independent, too. Thus there can be two types of SLR models known as — **Regression line y on x** (where x is Independent) and **Regression line x on y** (where y is Independent).

2

Regression Line y on x

General form: $y = a + bx$. Using statistical measures this equation is also written as $\mathbf{y - \bar{y} = b_{yx}(x - \bar{x})}$ where $\bar{x}$ and $\bar{y}$ are Means of x & y and b_{yx} = Regression Coefficient y on x = $r(\sigma_y/\sigma_x)$.

3

Regression Line x on y

General form: $x = a + by$. This equation is also expressed as $\mathbf{x - \bar{x} = b_{xy}(y - \bar{y})}$ where $\bar{x}$ and $\bar{y}$ are Mean values of x & y and b_{xy} = Regression Coefficient x on y = $r(\sigma_x/\sigma_y)$.

4

Intersection Point

Two Regression lines intersect at the point $(\bar{x}, \bar{y})$.

5

Sign of Coefficients

Sign of b_{yx} and b_{xy} should be same always — either both positive or both negative.

6

Relationship Between Coefficients

$b_{yx} \cdot b_{xy} = r^2$ Or, $r = \pm\sqrt{(b_{yx} \cdot b_{xy})}$ [r will take the sign same as b_{yx} and b_{xy}]

7

Slopes

Slope of Regression line y on x is b_{yx} and that of x on y is $1/b_{xy}$.

8

Perfect Correlation

Two Regression lines coincide if and only if there is a perfect correlation between x and y i.e., $r = \pm 1$. In such situation slopes of the two lines will be equal i.e., $b_{yx} = 1/b_{xy}$ Or, $b_{yx} \cdot b_{xy} = 1$.

9

Perpendicular Lines

Two Regression lines will be perpendicular to each other if $b_{yx} \cdot 1/b_{xy} = -1$ Or, $b_{yx} = -b_{xy}$.

CORRELATION COEFFICIENT FORMULAE

$$r = Cov(x,y) / (\sigma_x \cdot \sigma_y)$$

$$Cov(x,y) = \Sigma xy/N - (\Sigma x/N) \cdot (\Sigma y/N)$$

$$\sigma_x = \sqrt{[\Sigma x^2/N - (\Sigma x/N)^2]}$$

Multiple Linear Regression (MLR)

Multiple Linear Regression or MLR is the extension of Simple Linear Regression. Basic concepts behind MLR are same as those of SLR. The main difference however is that MLR has one response variable with more than one explanatory variable. In fact majority of the phenomenon around us — the demand for goods, growth of plants etc. — typically have more than just one variable related to them. Mathematically this also makes sense; the more variables are added to the model, the predictions from the model become better.

COMMON PROBLEMS IN MLR

CONCEPT	DEFINITION	RESULTING PROBLEM
Over-fitting	Adding too many predictors	The model is too closely related or fit to the sample dataset to the point that it introduces a lot of variability.
Under-fitting	Adding too few predictors	The model does not fit the data well enough because it is not complex enough to the point that it introduces bias.
Multi-collinearity	Pairs of explanatory variables are too highly correlated	Reduces the reliability of the model because it affects the variance.

MULTIPLE REGRESSION FORMULAE

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n + \varepsilon$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x_1} - \beta_2 \bar{x_2}$$

$$\beta_1 = [\Sigma(X_2)^2 \cdot \Sigma(X_1 y) - \Sigma(X_1 X_2) \cdot \Sigma(X_2 y)] \div [\Sigma(X_1)^2 \cdot \Sigma(X_2)^2 - (\Sigma X_1 X_2)^2]$$

$$\beta_2 = [\Sigma(X_1)^2 \cdot \Sigma(X_2 y) - \Sigma(X_1 X_2) \cdot \Sigma(X_1 y)] \div [\Sigma(X_1)^2 \cdot \Sigma(X_2)^2 - (\Sigma X_1 X_2)^2]$$

where $\Sigma(X_1)^2 = \Sigma(x_1)^2 - (\Sigma x_1)^2/N$; $\Sigma(X_2)^2 = \Sigma(x_2)^2 - (\Sigma x_2)^2/N$; $\Sigma X_1 y = \Sigma x_1 y - (\Sigma x_1 \cdot \Sigma y)/N$; $\Sigma X_2 y = \Sigma x_2 y - (\Sigma x_2 \cdot \Sigma y)/N$; $\Sigma X_1 X_2 = \Sigma x_1 x_2 - (\Sigma x_1 \cdot \Sigma x_2)/N$; N = Number of observation sets.

Illustration 2 — Multiple Regression Analysis

PROBLEM

In order to find out the effect of Educational Qualification and Experience on the Earnings of the workers of a CNC Machine Shop, a study has been conducted on 10 workers and the dataset below is provided to you.

WORKER	1	2	3	4	5	6	7	8	9	10
Education	11	11	12	12	13	14	14	16	16	16
Experience (Years)	10	6	10	5	5	6	5	8	7	2
Salary (₹ per month)	30000	27000	20000	25000	29000	35000	38000	40000	45000	28000

Conduct a Multiple Regression Analysis considering the following numerical equivalents for Educational Qualification — 11th Std. = 11, 12th Std. = 12, 1st Yr Bachelors = 13, 2nd Yr. Bachelors = 14, 1st Yr. Masters = 16. Also interpret the meaning of the coefficients of the variables and the constant term.

SOLUTION

Let the Multiple Regression Model be $\mathbf{y = \beta_0 + \beta_1 x_1 + \beta_2 x_2}$

Calculations yield:

- $\Sigma(X_1)^2 = 1859 - (135)^2/10 = 36.5$
- $\Sigma(X_2)^2 = 464 - (64)^2/10 = 54.4$
- $\Sigma(X_1 y) = 4374000 - (135)(317000)/10 = 94500$
- $\Sigma(X_2 y) = 2023000 - (64)(317000)/10 = -5800$
- $\Sigma(X_1 X_2) = 847 - (135)(64)/10 = -17$

Substituting values: $\beta_1 = [54.5 \times 94500 - (-17) \times (-5800)] / [36.5 \times 54.4 - (-17)^2] =$ **2977.51**

$\beta_2 = [36.5 \times (-5800) - (-17) \times 94500] / [36.5 \times 54.4 - (-17)^2] =$ **822.11**

$\bar{y} = 31700$, $\bar{x_1} = 13.5$, $\bar{x_2} = 6.4$

$\beta_0 = 31700 - 2977.51 \times 13.5 - 822.11 \times 6.4 =$ **–13757.75**

Required Multiple Regression Model: $y = -13757.75 + 2977.51x_1 + 822.11x_2$

Interpretation:

- **β_1 = 2977.51:** 1 unit increase in Education (x_1) will cause increase in Salary (y) by ₹2977.51 when x_2 = constant.
- **β_2 = 822.11:** 1 unit increase in Experience (x_2) will cause an increase in Salary (y) by ₹822.11 when Educational Qualification (x_1) is held constant.
- **β_0 = –13757.75:** When both the predictors (Education and Experience) are zero then the value of y is (–13757.75).

⚡ Quick Revision — Section 14.7

- **Regression Analysis:** Mathematical measure of the average relationship between two or more variables.
- **SLR:** Two variables (one Explanatory, one Response); Model: $y = a + bx$; OLS minimises sum of squares of residuals.
- **Regression Line y on x:** $y - \bar{y} = b_{yx}(x - \bar{x})$; $b_{yx} = r(\sigma_y/\sigma_x)$.
- **Regression Line x on y:** $x - \bar{x} = b_{xy}(y - \bar{y})$; $b_{xy} = r(\sigma_x/\sigma_y)$.
- **Relationship:** $b_{yx} \cdot b_{xy} = r^2$; Lines intersect at $(\bar{x}, \bar{y})$.
- **MLR:** One response variable, multiple explanatory variables; β_0, β_1, β_2 formulae.

14.8 Time Series Analysis

COMPONENTS · MEASUREMENT OF TREND · MEASUREMENT OF SEASONAL VARIATION

The term **"Time Series"** refers to a series of observations recorded in accordance with the time of occurrence. A study of time series data discloses that observed values of the variable are always fluctuating from time to time. The fluctuations are the result of the joint action of various forces, like changes in tastes and habits of people, increase in population, development of new technology resulting in lower cost of production, changes in environmental conditions etc.

The objects of time series analysis are to isolate and measure the effects of various components. Such an analysis helps understand the past behaviour, so that the future tendency may be predicted. To business executives who have to plan their production much ahead of sales, the analysis of time series is of great importance in planning for the hiring of personnel for peak periods, to accumulate an inventory of raw materials, to keep the equipment ready and finally in forecasting the future demand of their product.

Examples of Time Series Data

Profits earned by a company for each of the past ten years

Number of students registered for CMA Examination for the past decade

Percentage change in quarterly Consumer Price Index (CPI) for the last 45 years

Number of employees hired by a company for each of the last five years

Components of Time Series Data

The four components of Time Series are:

T

Secular Trend (T)

Smooth, regular and long term movement exhibiting the tendency of growth or decline over a period of time.

S

Seasonal Variation (S)

Type of periodic movement where the period is at the most one year.

C

Cyclical Fluctuation (C)

Type of periodic movement where the period is more than a year.

I

Irregular or Random Movement (I)

Variations caused by factors of erratic nature.

MODELS OF TIME SERIES

Multiplicative Model: $y_t = T \times S \times C \times I$

Additive Model: $y_t = T + S + C + I$

Although the additive model facilitates easier calculations, it has been found inappropriate in many practical situations and hence is not generally used.

Detailed Description of Components

Secular Trend or Trend

In a time series, observations are taken over successive periods. Though most of these data generally display some random fluctuations, the time series may still show gradual shifts to relatively higher or lower values over an extended period. This gradual shifting is referred to as the trend of the time series. A trend emerges due to one or more long term factors, such as changes in population size, changes in the demographic characteristics of the population, changes in tastes and preferences of customers etc. For example, manufacturers of automobiles in India may see substantial variations in automobile sales from one month to the next. But in reviewing auto sales in the past 15 to 20 years, the manufacturers may discover a gradual increase in annual sales volume.

Seasonal Variation

Business activities are found to have brisk and slack periods at different parts of the year. This ups and downs recurring with remarkable regularity year after year, is attributable to the presence of seasonal variation. For example, usage of swimming pools experience low values in the fall and winter months, but witness peak values during summer and springs. On the other hand manufacturers of woollen garments experience exactly opposite behaviour.

Cyclical Fluctuation

Such movements are fairly regular and oscillatory in nature. One complete period is called a cycle. Although a time series may often exhibit a trend over a long period, it may also display alternating sequences of points that lie above and below the trend line and last more than a year. The time series of the aggregate output in the economy (called real Gross Domestic Product or GDP) provides a good example of cyclical fluctuation in Time Series. While the trend line for GDP is sloping upwards, the output growth displays a cyclical behaviour around the trend line. This cyclical behaviour of GDP has been dubbed as business cycle by the economists.

Irregular or Random Movements

These are such variations which are caused by factors of erratic nature. These are completely unpredictable or caused by such unforeseen events as war, natural calamities, strike, lockout etc. Random Movements do not reveal any pattern of the repetitive tendency and may be considered as residual variation.

PURPOSE OF SEGREGATION

A very common practice for the purpose of forecasting is to segregate the above mentioned components from the observed dataset of Time Series. Segregation of the components facilitates to increase accuracy of the prediction being made for a future date.

Measurement of Trend

There are four methods of isolating Secular Trend from the Time Series data.

1

Free Hand Method

Given data plotted as points on a graph paper; a smooth free hand curve drawn through the scatter of plotted points. Not much used for actual forecasting because too much individual judgement involved.

2

Semi Average Method

Dataset divided into two equal parts; average of each found; these averages plotted against the mid-point of the time interval. Straight line joining the two points is the trend line. Simple but may lead to poor results.

3

Moving Average Method

Series of arithmetic means of successive observations calculated from overlapping groups. Each moving average based on values covering a fixed time interval called Period of Moving Average.

4

Fitting Mathematical Curves

Appropriate mathematical equation selected for trend; constants determined by Principle of Least Square. Most suitable for forecasting the trend.

Free Hand Method

The given data are plotted as points on a graph paper. The time series data (y_t) are taken along vertical axis and time (t) along horizontal. Then a smooth free hand curve is drawn through the scatter of the plotted points which appears

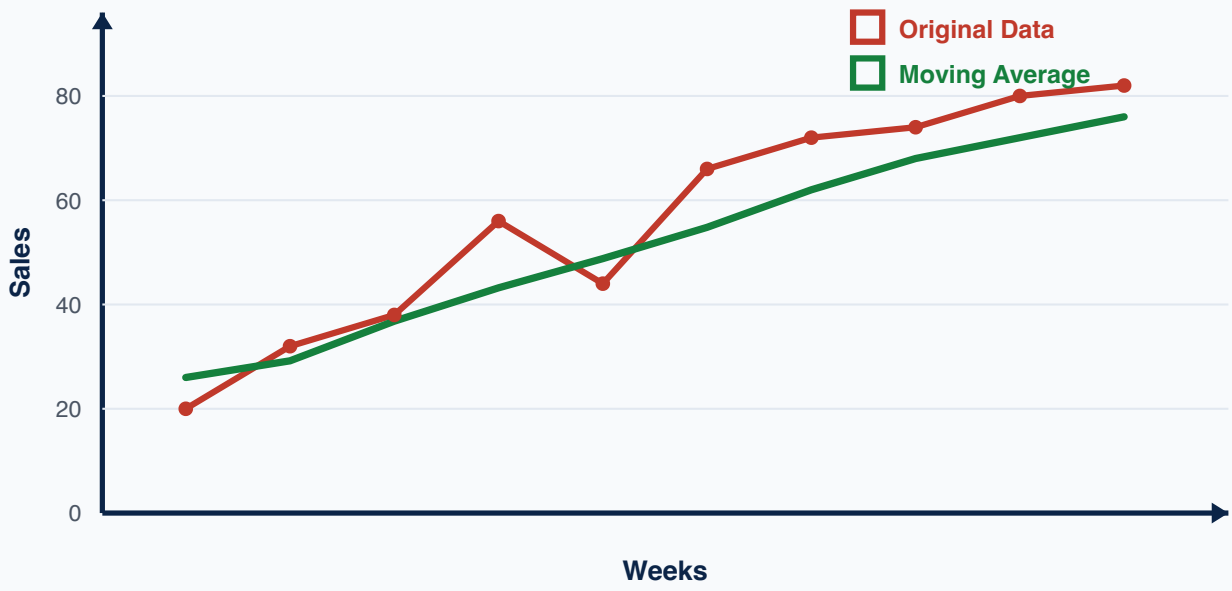
to represent their pattern of movement over time. The drawn curve is the Trend line and the value of y_t against any value of t can be obtained from this. This method is not much used for actual forecasting because too much of individual judgement is involved in it. It is used only to get a preliminary idea about the possible nature of the trend line.

Semi Average Method

This method deals with dividing the dataset into two equal parts and then finding average of each. These averages are plotted as points on a graph paper against the mid-point of the time interval covered by each part. The straight line joining the two points is considered to be the trend line. Although the method is simple to apply, it may lead to poor results when used indiscriminately.

Moving Average Method

This is very commonly used for the isolation of trend and in smoothing out fluctuations in time series. In this method a series of arithmetic means of successive observations known as Moving Averages are calculated from overlapping groups of successive elements of the given data and these moving averages are used as trend values. Each moving average is based on values covering a fixed time interval called **Period of Moving Average** and is shown against centre of the first.



WEEK	SALES	3 WEEKLY MOVING TOTAL	3 WEEKLY MOVING AVERAGE
1	17	—	—
2	28	75	25
3	30	116	38.7
4	58	138	46
5	50	171	57
6	63	—	—

ODD VS EVEN PERIOD OF MOVING AVERAGE

When period is ODD: Each moving average is shown against the middle most value of the concerned group of observations.

When period is EVEN: There are two middle periods and the moving average value is placed in between the two middle terms. The moving average value will not coincide with a period of the given time series. To synchronize, an average of two already calculated moving averages is computed and placed corresponding to the given time set. This is known as **Centring** and the moving averages are called **Centred Moving Averages**.

Fitting Mathematical Curves

This is perhaps the best and most objective method of determining trend. In this method an appropriate type of mathematical equation is selected for trend and the constants appearing in the trend equation are determined on the basis of the given time series data. The choice of the appropriate type of equation is facilitated by a graphical representation of the data.

1

Straight Line

If plotted data show approximately a straight line tendency on a graph paper, the equation used is $y = a + bx$.

2

Exponential Curve

If they show a straight line tendency on a semi-logarithmic graph paper, the equation used is $\log y = a + bx$.

3

Parabola

If graphical representation of data shows a parabola then the equation used is $y = a + bx + cx^2$.

PRINCIPLE OF LEAST SQUARE

The constants appearing in the equations mentioned above are obtained by applying Principle of Least Square. In fact it is a special category of Least Square Regression in which the independent variable is always time. Though this method involves considerable numerical calculations, it is found to be most suitable for forecasting the trend.

Illustration 3 — Moving Average Trend (EVEN Period)

PROBLEM

Find trend values of the following year wise data of Goods carried by a fleet of trucks of a Transport Company having pan India network using the Moving Average Method. [Assume a 4 yearly cycle]

YEAR	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Goods carried (Tons)	2204	2500	2360	2680	2424	2634	2904	3098	3172	2952	3248	3172

SOLUTION

Calculations for 4 Yearly Moving Average Trend values:

YEAR	GOODS CARRIED (TONS)	4 YEARLY MOVING TOTAL	4 YEARLY MOVING AVERAGE (NOT CENTRED)	2 ITEM MOVING TOTAL (CENTRED)	4 YEARLY MOVING AVERAGE (CENTRED)
2012	2204	—	—	—	—
2013	2500	—	—	—	—
2014	2360	9744	2436	4927	2463.50
2015	2680	9964	2491	5015.5	2507.75
2016	2424	10098	2524.5	5185	2592.50
2017	2634	10642	2660.5	5425.5	2712.75
2018	2904	11060	2765	5717	2858.50
2019	3098	11808	2952	5983.5	2991.75
2020	3172	12126	3031.5	6149	3074.5
2021	2952	12470	3117.5	6253.5	3126.75
2022	3248	12544	3136	—	—
2023	3172	—	—	—	—

Illustration 4 — Mathematical Curve Fitting (ODD Number of Years)

PROBLEM

The following table relates to the tourist arrivals in India during 2017 to 2023.

YEAR	2017	2018	2019	2020	2021	2022	2023
Tourist arrivals (lakhs)	18	20	23	25	24	28	30

Fit a Straight Line trend by the Method of Least Squares and estimate the number of tourists that would arrive in the year 2027.

SOLUTION

Let the best fit Trend line to the given data be **y = a + bx** (Origin at the year 2020 and x unit = 1 year)

Normal equations: $\Sigma y = a \cdot n + b \cdot \Sigma x$ ... (1) and $\Sigma xy = a \cdot \Sigma x + b \cdot \Sigma x^2$ ... (2) where n = No. of years = 7

YEAR	TOURIST ARRIVALS (Y IN LAKHS)	X	X^2	XY
2017	18	–3	9	–54
2018	20	–2	4	–40
2019	23	–1	1	–23
2020	25	0	0	0
2021	24	1	1	24
2022	28	2	4	56
2023	30	3	9	90
Total	**168**	**0**	**28**	**53**

Putting values in (1): 168 = a(7) + b(0) → **a = 24**

Putting values in (2): 53 = a(0) + b(28) → **b = 1.893**

Required equation of Straight Line Trend: **y = 24 + 1.893x** (Origin = 2020, x unit = 1 year)

For the year 2027, x = 7. Estimated number of tourists in 2027 = 24 + 1.893(7) = **37.25 lakhs**

Illustration 5 — Mathematical Curve Fitting (EVEN Number of Years)

PROBLEM

From the following past data of Sales (in lakhs Rupees) of a company estimate the same for the year 2029.

YEAR	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Sales (₹ Millions)	15.3	14.6	16.8	17.3	17.2	20.9	22.3	20.0	23.1	24.5

Assume the trend line to be linear. What is the monthly rate of increase of Sales?

SOLUTION

Let the best fit Linear Trend line be **y = a + bx** (Origin at the middle of the years 2018 & 2019 and x unit = 6 months)

Normal equations: $\Sigma y = a \cdot n + b \cdot \Sigma x$ ... (1) where n = 10; $\Sigma xy = a \cdot \Sigma x + b \cdot \Sigma x^2$ ... (2)

Using values: $\Sigma y = 192$, $\Sigma x = 0$, $\Sigma x^2 = 330$, $\Sigma xy = 177$

From (1): 192 = a(10) + b(0) → **a = 19.2**

From (2): 177 = a(0) + b(330) → **b = 0.536**

Required equation: **y = 19.2 + 0.536x** (Origin = Middle of 2018 & 2019, x unit = 6 months)

For the year 2029, x = 21. Estimated sales for 2029 = 19.2 + 0.536(21) = **30.456 Million ₹**

Yearly rate of increase in Sales = b = 0.536. Monthly rate of increase in Sales = b/12 = **0.0467 Million ₹**

Measurement of Seasonal Variation

Four methods are generally used for measuring Seasonal Variation or Short term fluctuation of time series.

- Method of Simple averages (monthly or quarterly)
- Moving Average Method
- Trend Ratio Method
- Link Relative Method

Method of Simple Averages

Generally applied to the time series data which do not contain trend or cyclical fluctuation to any appreciable extent. From the given quarterly data the averages (A_1, A_2, A_3 & A_4) for the four quarters are calculated and also Grand Average [$G = (A_1 + A_2 + A_3 + A_4)/4$] is calculated.

SEASONAL VARIATION FORMULAE

$$\text{Additive Model: } S_1 = A_1 - G;\ S_2 = A_2 - G;\ S_3 = A_3 - G;\ S_4 = A_4 - G$$

$$\text{Multiplicative Model: } S_1 = (A_1/G) \times 100;\ S_2 = (A_2/G) \times 100;\ S_3 = (A_3/G) \times 100;\ S_4 = (A_4/G) \times 100$$

Total of Seasonal Variations = 0 for Additive Model and Average Seasonal Index = 100 for Multiplicative Model. When monthly figures are given, then instead of Quarterly Averages we need to calculate Monthly Averages.

Illustration 6 — Seasonal Indices (Method of Simple Average)

PROBLEM

Calculate the Seasonal Indices for the following quarterly data in certain units. Appropriate method for finding the Indices has to be decided by you with due explanation.

YEAR	1ST QUARTER	2ND QUARTER	3RD QUARTER	4TH QUARTER
2020	39	21	52	81
2021	45	23	63	76
2022	44	26	69	75
2023	53	23	64	84

SOLUTION

The values in any quarter do not reveal any definite tendency to change. Thus there is no appreciable trend in the given dataset. So it is decided to use **Method of Simple Average (Quarterly)** to find out the Seasonal Indices. Also a Multiplicative Model is assumed for the data.

YEAR	1ST QUARTER	2ND QUARTER	3RD QUARTER	4TH QUARTER	TOTAL
2020	39	21	52	81	—
2021	45	23	63	76	—
2022	44	26	69	75	—
2023	53	23	64	84	—
Total	**181**	**93**	**248**	**316**	**838**

PARTICULARS	1ST QUARTER	2ND QUARTER	3RD QUARTER	4TH QUARTER	TOTAL
Arithmetic Mean	45.25	23.25	62	79	209.5
Seasonal Index	**86.4**	**44.4**	**118.4**	**150.8**	**400**

Calculations: Arithmetic Mean for any Quarter = Total for that quarter/4; Grand Average = Total of Arithmetic Means/4; Seasonal Index for any Quarter = (Arithmetic Mean of that Quarter/Grand Average) × 100

Illustration 7 — Seasonal Fluctuation (Moving Average Method)

PROBLEM

Calculate Seasonal Fluctuation from the following Time Series data obtained from a Mini Steel Plant:

YEAR	1ST QUARTER	2ND QUARTER	3RD QUARTER	4TH QUARTER
2020	65	58	56	61
2021	68	63	63	67
2022	70	59	56	52
2023	60	55	51	58

Use 4 Quarter Moving Average Method. Consider Additive Model for the Time Series data. Deseasonalise the given dataset.

SOLUTION

Calculations for 4 Quarter Moving Averages & Deviations from Trend:

YEAR	QUARTER	OUTPUT	4 QTR MOVING TOTAL (NOT CENTRED)	2 PERIOD MOVING TOTAL (CENTRED)	4 QTR MA (CENTRED)	DEVIATION FROM TREND
2020	1st	65	—	—	—	—
2020	2nd	58	—	—	—	—
2020	3rd	56	240	483	60.38	−4.38
2020	4th	61	243	491	61.38	−0.38
2021	1st	68	248	503	62.88	5.12
2021	2nd	63	255	516	64.50	−1.50
2021	3rd	63	261	524	65.50	−2.50
2021	4th	67	263	522	65.25	1.75
2022	1st	70	259	511	63.88	6.12
2022	2nd	59	252	489	61.12	−2.12
2022	3rd	56	237	464	58.00	−2.00
2022	4th	52	227	450	56.25	−4.25
2023	1st	60	223	441	55.12	4.88
2023	2nd	55	218	442	55.25	−0.25
2023	3rd	51	—	—	—	—
2023	4th	58	—	—	—	—

Calculations for Seasonal Fluctuations:

YEAR	1ST QUARTER	2ND QUARTER	3RD QUARTER	4TH QUARTER	TOTAL
2020	—	—	−4.38	−0.38	—
2021	5.12	−1.50	−2.50	1.75	—
2022	6.12	−2.12	−2.00	−4.25	—
2023	4.88	−0.25	—	—	—
Total	**16.12**	**−3.87**	**−8.88**	**−2.88**	**0.49**
Arithmetic Mean	5.37	−1.29	−2.96	−0.96	0.16
Adjustment	−0.04	−0.04	−0.04	−0.04	−0.16
Seasonal Fluctuation	**5.33**	**−1.33**	**−3.00**	**−1.00**	**0**

Deseasonalising given Time Series data:

YEAR	QUARTER	OUTPUT (Y_T)	SEASONAL FLUCTUATION (S)	DESEASONALISED DATA ($Y_T - S$)
2020	1st	65	5.33	59.67
2020	2nd	58	–1.33	59.33
2020	3rd	56	–3.00	59.00
2020	4th	61	–1.00	62.00
2021	1st	68	5.33	62.67
2021	2nd	63	–1.33	64.33
2021	3rd	63	–3.00	66.00
2021	4th	67	–1.00	68.00
2022	1st	70	5.33	64.67
2022	2nd	59	–1.33	60.33
2022	3rd	56	–3.00	59.00
2022	4th	52	–1.00	53.00
2023	1st	60	5.33	54.67
2023	2nd	55	–1.33	56.33
2023	3rd	51	–3.00	54.00
2023	4th	58	–1.00	59.00

Note: Deseasonalisation means elimination of Seasonal Variation from the Time Series data. For data in Additive Model it is done by subtracting Seasonal Variation from the original data. When Multiplicative Model is used then it is done by dividing the original data by Seasonal Effect (Seasonal Index/100).

Illustration 8 — Estimating Quarterly Sales

PROBLEM

On the basis of quarterly Sales (in ₹ Lakhs) of a certain commodity for the period 2019 to 2023 the following calculations were made:

Trend: Straight Line trend equation is y = 25 + 0.6t, Origin – 1st Quarter of 2001, t unit – 1 Quarter, y – Sales

Seasonal Variations:

QUARTER	I	II	III	IV
Seasonal Index	90	95	110	105

Estimate the Quarterly Sales figures for the year 2028.

SOLUTION

Given Trend equation: y = 25 + 0.6t (Origin at the 1st Quarter of 2001, t unit = 1 Quarter)

For the 1st Quarter of 2028, t = 36; 2nd Quarter, t = 37; 3rd Quarter, t = 38; 4th Quarter, t = 39

Trend for 1st Qtr = 25 + 0.6(36) = 46.6; 2nd Qtr = 25 + 0.6(37) = 47.2; 3rd Qtr = 25 + 0.6(38) = 47.8; 4th Qtr = 25 + 0.6(39) = 48.4

QUARTER OF 2028	TREND (T) IN ₹ LAKHS	SEASONAL INDEX	SEASONAL EFFECT (S) = SEASONAL INDEX/100	ESTIMATED SALES IN ₹ LAKHS (T × S)
I	46.6	90	0.90	41.94
II	47.2	95	0.95	44.84
III	47.8	110	1.10	52.58
IV	48.4	105	1.05	50.82

Note: For cases where the data follows Additive Model, the estimated Value should be calculated by adding Trend and Seasonal Variation.

⚡ Quick Revision — Section 14.8

- **Time Series:** Series of observations recorded in accordance with time of occurrence.
- **Four Components:** Secular Trend (T), Seasonal Variation (S), Cyclical Fluctuation (C), Irregular Movement (I).
- **Multiplicative Model:** $y_t = T \times S \times C \times I$; **Additive Model:** $y_t = T + S + C + I$.
- **Four Methods of Trend:** Free Hand, Semi Average, Moving Average, Fitting Mathematical Curves.
- **Moving Average:** Odd period — against middle value; Even period — requires centring.
- **Seasonal Variation Methods:** Simple Averages, Moving Average, Trend Ratio, Link Relative.

14.9 Exponential Smoothing

SMOOTHING CONSTANT · RECURRENCE RELATION · ILLUSTRATION

This method can be considered as a more sophisticated extension of Moving Average Method of forecasting technique which weighs past data in an exponential manner so that the most recent data carries more weight in the moving average. Unlike moving average method, where equal weightage is given to all the past observations, this method assigns **decreasing weightages to the past observations**. This is definitely more reasonable since older the observation the less relevance it holds for the future.

In Exponential Smoothing, the weights used are — α for the current observation, $\alpha(1-\alpha)$ for the immediately preceding observation, $\alpha(1-\alpha)^2$ for the still preceding observation and so on, where α is a constant known as **Smoothing Constant** and has a value lying between 0 and 1. It can be mentioned that the weights of observations from the current period backwards are diminishing with a common ratio $(1 - \alpha)$. In fact they form an infinite G.P. having Sum of all the terms equal to 1.

EXPONENTIAL SMOOTHING FORMULAE

$$u_t = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \alpha(1-\alpha)^3 y_{t-3} + \ldots \text{ to infinity}$$

$$\text{Recurrence Relation: } u_t = \alpha y_t + (1-\alpha)u_{t-1}$$

$$\text{Or, } u_t = u_{t-1} + \alpha(y_t - u_{t-1}) = u_{t-1} + \alpha e_t$$

Where u_t and u_{t-1} are the Forecasts at time t and (t–1) respectively; y_t is the observation for time t; α is the Smoothing Coefficient; $e_t = y_t - u_{t-1}$ is called Error or Discrepancy of the latest observation from the forecast in the previous period.

STEPS FOR NUMERICAL COMPUTATION

1. Find the Error or Discrepancy in the latest observation from the previous forecast using $e_t = y_t - u_{t-1}$
2. Multiply the Error by the Smoothing Coefficient to obtain the Correction (αe_t)
3. Add the Correction (αe_t) to the previous forecast (u_{t-1}) to get forecast (u_t) for the current period t.

In Exponential Smoothing the first forecast has to be obtained by some subjective method or by taking the average of the first few time periods. Subsequent forecasts are then obtained by repeatedly using the relationship $u_t = u_{t-1} + \alpha e_t$.

CHOICE OF SMOOTHING COEFFICIENT

When $\alpha = 0$, no Correction is necessary to the previous forecast. When $\alpha = 1$, the new forecast will always be the latest observation. Smoothing Coefficient α lies between 0 and 1 and it is selected on the basis of experiments with different values. Higher the value of α, sooner it discounts the effects of old observations. So when old observations are also of some significance, it is wise to take smaller values of α. For all practical purposes it is kept in the range 0.005 to 0.3 to smooth the forecast.

Illustration 10 — Exponential Smoothing (Shoe Manufacturer)

PROBLEM

M/S B.P. Leathers, a shoe manufacturer has modern outlook and they depend heavily on Business Forecasting methodology to plan their business activities like manufacturing, marketing, finance etc. At the beginning of the year 2023 they have forecasted data of demand of their shoes for the beginning of the month of March as 1000 pairs. But the actual demand turned out to be 900 pairs. Using a Smoothing Coefficient of 0.1 forecast the demand at the beginning of the 2nd week of March 2023.

Also forecast the demands using Exponential Smoothing technique at the beginning of each week till mid April 2023 when the actual demands are as follows — At the beginning of the 2nd week of March – 1010 pairs, At the beginning of the 3rd week of March – 1032 pairs, At the beginning of the 4th week of March – 976 pairs, At the beginning of the 1st week of April – 934 pairs, At the beginning of 2nd week of April – 1008 pairs & At the end of the 2nd week of April – 1020 pairs.

SOLUTION

As per the concept of Exponential Smoothing we have: $\mathbf{u_t = u_{t-1} + \alpha e_t}$ where $e_t = y_t - u_{t-1}$ = Forecast Error & $\alpha = 0.1$

BEGINNING OF	DEMAND OF SHOE (Y_T IN PAIRS)	PREVIOUS FORECAST (U_{T-1})	FORECAST ERROR ($E_T = Y_T - U_{T-1}$)	CORRECTION (AE_T) (A = 0.1)	NEW FORECAST ($U_T = U_{T-1} + AE_T$)
March 1st week	900	1000	–100	–10	990
March 2nd week	1010	990	20	2	992
March 3rd week	1032	992	40	4	996
March 4th week	976	996	–20	–2	994
April 1st week	934	994	–60	–6	988
April 2nd week	1008	988	20	2	990
April 2nd week end / Mid April	1020	990	30	3.0	**993**

Note: Except the 1st entry of the 3rd column, all the other entries are taken from the last column.

⚡ Quick Revision — Section 14.9

- **Exponential Smoothing:** More sophisticated extension of Moving Average Method; assigns decreasing weights to past observations.
- **Smoothing Constant (α):** Lies between 0 and 1; higher α discounts old observations more.
- **Formula:** $u_t = u_{t-1} + \alpha(y_t - u_{t-1}) = u_{t-1} + \alpha e_t$.
- **Practical Range:** α kept in range 0.005 to 0.3.

14.10 Input–Output Model

CONCEPT · ASSUMPTIONS · LIMITATIONS

Under this model a forecast of output is based on given inputs if the coefficients of input–output relationship are known. Similarly, input requirements can be forecast on the basis of final output with a given input–output relationship. Due to this mechanics of forecasting the model is named as **Input–Output Model**.

Assumptions of the Model

1

Interacting Industries

The economy consists of a number of interacting industries.

2

Single Product & Process

Each industry produces only one item and uses only one process of production.

3

Inputs from Other Industries

To produce an item, the industry requires as input the goods made by other industries, labour and perhaps imports. An industry may use some of its own goods. Such use may be taken as sale to itself.

4

Output as Input or Final Demand

The output of any industry becomes either the input to another industry or the final demand.

5

Fixed Proportions

In any productive process all inputs are used in fixed proportions and increase in input is in proportion with the level of output. Production takes place through processes with constant technical coefficients. Technical Coefficient shows the number of units of any industry's output needed to produce one unit of another industry's output.

6

Money Values

All transactions may be taken in terms of money values because it is a suitable common unit for aggregating inputs and outputs of industries.

UNIQUENESS OF THE MODEL

Uniqueness of Input–Output Model lies in the fact that the same is the only forecasting technique which is based on inter-industry flow of goods and services, given technical data on input usage by various industries. However the product quality, technology and industrial organisation undergo changes over time causing inter-industry relationship to change, too. The degree of accuracy of the forecasts through this model, therefore, depends on the extent to which the technical coefficients truly project the latest inter-industry flow relationship.

⚡ Quick Revision — Section 14.10

- **Input–Output Model:** Forecast of output based on given inputs if coefficients of input–output relationship are known.
- **Assumptions:** Interacting industries, Single product & process, Inputs from other industries, Output as input or final demand, Fixed proportions, Money values.
- **Technical Coefficient:** Number of units of any industry's output needed to produce one unit of another industry's output.

14.11 Qualitative Methods of Forecasting

HISTORICAL ANALOGY · EXECUTIVE OPINION · SURVEY · BAROMETRIC · DELPHI

1

Historical Analogy Method

Under this method forecast in regard to a particular phenomenon is based on some analogous conditions elsewhere in the past. For example, the forecast of demand for a product in India can be based on the demand for the same in some developed country in the past if it is found that the present conditions in India are very much like the same prevailing in that country in the past. In fact such a method is more useful for indicating qualitative changes in society. It is said social analogies have helped in indicating the trends of changes in the norms of corporate behaviour in terms of attitude of the worker against inequality etc. find similarities in various countries at different stages of the history of industrial growth. But it is difficult to quantify most of these phenomena and therefore this method does not have much relevance for statistical analysis.

2

Jury or Executive Opinion Method

This method attempts to pool the knowledge, experience and judgement of managers inside the organisation, by asking about their opinion on the likely sales of a product in future. This is particularly applicable for a new product which does not have any past data to facilitate forecasting.

3

Survey Technique

First hand data on the behaviour of certain desired variables can be obtained through selective surveys by means of questionnaires and interviews. The data so collected can be processed and analysed for purposes of testing some predetermined hypotheses and making predictions and future estimates of the behaviour of the variables under study.

4

Barometric Technique

The behaviour of certain economic or business variables can have an important effect on some other variables. For example, a shortfall in supply of a commodity may lead to a rise in its price. Here the shortfall is an indicator or a barometer of the likely increase in price. Indicators can be of three types — **Lead**, **Lag** and **Concurrent**. Issuance of Industrial License is a Lead Indicator of future industrial activity. Similarly demand for Household Furniture in a housing complex is a Lag Indicator because it becomes active only after the construction of houses are completed i.e., after a certain time lag. Concurrent Indicators move together i.e., no lead or lag time is involved.

5

Delphi Technique

This technique is used to make more realistic judgemental forecasts by minimizing bias. In this method a panel of experts (including decision makers, staff personnel and respondents) is asked sequential questions. It is a step by step procedure and final forecast is obtained by the common opinion of all the experts.

⚡ Quick Revision — Section 14.11

- **Five Qualitative Methods:** Historical Analogy, Executive Opinion, Survey Technique, Barometric Technique, Delphi Technique.
- **Barometric Indicators:** Lead (Industrial License), Lag (Household Furniture), Concurrent (move together).
- **Delphi Technique:** Panel of experts asked sequential questions; final forecast from common opinion.

14.12 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. In Exponential Smoothing Method which one of the following is true?

(a) $0 \leq \alpha \leq 1$ and high value of α is used for stable demand.

(b) $0 \leq \alpha \leq 1$ and high value of α is used for unstable demand.

(c) $\alpha \geq 1$ and high value of α is used for stable demand.

(d) $\alpha \leq 0$ and high value of α is used for unstable demand.

Answer: B

2. Which of the following is not a Casual Forecasting Method?

(a) Trend adjusted Exponential Smoothing

(b) Econometric models

(c) Linear Regression

(d) Multiple Regression

Answer: A

3. Which of the following is a Forecasting technique?

(a) PERT / CPM

(b) Exponential Smoothing

(c) Gantt Chart

(d) Control Chart

Answer: B

4. The number of averaging period in the Simple Moving Average Method of forecasting is increased for greater smoothing but at the cost of —

(a) Accuracy

(b) Stability

(c) Visibility

(d) Responsiveness to changes

Answer: D

5. In a Time Series forecasting model, the demands for five time periods are 10, 13, 15, 18 and 22. A linear regression fit resulted in the equation $y_t = 6.9 + 2.9t$, where y_t is the forecast for period t. The sum of the absolute deviations for the five data with respect to their corresponding forecasts (taking t = 1 for the first one) is —

(a) 2.3

(b) 0.2

(c) 1.2

(d) 2.2

Answer: D

6. Which of the following is not a part of Quantitative type of Forecasting Model?

(a) Moving Average

(b) Simple Average

(c) Delphi Method

(d) Exponential Smoothing

Answer: C

7. Which of the following Forecasting technique uses three types of participants: Decision Makers, Staff personnel and Respondents?

(a) Expert's Opinion (b) Sales Force Survey (c) Consumer Survey

(d) Delphi Method

Answer: D

8. Sales data for the numbers sold for a particular product during January to May 2007 shows the values 10, 11, 16, 19 and 25. Regarding forecast for the month of June which one of the following statement is true?

(a) Moving Average will forecast a higher value compared to regression. (b) Exponential Smoothing will forecast a higher value compared to regression (c) Regression will forecast a higher value compared to moving average.

(d) None of the above.

Answer: C

9. The Time Series forecasting method that gives equal weightage to each of the N most recent observations is —

(a) Moving Average Method (b) Exponential Smoothing with linear Trend (c) Triple Exponential Smoothing

(d) None of the above

Answer: A

10. Which of the following is not a forecasting technique?

(a) Trend line estimate (b) Delphi Method (c) Hungarian Method

(d) Judgemental technique

Answer: C

11. In Simple Exponential Smoothing forecast, to give higher weightage to recent demand information, the smoothing constant must be close to —

(a) –1 (b) 0 (c) 0.5

(d) 1

Answer: D

12. Which of the following is not true for forecasting?

(a) Forecasts are rarely perfect.

(b) The underlying casual system will remain same in the future.

(c) Forecast for group of items is accurate than individual item

(d) Short range forecasts are less accurate than long range forecasts.

Answer: D

13. In which of the following forecasting technique, data obtained from past experience is analysed?

(a) Judgemental forecast

(b) Time Series forecast

(c) Associative model

(d) All of the above

Answer: B

14. Delphi Method is used for —

(a) Judgemental forecast

(b) Time Series forecast

(c) Associative model

(d) All of the above

Answer: A

15. Short term regular variations related to the calendar or time of the day is known as —

(a) Trend

(b) Seasonality

(c) Cycles

(d) Random variations

Answer: B

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
B	A	B	D	D	C	D	C	A	C	D	D	B	A	B

State True or False

SL	STATEMENT	ANSWER
1	Surveys and Opinion Polls are Qualitative Techniques.	**True**
2	The Delphi Method generates forecasts by surveying consumers to determine their opinions.	**False**
3	Irregular or random influences on time series data give rise to secular trend.	**False**
4	The linear trend equation can be estimated by ordinary least squares Regression Analysis.	**True**
5	The ratio to trend method is used to find a linear trend equation.	**False**
6	Forecasts based on lead indicators are qualitative.	**True**
7	The long run increase or decrease in time series data is referred to as Cyclical Fluctuation.	**False**
8	The choice of a forecasting method should be based on an assessment of costs and benefits of each method in a specific application.	**True**
9	Barometric forecasting methods are most useful for long term forecasts.	**False**
10	Qualitative forecasts based on surveys tend to perform particularly well during periods of unexpected international political upheaval.	**False**
11	Councils of distinguished foreign dignitaries and business people are used to obtain qualitative forecasts with a foreign perspective.	**True**
12	Time analysis generates forecasts by identifying cause and effect relationship between variables.	**True**
13	Time Series data are observations on a variable at different points of time.	**True**
14	A time series that displays regular seasonal variation is said to exhibit cyclical fluctuation.	**False**
15	Smoothing techniques are most useful for time series data that is primarily influenced by irregular variation.	**True**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	Shortfall in supply of a commodity may lead to a rise in its price. Here the shortfall is a _____ of the likely increase in price.	**Barometer**
2	Multiple linear regression or ___ is an extension to the Simple linear regression.	**MLR**
3	Deseasonalisation is ____ of Seasonal Variation component from Time Series data.	**Removal**
4	Two regression lines _____ if $b_{yx} \cdot b_{xy} = 1$.	**Coincide**
5	Signs of Regression Coefficients and Correlation Coefficient should be ____ always.	**Same**
6	In the regression line x on y the independent variable is ___.	**y**
7	Coordinates of the point of intersection of two regression lines give ___ values of the variables.	**Mean**
8	$r(\sigma_y/\sigma_x)$ is known as the ____ of the regression line y = a + bx.	**Slope**
9	Forecasting is based on current and future assets and liabilities as well as predictions for liquid capital and cash flow estimates.	**Capital**
10	Business _____ allows a company to make long term plans and prepare for any changes in the market.	**Forecasting**

14.13 Essay Type Questions & Numerical Problems

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

What is Secular Trend in Time Series? Name the methods by which it can be measured.

2

Define Regression. What are the estimators in a linear regression equation?

3

What do you mean by Demand Forecasting?

4

Name the different models of Qualitative Forecasting.

5

What is the speciality of Delphi technique?

6

"To deseasonalise time series data firstly trend component of it needs to be removed" — discuss.

Essay Type Questions

1

What are the limitations of Business Forecasting?

2

What are the assumptions on which Input–Output Model is established?

3

Briefly describe the different models of time series data.

4

Write short notes on SLR and MLR.

5

Describe the importance of Business Forecasting in different areas of an organisation.

Comprehensive Numerical Problems

PROBLEM 1 — EXPONENTIAL SMOOTHING (SHIP UNLOADING)

A company dealing in logistics business has a business wing named Ship Unloading. During the years 2022 and 2023 they have the following quarterly figures (in tonnage) of material unloaded from Ships:

YEAR	2022 I	2022 II	2022 III	2022 IV	2023 I	2023 II	2023 III	2023 IV
Material Unloaded	180	168	159	175	190	205	180	182

If the forecast for the first quarter of 2022 is 175 tons then what is the forecast figure for the first quarter of 2024? Use a Smoothing coefficient of 0.1

ANSWER

178.6 Tons

PROBLEM 2 — SEASONAL INDICES (TREND RATIO METHOD)

The number of quarterly traffic accidents in a Metro city during 2021-2023 are as below:

YEAR / QUARTER	1ST	2ND	3RD	4TH
2021	165	135	140	180
2022	152	121	127	163
2023	140	100	105	158

Find the Seasonal Indices by Trend Ratio Method, assuming a linear trend for the data.

ANSWER

105 for 1st quarter, 83 for 2nd Quarter, 89 for 3rd Quarter and 123 for 4th Quarter.

PROBLEM 3 — CENTRED MOVING AVERAGE VERIFICATION

For the following series of observations verify that the 4 yearly Centred Moving Average is equivalent to the 5 yearly Weighted Moving Average with weights 1, 2, 2, 2, 1 respectively.

YEAR	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Sales ('0000 ₹)	2	6	1	5	3	7	2	6	4	8	3

ANSWER

Detailed calculations show that the 4 yearly Centred Moving Average yields the same values as the 5 yearly Weighted Moving Average with weights 1, 2, 2, 2, 1 — both being equivalent methods of trend measurement.

PROBLEM 4 — MULTIPLE LINEAR REGRESSION

In the following dataset consider y as the Response Variable and x_1 and x_2 as the Predictor Variables to find the Linear Regression model. Interpret the estimators of the model.

Y	140	155	159	179	192	200	212	215
x_1	60	62	67	70	71	72	75	78
x_2	22	25	24	20	15	14	14	11

ANSWER

$y = -6.867 + 3.148\, x_1 - 1.656\, x_2$

PROBLEM 5 — CORRECT REGRESSION LINES

While calculating the Coefficient of Correlation between two variables x and y, the following results were obtained: N = 25, $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 650$, $\Sigma y^2 = 460$, $\Sigma xy = 508$. It was however discovered later on that two pairs of observation (x,y) were copied as (6,14) and (8,6) instead of (8,12) and (6,8) respectively.

Required: Determine the correct equations of the two Regression Lines.

ANSWER

$9x - 5y - 25 = 0$ and $4x - 5y = 0$

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Business Forecasting

Technique of a business's future developments based on trends, patterns and analysis of data — both current and historical.

Regression Analysis

A mathematical measure of the average relationship between two or more variables in terms of the original units of data.

Simple Linear Regression (SLR)

Regression involving two variables — one Explanatory (Independent) and one Response (Dependent).

Multiple Linear Regression (MLR)

Regression with one Response variable and more than one Explanatory variable.

Time Series

Series of observations recorded in accordance with the time of occurrence.

Secular Trend

Smooth, regular and long term movement exhibiting the tendency of growth or decline over a period of time.

Seasonal Variation

Type of periodic movement where the period is at the most one year.

Cyclical Fluctuation

Type of periodic movement where the period is more than a year.

Irregular Movement

Variations caused by factors of erratic nature; completely unpredictable.

Moving Average

Series of arithmetic means of successive observations calculated from overlapping groups of successive elements of data.

Exponential Smoothing

Method that weighs past data in an exponential manner so that the most recent data carries more weight.

Smoothing Constant (α)

Constant between 0 and 1 used in Exponential Smoothing; higher value discounts old observations more.

Delphi Technique

Qualitative forecasting method using a panel of experts asked sequential questions; final forecast from common opinion.

Barometric Technique

Forecasting based on economic or business indicators — Lead, Lag, or Concurrent.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 14

ABBREVIATION	FULL FORM
SLR	Simple Linear Regression
MLR	Multiple Linear Regression
OLS	Ordinary Least Squares
MSE	Mean Squared Error
MAPE	Mean Absolute Percentage Error
ARIMA	Auto-Regressive Integrated Moving Average
CPI	Consumer Price Index
GDP	Gross Domestic Product
MA	Moving Average
α	Smoothing Constant
y_t	Observation for time t
u_t	Forecast at time t
e_t	Forecast Error at time t
b_{yx}	Regression Coefficient y on x
b_{xy}	Regression Coefficient x on y

14.10 Input–Output Model

14.11 Qualitative Methods

14.12 MCQs, True/False, Fill in the Blanks

14.13 Essay Type Questions & Numerical Problems

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Introduction to Tools for Data Analytics

Module 15 · Data Analytics, Types & Tools

EDITION	PAPER	SECTION	MODULE WEIGHTAGE
December 2026	Paper 16 · Strategic Cost Management	Section B · Weightage 40%	Part of Section B

✓ Website Publication · ✓ PDF Conversion · ✓ A4 Printing · ✓ Mobile & Desktop Reading · ✓ Self Study · ✓ Grand Revision · ✓ Final & Last Day Revision · ✓ Personal Reference Book

15

Module Overview & Syllabus Mapping

WHAT THIS MODULE INCLUDES · LEARNING OBJECTIVES · EXAM RELEVANCE

MODULE 15 INCLUDES

Introduction to Data Analytics — Meaning of Data Analytics · Role of Data Analytics for a Business · **Steps in Data Analytics** — Data Requirements, Collection, Organisation, Cleaning (Data Wrangling), Analysis & Reporting · **Types of Data Analytics** — Descriptive, Diagnostic, Predictive, Prescriptive, Exploratory & Confirmatory · **Tools for Data Analytics** — Data Transformation Tools (ETL/ELT), Data Analysis Tools (Spreadsheets, BI Tools, Financial Data Analytics Tools, Programming Languages, Statistical Data Analysis Tools, Industry Specific Tools), Visualisation Tools.

Module Learning Objectives

After studying this module, the students will be able to:

1

Meaning of Data Analytics

Understand the meaning of Data Analytics.

2

Steps of Data Analytics

Understand the steps of Data Analytics.

3

Types of Data Analytics

Know about the various types of Data Analytics.

4

Tools for Data Analytics

Know about the different Tools used for Data Analytics.

Subject Learning Objectives (SLOB) Mapped

SLOB MAPPED AGAINST THE MODULE

To equip oneself with application-oriented knowledge of Tools for Data Analytics to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

4	5	6+	30
CORE TYPES OF ANALYTICS	STEPS IN DATA ANALYTICS	CATEGORIES OF TOOLS	MCQS
Descriptive, Diagnostic, Predictive, Prescriptive	Requirements to Reporting	ETL, BI, Financial, Programming, Statistical, Visualisation	With answers

SYLLABUS STRUCTURE NOTE

Module 15 is part of **Section B: Quantitative Techniques in Decision Making** which carries **40% weightage**. This module introduces the contemporary field of Data Analytics — its meaning, steps, types, and the various tools used. The module is highly relevant in today's digital business environment and requires understanding of both conceptual frameworks and practical tools like ETL, BI, R, Python, SAS, and Spreadsheets.

📖 Table of Contents — Module 15

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2. Steps in Data Analytics
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4. Tools for Data Analytics — Overview
5. Data Transformation Tools (ETL/ELT)
6. Spreadsheet Tools
7. Business Intelligence (BI) Tools
8. Financial Data Analytics Tools

9. Programming Languages (R, Python, SAS)
10. Statistical Data Analysis Tools
11. Industry Specific & Visualisation Tools
12. MCQs, True/False, Fill in the Blanks
13. Essay Type Questions

15.1

Introduction to Data Analytics

MEANING · ROLE OF DATA ANALYTICS FOR A BUSINESS

Tools of Data Analytics actually refer to different software used for the purpose of Data Analytics. Before discussing about these, it is felt that a few words about **Analytics** is going to be helpful to understand the necessity of various types of tools related to it.

Analytics is the scientific process of discovering and communicating the meaningful patterns which can be found in data. In other words it is the science that analyses raw data to fetch information contained in it. It is concerned with turning raw data into insight for making better decisions. Analytics relies on the application of **statistics, computer programming and operations research** in order to quantify and gain insight to the meanings of data. It is especially useful in areas which record a lot of data or information.

In today's information age, data is everywhere and ever increasing. As soon as internet is used by one (which may be from a desktop computer or laptop or tab or from a mobile phone etc.) digital foot print is left which gives rise to data of some kind.

Thus **Data Analytics** is a broad term that encompasses many diverse types of Data Analysis. Any type of data or information can be subjected to the techniques of data analytics to get insight that can be used to improve things. These techniques can reveal trends and metrics that would otherwise be lost in the mass of information. The output of data analytics can then be used to optimize processes to increase the overall efficiency of a system, including that of a business. Implementing it into business models means companies can help reduce costs by identifying more efficient ways of doing business and by storing large amounts of data. Actually this helps to make better business decisions by analysing customer trends and level of satisfaction which can lead to development of new and better products and services.

Role of Data Analytics for a Business

1

Gather Hidden Insights

Hidden insights from data are gathered and then analysed with respect to the requirements of the business.

2

Generate Reports

Reports are generated and passed on to the respective teams and individuals to deal with further actions for a high rise in business.

3

Perform Market Analysis

Market analysis is performed to understand the strengths and weaknesses of the competitors.

4

Improve Business Requirements

Analysis of data facilitates improvement in business by meeting the actual requirements of the customer.

KEY INSIGHT

Data Analytics is not just about analysing data — it is about **turning raw data into actionable insights** that can drive business decisions, reduce costs, and create competitive advantage.

⚡ Quick Revision — Section 15.1

- **Analytics:** Scientific process of discovering and communicating meaningful patterns in data.
- **Data Analytics:** Broad term encompassing many diverse types of data analysis to get insights for improving business.
- **Role:** Gather hidden insights, generate reports, perform market analysis, improve business requirements.
- **Relies on:** Statistics, Computer Programming, Operations Research.

15.2 Steps in Data Analytics

FIVE-STEP PROCESS FROM REQUIREMENTS TO REPORTING

Data Analytics follows a systematic step-by-step approach. Each step is crucial for ensuring that the final insights are accurate, meaningful and actionable. The five steps are as follows:

Step 1 — Determine Data Requirements

The first step is to determine the data requirements or how the data is grouped. Data may be separated by **age, demography, income, gender** etc. Data values may be numerical or be divided by category.

Step 2 — Collect the Data

The second step in Data Analytics is the process of collecting it. This can be done through a variety of sources — **computers, on-line sources, cameras, satellites or through personnel**.

Step 3 — Organise the Data

Once the data is collected, it must be organized so that it can be analysed. This may take place on a **Spread-sheet** or other form of software that can take statistical data.

Step 4 — Clean the Data (Data Wrangling)

The data is then cleaned up before analysis. This means it is scrubbed and checked to ensure there is **no duplication or error** and that it is not incomplete. This step helps correct any error before being analysed. This process is also known as **Data Wrangling**.

Step 5 — Analyse Data and Generate Report

The final step is to analyse the cleaned data using appropriate tools and techniques, and generate reports that provide actionable insights for business decision-making.



⚡ Quick Revision — Section 15.2

- **Five Steps:** Determine requirements → Collect → Organise → Clean (Data Wrangling) → Analyse & Report.
- **Data Wrangling:** Cleaning, scrubbing, checking for duplication, errors and incompleteness.
- **Sources of Data Collection:** Computers, on-line sources, cameras, satellites, personnel.

15.3 Types of Data Analytics

DESCRIPTIVE · DIAGNOSTIC · PREDICTIVE · PRESCRIPTIVE · EXPLORATORY & CONFIRMATORY

There are **four main types** of Data Analytics depending on the type of data available and the type of knowledge one is looking for from the available data. These are:

D

Descriptive Analytics

Answers **"What happened?"** — Basic type that examines, understands and describes something that has already happened.

Dg

Diagnostic Analytics

Answers **"Why did it happen?"** — Goes deeper to find the root cause of what happened.

P

Predictive Analytics

Answers **"What will happen?"** — Uses historical data, past trends and assumptions to predict future.

Pr

Prescriptive Analytics

Answers **"What actions should be taken?"** — Recommends specific actions to reach future targets.

Detailed Description of the Four Types

1. Descriptive Analytics

This is the **most basic type** of Analytics. It looks at data to examine, understand and describe something that has already happened. It provides quantitative information on "What happened" by analysing the data with the help of statistical techniques like calculation of Mean, Median, Mode etc. **Example:** Daily report of Cost of Production of an item being produced continuously in an automatic machine.

2. Diagnostic Analytics

It goes deeper than Descriptive Analytics by seeking to understand the **"Why"** behind what happened. While the production report may show an increase in the Cost of Production of the item, the reason behind the same is not mentioned. Diagnostic Analytics will look into the patterns and deviations to find the root cause. It involves **correlation of two different datasets**.

3. Predictive Analytics

This relies on **historical data, past trends and assumptions** to answer questions about "What will happen" in future. This is of immense importance as far as the situations of future prediction (like Sales forecasting) are concerned. Predictive Analytics correlates the results of Descriptive and Diagnostic Analytics with external datasets.

4. Prescriptive Analytics

This identifies the **specific actions** an individual or organisation should take to reach future targets or goals. In other words it comes up with recommendations of "What actions should be taken". Different possible scenarios are taken into account and the corresponding outcomes of each is considered of which the best possible one is recommended. **Huge computing power** is necessary. This type of Analytics skill is observed in **Artificial Intelligence (AI)** and **Machine Learning (ML)**.

Additional Types — Exploratory & Confirmatory Data Analytics

EXPLORATORY DATA ANALYTICS

- Explores the dataset to find answer as to which describes the data well.
- **No hypothesis** is stated before analysing the data.
- Starts by framing questions about what to do with the data.
- Data is manipulated to the best possible extent to find answers.
- Involves understanding data structure, identifying erroneous/missing data, establishing margin of error, and figuring out hypothesis.

VS

CONFIRMATORY DATA ANALYTICS

- Deals with some **stated hypothesis**.
- Same is tested for the dataset using statistical tools.
- Inference is drawn about its correctness.
- **Example:** "I predict that a person's likelihood of recommending our product is directly proportional to their reported satisfaction with the product."
- Availability of stated hypothesis gives a **clear cut direction** towards which data analysis is done.

COMPLEXITY & VALUE PROGRESSION

The four main types of analytics progress in **complexity and value** as follows:
Descriptive (What happened?) → **Diagnostic** (Why did it happen?) → **Predictive** (What is likely to happen?) → **Prescriptive** (What action is to be taken?)

⚡ Quick Revision — Section 15.3

- **Four Types:** Descriptive, Diagnostic, Predictive, Prescriptive.
- **Descriptive:** What happened? — Mean, Median, Mode.
- **Diagnostic:** Why did it happen? — Root cause, correlation.
- **Predictive:** What will happen? — Historical data, trends, forecasting.
- **Prescriptive:** What action to take? — AI, ML, scenario analysis.
- **Exploratory:** No prior hypothesis. **Confirmatory:** Tests stated hypothesis.

15.4 Tools for Data Analytics — Overview

CATEGORIES OF SOFTWARE USED IN DATA ANALYTICS

From the above discussions, it is clear that acquiring and storing data, processing the same and finally report generation are the steps in Data Analytics in a nutshell. For these, the tools used are nothing but different **Software**. Depending on the purpose for which a tool is used, one can classify them as follows:

1

Data Transformation Tools

Used for cleansing of available data — ETL (Extract, Transform, Load) or ELT tools.

2

Data Analysis Tools

Used for actual analysis — Spreadsheets, BI Tools, Financial Analytics Tools, Programming Languages, Statistical Tools, Industry Specific Tools.

3

Visualisation Tools

Used for presenting results — Charts, graphs, dashboards, reports.

⚡ Quick Revision — Section 15.4

- **Three Main Categories:** Data Transformation Tools, Data Analysis Tools, Visualisation Tools.

15.5 Data Transformation Tools (ETL/ELT)

EXTRACT, TRANSFORM, LOAD · DATA WAREHOUSE · DATA MART · DATA LAKE

These are basically used for the purpose of **cleansing of the available data**. A prime requirement of getting meaningful output from any software is to provide it with the data in specific field types and structures. Normally lot of issues are noticed in the datasets as far as formats, structures, spellings etc. are concerned. To take care of these, **ETL (Extract, Transform and Load)** tools are used. It can be mentioned here that according to some other line of thinking, the tools are **ELT (Extract, Load and Transform)** type and not ETL type which works better. As far as our purpose is concerned both work perfectly alright.

Such tools facilitate building a **pipeline** through which the data are brought together from multiple sources, cleansed and stored in a place called **data warehouse** for analysing. This is applicable when the data is **structured**. In case of **unstructured or semi structured data**, the storage is called **Lake** instead of Warehouse.

The ETL Process

1. Data Extraction

Data Stores filling start by data extraction from primary sources. Various data inconsistency is removed during this phase. Before their transformation into the data schema, extracted data can be loaded in a temporary dumping ground known as **Data Staging Area (DSA)**. The data component in DSA used to be most frequently a part of those solutions of data stores which has a source in heavy transaction systems. Use of DSA will reduce requirement of transaction system's utilization in the ETL Process. Use of DSA is possible also in the case when it is necessary to transfer data from a text file into the required database format.

2. Data Transformation

After the extraction follows data transformation which will convert data obtained from single data source into **unified data model**. This model makes it possible to create aggregation and clustering.

3. Data Loading

The final phase of ETL is data transmission from source data memories or temporary dumping ground to database tables of the data store. At the primary filling it can be a gigantic quantity of data. Because ETL works in **batch mode**, such amount of data will be brought again at the time of next regular updating. The time interval of updating could be a day, a week, a month or a year.

Data Warehouse vs Data Mart vs Data Lake

Data Warehouse

The transformed data coming from various service systems and external databases are saved in Data Warehouse which is an extensive **central business database**. Actually the data in the Data Warehouses are not optimized for quick transaction processing but meant for quick administration of analytical information obtained from big amount of data.

Data Mart

When the concept of **decentralization** is applied to Data Warehouse then we get Data Marts. Basically these are decentralized, based on certain themes. Data Marts are meant to provide analytical information to specific groups like **Marketing, Sales, Finance** etc.

Data Lake

In case of **unstructured or semi structured data**, the storage is called Lake instead of Warehouse. Data Lake stores raw data in its native format until it is needed.

COMMON ETL TOOLS

Many software for visualization and analysis have such ETL (or ELT) tools built in. Besides, there are many special tools which are capable of handling this cleansing process. **Xplenty, Stitch, ABS Glue, Skyvia** etc. are some of the commonly used ETL Tools for data cleansing.

⚡ Quick Revision — Section 15.5

- **ETL:** Extract, Transform, Load. **ELT:** Extract, Load, Transform.
- **Data Staging Area (DSA):** Temporary dumping ground for extracted data.
- **Data Warehouse:** Central business database for analytical information.
- **Data Mart:** Decentralized, theme-based subset of Data Warehouse.
- **Data Lake:** Storage for unstructured/semi-structured data.
- **ETL Tools:** Xplenty, Stitch, ABS Glue, Skyvia.

15.6 Spreadsheet Tools

MS-EXCEL · GOOGLE SHEETS · QUIP · PIVOT TABLES

Although there are several Spread-sheet database software solutions that offer more than a traditional spread-sheet, Spread-sheet software is still indispensible for many businesses. From generating reports and creating a budget to becoming a glorified to-do list, spread-sheets have the flexibility to meet just about every need of user. The right

Spread-sheet software can not only help the user to shift through thousands of datasets, but also keeps the user on track for setting and completing goals.

Of the several available Spread-sheet tools of today, **MS-Excel** remains the most common. The simplest and most obtainable analysis proceedings of business data is offered by MS-Excel. Certainly it is the cheapest option available because there is no user having MS-Excel not installed in the computer system. So there is no necessity to buy license for specialized software. Users can straightway go for creation of analytical reports and graphs. Data analyses created by MS-Excel are very dynamic and effective. They enable a lot of different views and graphical representations.

Ways of Feeding Data to MS-Excel

- **Manual data feeding from business reports**
- **Data import from business information system**
- **Direct connection to database of business information system**

Pivot Tables

Pivot Tables are one of the most powerful tools of MS-Excel. This enables **data summarization, filtration and ordering**. It is possible to create a lot of different views, reports and graphs from one data source. Created Pivot Table is easily editable — we can add or delete data, columns, rows or change summaries without the influence of Data Source.

Why MS-Excel Finds Utility as a Data Science Tool

1

Awareness

Irrespective of the demographic background, MS-Excel is a widely known tool to almost all people who work with data, and prepare reports. The tables, pivots, VBA-scripts and charts are having high awareness. MS-Excel is treated as a personal productivity tool used extensively in business organisations for preparing dashboards and reporting.

2

User-friendliness

Microsoft Excel is incredibly easy to use. Especially for beginner data scientists, the ease of use and the wide-scale availability of Microsoft Excel makes it an excellent introductory program for both students and professionals.

3

Data Visualisation

MS Excel is one of best editors for data visualization in 2D format. The tables are easily edited, formatted, colourized, and shared. MS Excel can handle as many as **1,048,576 rows and 16,384 columns** per sheet. Therefore, other than a set of complex real-life problems dealing with truly big data, MS-Excel can solve the purpose in many cases.

4

Data Analysis

Microsoft Excel has multiple functions programmed into the software which makes it simple to explore a dataset through sorting, filtering, and pivot tables. Tools like Scenario Manager also make it easy to create and compare hypothetical scenarios across datasets. Excel comes with various analytical tool packs that can be activated to unleash more advanced calculations.

5

Comparability with Other Software

This is one of the very powerful features of MS Excel as a tool. The other tools like SPSS, JAMOVI, R, Python etc. directly allow to import the excel sheet (in CSV form) as the input data for further analysis.

Other Spreadsheet Tools

Google Sheets

Google Sheets is a **free alternative** to Excel. It requires a Google account to begin work. It offers **real-time collaboration, commenting and version history** to accurately track all progress between contributors. Google Sheets can immediately sync with G Suite apps like Google Analytics and Google Data Studio. It can also populate a spread-sheet with data pulled from Google Forms. Available in web, Android and IOS systems. Google Sheets and all of its features are available up to **15 GB storage**.

Quip

Quip is a unique productivity tool that combines spread-sheets, documents and team chat all into one app. The tool features flexibility and customization. For instance you can toggle between document and spread-sheet layout as you work. Quip also boasts more than **400 built-in functions** and offers key board shortcuts for most menu items. Creating a Graph or Chart is as easy as selecting a data range and clicking a button.

⚡ Quick Revision — Section 15.6

- **MS-Excel:** Most common spreadsheet tool; 1,048,576 rows × 16,384 columns; Pivot Tables; Scenario Manager.
- **Google Sheets:** Free, real-time collaboration, 15 GB storage, syncs with G Suite.
- **Quip:** Combines spreadsheets, documents and team chat; 400+ built-in functions.
- **Pivot Tables:** Data summarization, filtration and ordering.

15.7 Business Intelligence (BI) Tools

MEANING · DATA STORAGE · DATA MINING · PREDICTIVE ANALYTICS · NLP · OLAP · DASHBOARDS

Business Intelligence (BI) Software is a set of business analytics solutions used by companies to retrieve, analyse and transform data into useful business insights usually within easy-to-read visualization — like charts, graphs and dashboards. Examples of the best BI Tools include data visualization, data warehouses, interactive dashboards and BI reporting tools. A BI Solution pulls internal data produced by a company, into an Analytics platform for deep insights as to how different parts of a business affect one another.

As Big Data has gained in prominence, the tendency for companies to collect, store and mine their business data has increased many times and so has the popularity of BI Software. Companies generate, track and compile business data at a scale never seen before. The ability to integrate cloud software directly with proprietary systems has further driven the need to combine multiple data sources and take advantage of data preparation tools. But all this data is nothing if we can't make sense of it and use it to improve business outcomes.

WHY BI MATTERS

To make informed choices, businesses need to make their decisions on evidence. The mountains of data that businesses and their customers produce contain evidence of purchasing patterns and market trends. By aggregating, standardising and analysing that data, businesses can better understand their customers, better forecast their revenue growth and better protect themselves against business pitfalls.

Data Storage for BI

Throughout an organisation data is available in a number of systems. Companies should ensure **standardised formatting** across data types from each of these systems so that most accurate analysis is possible. A large enterprise can have information about the customers in the **Customer Relationship Management (CRM)** application and have financial data in the **Enterprise Resource Planning (ERP)** application and several other key revenue datasets in various cloud software applications. These separate programs may label and categorise data differently and the company will need to standardise the data before analysis.

Some BI Platforms pull data for analysis directly from the source applications through a **native API connection** or **Webhook**. Other BI Tools require the use of a **cloud data storage system** to aggregate diverse data sets in a common

location. Small businesses, single departments or individual users may find that a native connection works well, but big organisations which generate large datasets will need a more comprehensive BI setup.

Key BI Capabilities

1

Data Mining

This is the activity of **"Data Discovery"** because here the patterns and inconsistencies of data unveiled through automated or semi-automated data analysis. Common correlations drawn from Data Mining include grouping specific sets of data, finding outliers in data and drawing connections and dependencies from disparate datasets.

2

Predictive Analytics

One of the most exciting aspects of BI, advanced analytics features like Predictive and Prescriptive analytics function as a subset of Data Mining. The tools use existing datasets and algorithmic models to help companies make better decisions. Involves **Predictive Modelling, Descriptive Modelling and Decision Analytics**.

3

Natural Language Processing (NLP)

Also known as Text Analytics software — combs large sets of **unstructured data** to find hidden patterns. Particularly interesting for businesses that work with social media. NLP Tools measure customer sentiment, provide actionable insight into lifetime customer value and learn customer trends.

4

Online Analytical Processing (OLAP)

Uses **multidimensional databases** to enable users to query data warehouses and create reports that view data from multiple perspectives. Gives BI Software the ability to combine data, drill down into single metrics and view data for combinations of single metrics that are unobtainable in a traditional spread-sheet setup.

5

Data Visualisation

One of the most popular trends of BI — allows companies to graphically present the results of data mining or other analytics. Presenting findings in a visual format like a graph, chart or on a map provides immediate insights into the most important metrics.

6

Dashboards

Every user of a system does not need full access to everything available in the dashboard. Most employees only need access to a dashboard of their most important metrics. It gives at a glance access to a range of predefined visualisations. Types include Sales Dashboards, Marketing Dashboards, Customer Success Dashboards, IT Dashboards.

7

Alerts and Notifications

Alerts notify users who do not spend most of their time in the tool to data changes that need immediate action. When companies set alerts for threshold of high and low performance, they can track when they need to mobilize a response or investigate an issue before it becomes an emergency.

8

Embedded Analytics

BI applications promise to clarify data analytics tools for the most non-technical of employees, which has driven the demand for Embedded BI. These features let companies build data visualisations within their cloud BI Software and dynamically serve those visualisations to internal and external customers within company apps.

Top BI Tool Software Vendors

SELF SERVICE	DATA VISUALISATION	DATA WAREHOUSING	BI PLATFORMS
SAP Crystal Reports	iDashboards	Sisense	Tableau
Chartio	Dundas	Oracle BI	InsightSquared
Alteryx	Segment	SAS	Domo
Jaspersoft	Geckoboard	Birst	SAP Lumira

Important BI Tools

Tableau

Specialises in making beautiful visualisations. It is a very advanced tool and one of the early entrants in the market. It is considered to be the industry leader among BI Tools. In addition to easy-to-use Reporting, a full API, Report Sharing and good customer support are the plus points. But it is quite costly and affordable for corporate business houses. Tableau has a public (free) version but with limited capabilities. Tableau allows to pull data from an array of cloud software and data preparation sources, including but not limited to Excel, SQL and Oracle.

InsightSquared

Built specifically to help with Sales, Marketing, Finance, Staffing and Support Services. It is perfect for detailed insights about specific aspects of a company's operations. In addition to data analysis and reports, InsightSquared also allows one to get an idea about Closed business deals, Sales success rates of certain salesperson and more. Integrating InsightSquared with Salesforce gives users access to predictions about which prospects are most likely to lead to a sale.

Domo

The first BI Tool to run completely on the **cloud**. Domo is a quickly growing system that allows the user to access insights about the business from anywhere. Domo offers over **500 data connectors** and its own app store allows the user to plug in apps specifically tailored for the industry to which the user's business belong.

OTHER BI SOFTWARES

Names of few more important BI Softwares are — **QlikView, Birst, GoodData, Hubble, Looker, MicroStrategy, BOARD, Microsoft PowerBI, Oracle BI, Logi Analytics, Sisense, Alteryx** etc.

⚡ Quick Revision — Section 15.7

- **BI Software:** Set of business analytics solutions to retrieve, analyse and transform data into useful business insights.
- **Key Capabilities:** Data Mining, Predictive Analytics, NLP, OLAP, Data Visualisation, Dashboards, Alerts, Embedded Analytics.
- **Data Mining:** Data Discovery — patterns, outliers, correlations.
- **OLAP:** Multidimensional databases for querying data warehouses.
- **Important BI Tools:** Tableau, InsightSquared, Domo, PowerBI, QlikView, Looker, etc.

15.8 Financial Data Analytics Tools

TYPES · FINANCIAL ANALYTICS TOOLS · JEDOX · ZOHO ANALYTICS · QUICKBOOKS

Harnessing the power of technology, Financial Analytics tools are transforming business operations and planning. Financial Analytics, a subset of Business Intelligence (BI) and Enterprise Performance Management (EPM), has become indispensible in the modern business environment. Using financial analytics tools companies can examine huge volumes of financial and other data to identify patterns and predict trends, boost revenues and slash costs.

Financial Analytics involves the use of software applications to examine all the data generated by an organisation. This includes Financial and Accounting information as well as other data like capturing customer interactions, monitoring supply chains, deep diving into historical trends and more.

As the complexity of business world has increased, companies need to deploy more sophisticated tools to understand what is going on and to plan. Financial Analytics improves on traditional financial analysis because of its ability to capture larger volumes of data as well as to drill down to a highly detailed level. Financial Analytics does not only reveal historical trends, but using Predictive Analytics it can peer into the future.

Critical Issues Addressed by Financial Analytics Tools

- What is the profitability of different products or services?
- What are the most valuable customer segments?
- What risks are the business currently facing and what are the potential future challenges?
- How are different sales channels performing?
- What future events could impact stock price performance?

Types of Financial Analytics

1. Predictive Sales Analysis

Enables an organisation to assess and adjust Sales Forecasts more rapidly than when using Excel based models. Predictive Sales Analysis can incorporate many more data points to predict future sales because of the ability to drill down deeper, for example, into the performance of different sales channels and different product lines.

2. Product Profitability Analytics

A company's overall profit is the money left at the end of an accounting period after subtracting total costs from the total revenue. Product Profitability Analysis is used to determine the profitability of each product or service. Product revenues are stripped off all associated costs — production costs, logistics, discounts, rebates, marketing and commissions — to reveal the profit derived from each product.

3. Value Driver Analytics

In business Value Driver are the elements that increase the worth of a product, service, asset or business. What value driver analytics provides is a way to quantify, measure and assess these drivers using transparent financial metrics.

4. Financial Ratio Analytics

Ratio Analysis is simply the analysis of different financial metrics in a business' financial statements. Financial ratios are used in six main areas — **Liquidity, Coverage, Solvency, Profitability, Efficiency and Market Prospects**. Analytics allows us not only to see different ratios more quickly and accurately than in the past but also let Data Analysts visualise this data in more understandable formats.

5. Scenario and Sensitivity Analysis

Business leaders make business decisions based on an analysis of risks and benefits. **Scenario Analysis** examines the results of all the assumptions — while **Sensitivity Analysis** examines the impact of changing a single variable. Analytics takes this to another level by allowing for the development of more complex models that can be updated more rapidly.

6. Growth Rate Analysis

Growth Rates are the most common financial metrics used in business. They are used by business executives to track performance. Businesses typically track revenue and profitability growth, sales and cost evolution and so on. Investors and financial analysts typically use growth rate analysis to assess potential investments including revenues and earnings, price to earnings ratio (P/E), price to earnings to growth ratio (PEG) and return on equity (ROE).

7. Cash Flow Analysis

Cash is most important resource in business. Being able to determine at any point in time how much cash a company has or will have in a specific future period is essential. Cash flow analysis reveals how much money is coming into the business from sales, services or investments or how much money is going out in the form of costs and interest payments. Cash flow is divided into — (1) Operating activities, (2) Investing activities, and (3) Financing activities.

8. Variance Analysis

Variance Analysis is an important aspect of budgeting activities. At the most basic level, we use it to compare actual performance to what was predicted in company forecasts or budgets. Variance Analysis can highlight various issues. Calculating and analysing variances can identify areas where management needs to put more focus — for example in optimising costs or increasing operational efficiency.

Financial Analytics Tools

Jedox

- SAAS based solution Business Intelligence within seamlessly integrated software platform.
- Deployable in the cloud as well as on-premise server or hybrid.
- Facilitates business planning, reporting, data consolidation and financial data analytics.
- Offers Excel add-in enabling data modelling, ad-hoc and predictive analytics, KPI reporting, budgeting, forecasting activities within the familiar Excel environment.

Zoho Analytics

- BI and Analytics platform.
- Deployable in the cloud as well as on-premises server.
- Can be used across various functional areas and by a wide range of users for their reporting and analytics needs.
- Allows users to easily create and share powerful reports within minutes.
- Current version has additional features like Zoho DataPrep, a Unified Data Management and Analytics Platform, Deep Augmented capabilities like Zia insights and Conversational BI, Data Story telling features.

QuickBooks

- One of the most feature rich accounting solutions for small businesses.
- User friendly and accessible platform, providing all the tools needed for day to day financial management.
- Ability to track and record business information at high level of detail — providing treasure trove of "Small" Data.
- Can generate reports but requires integration with other data analysis/reporting tools to provide more advanced Financial Analytics.

OTHER FINANCIAL ANALYTICS TOOLS

Few other tools for Financial Data Analytics are **Hyper Anna, NetSuite, FICO** etc.

⚡ Quick Revision — Section 15.8

- **Types of Financial Analytics:** Predictive Sales, Product Profitability, Value Driver, Financial Ratio, Scenario & Sensitivity, Growth Rate, Cash Flow, Variance.
- **Financial Analytics Tools:** Jedox, Zoho Analytics, QuickBooks, Hyper Anna, NetSuite, FICO.
- **Financial Ratios:** Liquidity, Coverage, Solvency, Profitability, Efficiency, Market Prospects.

15.9 Programming Languages for Data Analytics

R PROGRAMMING · PYTHON · SAS

Data scientists use several Programming Languages in their work. Programming allows the creation of specific analytical solutions which may not be available in other ready to use software packages. Some of the most popular

languages for this purpose are — **R Programming, Python, SAS** etc.

R Programming

R is an **open source programming language** that is widely used as a statistical software and data analysis tool. It generally comes with command line interface. R is available across widely used platforms like Windows, Linux, Mac-OS. It is the latest cutting edge tool. Due to its expressive syntax and easy-to-use interface, it has grown in popularity in recent years. In fact this is one of the most popular languages amongst statisticians, data analysts, researchers and marketers to retrieve, clean, analyse, visualize and present data.

It was designed by **Ross Ihaka and Robert Gentleman** at the University of Auckland, New Zealand and currently developed by the R Development Core team. R programming language is an implementation of the S programming language. The project was conceived in 1992, with an initial version released in 1995 and a stable beta version in 2000.

Reasons for Using R

1

Vast Community

The R Programming language has a vast community of users and it's growing day by day. It is being used by the biggest tech giants which is a sign of the potential of the language.

2

Free Installation

It's an Open Source language. It can be installed for free because it is licensed under the GNU General Public License. There are many R Packages available under the same license.

3

Hottest Trend

R is currently the most wanted programming language in the Data Science jobs. Hence it is considered to have the hottest trend in the job market.

4

Platform Independence

R Programming is Platform Independent language. It is compatible with all the popularly used Operating Systems like Windows, Linux and Mac-OS.

5

Integrate with Other Languages

It allows the user to integrate it with other languages like C, C++ etc. As a result, easy interaction with many data sources and statistical packages is possible.

6

Latest Cutting Edge Technology

It is used as a leading tool for Machine Learning, Statistics and Data Analysis. R can easily create objects, functions and packages based on user's requirements.

Applications of R Programming in the Real World

Data Science

With the advent of "Internet of things" (IoT) devices creating terabytes of data, Data Science is a field that has no other way but to go up. Programming languages like R give a data scientist superpowers that allow them to collect data in real time, perform statistical and predictive analysis, create visualisation and communicate actionable results to the stakeholders.

Statistical Computing

R is the most popular programming language among statisticians. In fact it was initially built by statisticians for carry work related to statistical data. It has a rich package repository with over **9000 packages** having every statistical function one can think of. R's expressive syntax allows researchers to quickly import, clean and analyse data from various data sources.

Machine Learning

R has found a lot of use in predictive analytics and machine learning. It has various packages for common ML tasks like linear and non linear regression, decision trees, linear and non-linear classification and many more. Everyone from machine learning enthusiasts to researchers use R to implement machine learning algorithms in fields like finance, genetics research, retail, marketing and health care.

Features of R Programming Language

STATISTICAL FEATURES

- **Basic Statistics:** Mean, Median and Mode — Measures of Central Tendency.
- **Static Graphics:** Rich facilities for creating and developing interesting Static Graphics — graphic maps, mosaic plots, bi-plots.
- **Probability Distributions:** Binomial, Normal, Student's t, Chi Square distributions.
- **Data Analysis:** Large, coherent and integrated collection of tools for data analysis.

VS

PROGRAMMING FEATURES

- **R Packages:** Wide availability of libraries — CRAN (Comprehensive R Archive Network) holds more than 10,000 packages.
- **Distributed Computing:** Packages like ddR and multidplyr are used for distributed programming in R.

Example: R Program to Generate Random Numbers and Bar Chart

Problem

Write R program to generate 10,000 numbers in a Random distribution, organise them based on the frequency and create a Bar Chart.

Solution

R CODE

```
n <- floor(rnorm(10000, 500, 100))
```

```
t <- table(n)
```

```
barplot(t)
```

Explanation: The first line generates 10,000 random numbers with Mean = 500 and Standard Deviation = 100. The "floor" function removes decimal points. The "table" function counts frequency of each number. The "barplot" function creates the chart. The output follows a Normal Distribution (bell shape).

Advantages & Disadvantages of R

ADVANTAGES

- Most comprehensive statistical analysis package.
- Open source — can be used anywhere at any time.
- Cross platform language — works with any popular OS.
- Everyone is welcome to provide new packages, bug fixes and code enhancements.

VS

DISADVANTAGES

- Standard of some packages is less than perfect.
- May consume all available memory.
- Nobody to complain if something doesn't work.
- Much slower than other programming languages such as Python & MATLAB.

Python

Python is a widely used general purpose, high level programming language. It was created by **Guido van Rossum** in 1991 and further developed by Python Software Foundation. It was designed with an emphasis on **code readability**.

Its syntax allows programmers to express their concepts in fewer lines of code. It allows the users to work quickly and integrate systems more efficiently.

There are two major Python versions — **Python 2 and Python 3**. Both are quite different.

Reasons for the Increasing Popularity of Python

- **Emphasis on code readability, shorter codes and ease of writing**
- **Programmers can express logical concepts in fewer lines of code**
- **Supports multiple programming paradigms**
- **Inbuilt functions for almost all frequently used concepts**
- **Philosophy is "Simplicity is the best"**

Key Features of Python

1

Interpreted

No separate compilation and execution steps like C and C++. Directly run the program from the source code. Internally, Python converts the source code into an intermediate form called bytecodes which is then translated into native language.

2

Platform Independent

Python programs can be developed and executed on multiple operating system platforms — Linux, Windows, Macintosh, Solaris and many more.

3

Free and Open Source

Redistributable.

4

High Level Language

No need to take care about low level details such as managing the memory used by the program.

5

Simple

Closer to English language. Hence easy to learn. More emphasis on the solution to the problems rather than the syntax.

6

Embeddable

Python can be used within C and C++ program to give scripting capabilities for the program's user.

7

Robust

Exceptional handling features. Inbuilt memory management techniques.

8

Rich Library Support

The Python standard library is very vast — known as the "Batteries included" philosophy. Helps in regular expressions, documentation generation, unit testing, threading, databases, web browsers, CGI, email, XML, HTML, WAV files, cryptography, GUI and many more.

Current Applications of Python

- Linux distributions installers (Ubuntu's UBIQUITY)
- Information Security Industry
- Raspberry Pi — principal user programming language
- Game Development
- GNU Debugger — "Pretty Printer" for C++ Containers
- Artificial Intelligence
- Natural Language Processing

Advantages & Disadvantages of Python

ADVANTAGES

- Presence of third party modules.
- Extensive support libraries (NumPy, Pandas).
- Open source and community development.
- Versatile, easy to read, learn and write.
- User friendly data structures.
- High level language.
- Dynamically typed language.
- Object oriented language.
- Portable and interactive.
- Ideal for prototypes.
- Highly efficient.
- IoT opportunities.
- Interpreted language.
- Portable across operating systems.

VS

DISADVANTAGES

- Slow speed of execution compared to C and C++.
- Absence from mobile computing and browsers.
- For C & C++ Programmers switching to Python can be irritating as the language requires proper indentation of code.
- Certain variable names commonly used, like SUM are functions in Python.

SAS (Statistical Analysis System)

SAS is a powerful software that has been the first choice of the private enterprises for their analytics needs for a long time. Its GUI and comprehensive documentation coupled with reliable technical support made it a very good tool for companies.

It is a **command driven software** developed by the SAS institute. It is mainly used as a tool for all kinds of statistical modeling, data management and predictive analysis. It can be used to import data from a variety of data sources, analyze and perform analysis on the data and export the outcome in HTML, PDF, Excel, RTF and other formats. SAS also provides a **graphical point-and-click user interface** for non-technical users. SAS programs have **DATA steps**, which retrieve and manipulate data, and **PROC steps**, which analyze the data. Each step consists of a series of statements. We can also write **SAS macros** which are pieces of code or variables that are coded once and referenced to perform repetitive tasks.

Types of SAS Software

- Windows or PC SAS
- SAS EG (Enterprise Guide)
- SAS EM (Enterprise Miner — for Predictive Analysis)
- SAS Stats

Key Features of SAS

1

Data Access

Easily accessible raw data files & data in from of an external database. Read and write almost any data format!

2

Data Management

Manage data using tools for data entry, editing retrieval, formatting & conversion.

3

Data Analysis

Analyze data using descriptive, statistics, multivariate techniques, forecasting, modeling, linear programming.

4

Advanced Analytics

Helps you to make changes and improvements in business practices.

5

Report Formation

Report formation with perfect graphs.

6

Operations Research

Operations research and project Management.

7

Data Handling

Data updating and modification. Powerful data handling language. Excellent data cleansing functions.

8

Multiple Host Systems

Interact with multiple host systems.

⚡ Quick Revision — Section 15.9

- **R:** Open source, statistical software, CRAN 10,000+ packages, platform independent, used for ML, statistics, data science.
- **Python:** General purpose, high level, interpreted, platform independent, rich library support, used in AI, NLP, IoT.
- **SAS:** Command driven, DATA steps and PROC steps, SAS macros, used for statistical modeling and predictive analysis.

15.10 Statistical Data Analysis Tools

SAS · SPSS · EVIEWS

Statistical data analysis tools are specialised software packages designed for advanced statistical analysis, econometric modelling, and quantitative research. Some of the key tools are:

SAS (Statistical Analysis System)

As discussed in the previous section, SAS is a powerful software for statistical modeling, data management and predictive analysis. It is command driven with DATA and PROC steps. It also provides a graphical point-and-click user interface for non-technical users. SAS is extremely popular in commercial analytics.

SPSS (Software Package for Statistical Analysis)

SPSS is another popular Statistical tool. It is used most commonly in the **Social Sciences** and is considered as easiest to learn among enterprise statistical tools. It is loved by the non-statisticians because it is similar to Excel. As almost all the users of computer are familiar with Excel, they find SPSS very easy to use. But it has the same downside as SAS — it is **expensive**.

EViews

Statistical and econometric data analysis is done by this tool. Econometrics deals with three types of data: **cross-sectional data, time series data, and panel (longitudinal) data**. EViews allows you to work with all three types of data. EViews is most commonly used for time series analysis in academics, business, and government. It allows you to save results within a program and to "retrieve" these results for further calculations later.

TYPES OF DATA IN ECONOMETRICS

- **Time Series Data:** Observe the behavior of a single entity over multiple time periods (e.g., financial data, industrial production, inflation, GDP).
- **Cross-Sectional Data:** Analyze data from multiple entities at a single point in time.
- **Panel (Longitudinal) Data:** Combination of time series and cross-sectional data — multiple entities observed at multiple time periods.

⚡ Quick Revision — Section 15.10

- **SAS:** Command driven, DATA/PROC steps, SAS macros, commercial analytics.
- **SPSS:** Social Sciences, easy to learn, similar to Excel, expensive.
- **EViews:** Econometric analysis — time series, cross-sectional, panel data.

15.11 Industry Specific & Visualisation Tools

ERP TOOLS · VISUALISATION TOOLS · DASHBOARDS

Industry Specific Tools

There are several Industry specific tools available or built into industry specific ERPs. Such as **SAP, Oracle, Microsoft Dynamics, Sage, Tally** etc.

Visualisation Tools

Visualisation of the results of Data Analytics is the most important part from the point of view of the users. Visuals make communication about the insights of the data much easy. It makes a lot of sense to see the visual trends, charts, graphs etc. than to go through huge chunks of data tables. Thus in the categorisation of Data Analytics tools there is a special place for Visualisation Tools. These can be taken as **Reporting Tools** also. In reality there is no such tool in data analytics which is meant only to prepare reports with visuals for the analysis of data done by some other tool. Actually all the BI tools used for Data Analytics have this capability. But of course there is some difference among these tools from the point of view of capability of visual preparation.

Common Features of Visualisation Tools

1

Ease of Using Multiple Databases

Visualisation tools should have the option for picking the data from multiple sources like Excel, CSV, SQL Server, MySQL etc. This facilitates picking up of the relevant data without much of transformation.

2

Capability to Build Relationships

Capability to build relationship between various datasets is one of the most important criteria for any Visualisation tool. This makes the data interactive which results in ease of preparing reports using a variety of datasets.

3

Capability to Attract the Users

Attractive depiction of the information using various colours in different types of charts (Pie chart, Bar chart etc.) is one of the prime requirements of visuals. Due to this, the user can pay proper attention to the appropriate information.

4

Ease of Interaction and Dynamism

Dynamism in Visualisation tools allow the users to drill information to different dimensions. The charts have become dynamic in Visualisation Tools and the users can interact with the charts as and when he or she hovers over those. Drag and drop of the items of different charts to the respective dashboards can be possible dynamically.

5

Graphical User Interface

The Visualisation Tools should have the option to view the data in different forms like a Geographical Map, Area Chart, Histogram etc. This facilitates the users to get more insights and the data conveys more important messages.

POPULAR VISUALISATION TOOLS

Some of the popular tools with great capability of generating attractive visuals are **Tableau, Power BI, Zoho Reports** etc. All these are primarily BI Tools with very good capability to generate colourful reports.

⚡ Quick Revision — Section 15.11

- **Industry Specific Tools:** SAP, Oracle, Microsoft Dynamics, Sage, Tally.
- **Visualisation Tools Features:** Multiple database support, relationship building, attractive visuals, dynamism, GUI.
- **Popular Visualisation Tools:** Tableau, Power BI, Zoho Reports.

15.12 Multiple Choice Questions, True/False & Fill in the Blanks

WITH ANSWERS AND REASONING

Multiple Choice Questions

1. Tableau is a —

(a) Business Intelligence Tool (b) Visualisation Tool (c) Both (a) and (b)

(d) None of the above

Answer: C

2. Which of the following statement is correct?

(a) Functioning of ETL Tool is same as that of ELT Tool. (b) For Data Analytics the purpose of ETL Tool is same as that of ELT Tool. (c) Both (a) and (b)

(d) None of the above

Answer: B

3. Which of the following statement is incorrect?

(a) Microsoft Excel is most popular among all the available spreadsheets. (b) Zoho Analytics is a tool used for Financial Data analysis. (c) Visualisation Tools are the Reporting Tools.

(d) None of the above.

Answer: D

4. Prescriptive Analytics is very important because —

(a) It tells about the action to be taken. (b) It tells about what is likely to happen. (c) It tells about how something has happened.

(d) It tells about what has happened.

Answer: A

5. Which of the following has no relation to Business Intelligence?

(a) A set of business analytics solutions to retrieve, analyse and transform data into useful business insights

(b) Visualisation Tools are primarily BI Tools.

(c) ABS Glue is a tool used for the purpose of Business Intelligence.

(d) Embedded Analytics is an important part of any Business Intelligence tool.

Answer: C

6. Which of the following is related to Financial Data Analytics?

(a) Value driver analytics

(b) Financial ratio analytics

(c) Predictive sales analysis

(d) All the above

Answer: D

7. Analysis of a dataset has revealed the fact that profit of a business has reduced for the financial year 2023-24. What category of data analytics it comes under?

(a) Descriptive Analytics

(b) Predictive Analytics

(c) Diagnostic Analytics

(d) Prescriptive Analytics

Answer: A

8. #Script Ends — is related to which type of programming language?

(a) R Programming

(b) SAS

(c) Python

(d) SPSS

Answer: C

9. Which one of the following is a Key feature of SAS language?

(a) Capability of handling data analysis related to Operations Research and Project Management.

(b) Capability of report formation with perfect graphs.

(c) Capability to interact with multiple host systems

(d) All the above

Answer: D

10. Which one of the following is not a spreadsheet?

(a) Google Sheets (b) MS Excel (c) EViews

(d) Quip

Answer: C

11. Descriptive Analytics answers which of the following questions?

(a) What happened? (b) Why did it happen? (c) What will happen?

(d) What action should be taken?

Answer: A

12. Which type of analytics uses Artificial Intelligence (AI) and Machine Learning (ML)?

(a) Descriptive Analytics (b) Diagnostic Analytics (c) Predictive Analytics

(d) Prescriptive Analytics

Answer: D

13. In ETL, what does T stand for?

(a) Transfer (b) Transform (c) Transpose

(d) Track

Answer: B

14. Which of the following is a temporary dumping ground for extracted data?

(a) Data Warehouse (b) Data Mart (c) Data Staging Area (DSA)

(d) Data Lake

Answer: C

15. CRAN is related to —

(a) Python (b) SAS (c) R Programming

(d) SPSS

Answer: C

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
C	B	D	A	C	D	A	C	D	C	A	D	B	C	C

State True or False

SL	STATEMENT	ANSWER
1	Data Analytics encompasses many diverse types of data analysis.	**True**
2	Purpose of Predictive Analytics and Prescriptive Analytics are same.	**False**
3	There is no hypothesis stated in Confirmatory data analytics.	**False**
4	PowerBI is a business intelligence tool but more popular as data visualisation tool.	**True**
5	The process of Data Mining is also known as the process of data discovery.	**True**
6	Ease of using multiple databases is not a necessity for Visualisation Tools.	**False**
7	Use of cloud data storage system to aggregate diverse datasets in a common location is common for Business Intelligence software.	**True**
8	Rating of different BI Software products are based on their capability of Data Analytics, Natural language processing, Real time reporting and Embedded analytics.	**True**
9	In data analytics the term Mart is related to a place where purchase of data is done.	**False**
10	R Programming is incapable of handling any statistical data analysis and that's why SAS is developed.	**False**

Fill in the Blanks

SL	STATEMENT	ANSWER
1	EViews is a statistical and _____ tool.	**Econometric**
2	Python is closer to _____ language.	**English**
3	QuickBooks is a _____ Analytics tool.	**Financial**
4	Tools handling extract, transform and load functions of data are known as _____ tool.	**ETL**
5	CRAN is related to _____.	**R Programming**
6	Descriptive Analytics answers the question "_____".	**What happened?**
7	Diagnostic Analytics answers the question "_____".	**Why did it happen?**
8	Predictive Analytics answers the question "_____".	**What will happen?**
9	Prescriptive Analytics answers the question "_____".	**What action should be taken?**
10	The process of cleaning data is also known as _____.	**Data Wrangling**

15.13 Essay Type Questions

FOR COMPREHENSIVE EXAMINATION PREPARATION

Short Essay Type Questions

1

Which type of Data Analytics is necessary to know why there is a decline in certain business?

2

"Data Mining is nothing but Data Discovery" — discuss.

3

What is meant by Data Wrangling?

4

Name the different types of Data Analytics.

5

What is the role of Data Analytics for a business?

Essay Type Questions

1

What are the different types of Data Analysis Tools? Discuss each briefly.

2

Write short notes on — Exploratory and Confirmatory Data Analytics, Embedded Analytics facility of BI Tools.

3

Discuss the steps involved in Data Analytics.

4

Explain the various types of Data Analytics with suitable examples.

5

Write a detailed note on Business Intelligence Tools.

6

Discuss the features, advantages and disadvantages of R Programming and Python.

7

What are the different types of Financial Analytics? Explain any four.

8

Explain the ETL process in detail.

Terms to Master

KEY DEFINITIONS FOR LAST-MINUTE REVISION

Analytics

Scientific process of discovering and communicating the meaningful patterns which can be found in data.

Data Analytics

Broad term encompassing many diverse types of data analysis to get insight that can be used to improve things.

Descriptive Analytics

Most basic type — examines, understands and describes something that has already happened. Answers "What happened?"

Diagnostic Analytics

Goes deeper than Descriptive Analytics — seeks to understand "Why" behind what happened.

Predictive Analytics

Relies on historical data, past trends and assumptions to answer "What will happen?" in future.

Prescriptive Analytics

Identifies specific actions to reach future targets — recommends "What actions should be taken".

Data Wrangling

Process of cleaning, scrubbing and checking data to ensure no duplication or error and that it is not incomplete.

ETL

Extract, Transform, Load — tools used for cleansing of available data.

Data Warehouse

Extensive central business database for analytical information.

Data Mart

Decentralized, theme-based subset of Data Warehouse for specific groups like Marketing, Sales, Finance.

Data Lake

Storage for unstructured or semi-structured data.

Business Intelligence (BI)

Set of business analytics solutions to retrieve, analyse and transform data into useful business insights.

Data Mining

Activity of "Data Discovery" — patterns and inconsistencies of data unveiled through automated or semi-automated analysis.

OLAP

Online Analytical Processing — uses multidimensional databases to query data warehouses and create reports.

NLP

Natural Language Processing — combs large sets of unstructured data to find hidden patterns.

R Programming

Open source programming language used as statistical software and data analysis tool.

Python

Widely used general purpose, high level programming language created by Guido van Rossum in 1991.

SAS

Command driven software for statistical modeling, data management and predictive analysis.

Pivot Table

Powerful MS-Excel tool enabling data summarization, filtration and ordering.

Visualisation Tools

Tools for graphically presenting results of data mining or analytics — charts, graphs, dashboards.

Ab

Abbreviations

QUICK REFERENCE FOR ALL ACRONYMS USED IN MODULE 15

ABBREVIATION	FULL FORM
ETL	Extract, Transform, Load
ELT	Extract, Load, Transform
DSA	Data Staging Area
BI	Business Intelligence
OLAP	Online Analytical Processing
NLP	Natural Language Processing
AI	Artificial Intelligence
ML	Machine Learning
IoT	Internet of Things
CRM	Customer Relationship Management
ERP	Enterprise Resource Planning
EPM	Enterprise Performance Management
CRAN	Comprehensive R Archive Network
SPSS	Software Package for Statistical Analysis
SAS	Statistical Analysis System
KPI	Key Performance Indicators
SAAS	Software As A Service
P/E	Price to Earnings Ratio
PEG	Price to Earnings to Growth Ratio
ROE	Return on Equity

15.3 Types of Data Analytics

15.4 Tools for Data Analytics — Overview

15.5 Data Transformation Tools (ETL/ELT)

15.6 Spreadsheet Tools

15.7 Business Intelligence (BI) Tools

15.8 Financial Data Analytics Tools

15.9 Programming Languages (R, Python, SAS)

15.10 Statistical Data Analysis Tools

15.11 Industry Specific & Visualisation Tools

15.12 MCQs, True/False, Fill in the Blanks

15.13 Essay Type Questions

ETL Process Flow

BI Tool Capabilities

Financial Analytics Types

R / Python / SAS Comparison

Formula Boxes

MCQs with Reasoning

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